Computation of Exact Transfer Maps from Magnet Simulation Data

Marco Venturini

LBNL

(A sketch of the method and progress was given by J. G. Wang)

Outline by J.G. Wang

- Slide 1: Marco and Alex published a paper in which they established the frame work for map calculation from magnetic data.
- Slide 2: J.G. and Marco in collaboration to study the SNS magnets for their fringe field and interferences.
- Slide 3: Procedures of transfer map computation – a different approach with the Lie algebra.
- Slide 4: Magnetic vector potential is needed, and the way of its calculation is from the 3D multipole expansion.
- Slide 5: An example of the magnetic vector potential calculated for the SNS ring quadrupole 21Q40.

- Work continues.

Slide 1: An early paper by M. Venturini & A. Dragt

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

Accurate computation of transfer maps from magnetic field data

Marco Venturini*, Alex J. Dragt

Physics Department, University of Maryland, College Park, MD 20742, USA

Abstract

Consider an arbitrary beamline magnet. Suppose one component (for example, the radial component) of the magnetic field is known on the surface of some imaginary cylinder coaxial to and contained within the magnet aperture. This information can be obtained either by direct measurement or by computation with the aid of some 3D electromagnetic code. Alternatively, suppose that the field harmonics have been measured by using a spinning coil. We describe how this information can be used to compute the exact transfer map for the beamline element. This transfer map takes into account all effects of real beamline elements including fringe-field, pseudo-multipole, and real multipole error effects. The method we describe automatically takes into account the smoothing properties of the Laplace–Green function. Consequently, it is robust against both measurement and electromagnetic code errors. As an illustration we apply the method to the field analysis of high-gradient interaction region quadrupoles in the Large Hadron Collider (LHC). © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Transfer maps; Magnetic field data; Laplace–Green functions; High-gradient interaction region

1. Introduction

The motion of charged particles through any beam-line element is described by the transfer map $\mathcal{M}$ for that element. Through aberrations of order $(n-1)$ such a map has the Lie representation [1,2]

$$\mathcal{M} = \mathcal{R}_2 \exp(:f_3:) \exp(:f_4:) \cdots \exp(:f_n:) \qquad (1)$$

where $\mathcal{R}_2$ describes the linear part of the map. The linear map $\mathcal{R}_2$ and the Lie generators f_ℓ are determined by solving the equation of motion $\dot{\mathcal{M}} = \mathcal{M}:-H:$ where $H = H_2 + H_3 + H_4 + \cdots$ is the Hamiltonian expressed in terms of deviation variables and expanded in a homogeneous polynomial series. The deviation variable Hamiltonian H is determined in turn by the Hamiltonian K. In Cartesian coordinates with z taken as the independent variable, and in the absence of electric fields, K is given by the relation

$$K = -[p_t^2/c^2 - m^2c^2 - (p_x - qA_x)^2 - (p_y - qA_y)^2]^{1/2} - qA_z.$$

Here A is the magnetic vector potential. We conclude that what we need is a Taylor expansion for

*Corresponding author. Tel.: +1-301-405-6188; fax: +1-301-314-9525.
E-mail address: venturin@physics.umd.edu (M. Venturini)

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Slide 2: A paper for PAC2005

3D-Simulation Studies of SNS Ring Doublet Magnets*

J. G. Wang#, SNS/ORNL, Oak Ridge, TN 37831-6471, U.S.A.
N. Tsoupas, BNL, Upton, NY 11973, U.S.A.
M. Venturini, LBNL, Berkeley, CA 94720, U.S.A.

Abstract

The accumulator ring of the Spallation Neutron Source (SNS) at ORNL employs in its straight sections closely packed quadrupole doublet magnets with large aperture of R=15.1 cm and relatively short iron-to-iron distance of 51.4 cm. These quads have much extended fringe field, and magnetic interferences among them in the doublet assemblies is not avoidable. Though each magnet in the assemblies has been individually mapped to high accuracy of lower 10-4 level, the experimental data including the magnet interference effect in the assemblies will not be available. We have performed 3D computing simulations on a quadrupole doublet model in order to assess the degree of the interference and to obtain relevant data for the SNS ring commissioning and operation.

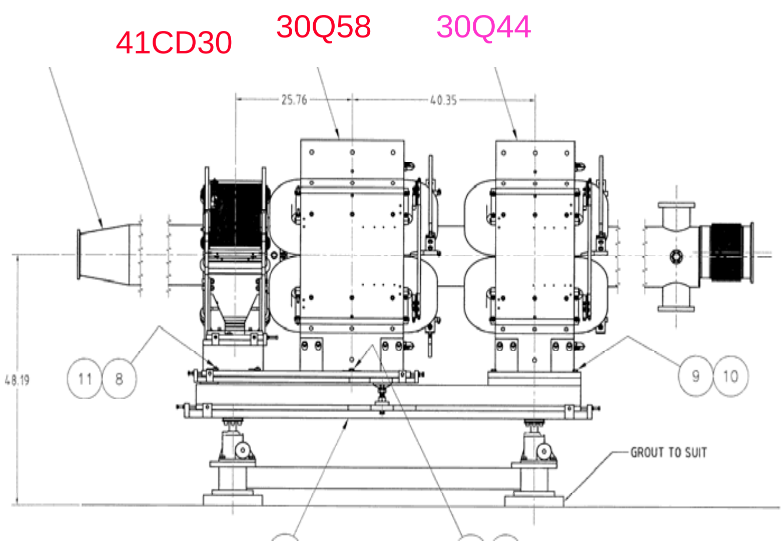

Figure 1: SNS ring doublet assembly [4].

Slide 3: Transfer Map Computation

- **Hamiltonian for charged particles with z as independent variable in magnetic field:**

$$H_z(x, p_x, y, p_y, t, p_t, z) = -\sqrt{\left(\frac{p_t}{c}\right)^2 - (mc)^2 - (p_x - qA_x)^2 - (p_y - qA_y)^2} - qA_z$$

- **Canonical equations in terms of Poisson brackets:**

$$\dot{f} = -[H_z, f] = \sum_{i=1}^{3}\left(\frac{\partial H_z}{\partial q_i}\frac{\partial f}{\partial p_i} - \frac{\partial H_z}{\partial p_i}\frac{\partial f}{\partial q_i}\right) \equiv -:H_z:f$$

- **Dynamic variables $f(q_i,p_i,z)$ can be found by a "simple" integration:**

(for autonomous Hamiltonian)

$$f\left(Z^1, z^1\right) = \mathrm{Exp}\left[\left(z^1 - z^0\right): -\mathrm{H}_z :\right] f\left(Z^0, z^0\right) \equiv M^{z^0 \to z^1} f\left(Z^0, z^0\right)$$

$$where \quad Z = (q_1, p_1, q_2, p_2, q_3, p_3)$$

Slide 4: Magnetic Vector Potential (1)

- **Free choices of the gauge conditions**
- **Chose** $A_\theta(r,\theta,z)=0$
- **Then** $A_r(r,\theta,z)=\sum_{m=1}^{\infty}\frac{r}{m}\left[B_{z,s}^{(m)}Cos(m\theta)-B_{z,c}^{(m)}Sin(m\theta)\right]$

$$A_z(r,\theta,z)=\sum_{m=1}^{\infty}\frac{r}{m}\left[-B_{r,s}^{(m)}Cos(m\theta)+B_{r,c}^{(m)}Sin(m\theta)\right]$$

Where B is obtained from 3D multipole expansion

- **In Cartesian coordinates for a quad**

$$A_x \approx \left[\frac{1}{2}C_{2,s}^{[1]}-\frac{1}{24}C_{2,s}^{[3]}\left(x^2+y^2\right)\right]\left(x^2-y^2\right)x+\frac{1}{6}C_{6,s}^{[1]}\left(x^6-15x^4y^2+15x^2y^4-y^6\right)x$$

$$A_y \approx \left[\frac{1}{2}C_{2,s}^{[1]}-\frac{1}{24}C_{2,s}^{[3]}\left(x^2+y^2\right)\right]\left(x^2-y^2\right)y+\frac{1}{6}C_{6,s}^{[1]}\left(x^6-15x^4y^2+15x^2y^4-y^6\right)y$$

$$A_z \approx -\left[C_{2,s}-\frac{1}{6}C_{2,s}^{[2]}\left(x^2+y^2\right)+\frac{1}{128}C_{2,s}^{[4]}\left(x^2+y^2\right)^2\right]\left(x^2-y^2\right)-C_{6,s}\left(x^6-15x^4y^2+15x^2y^4-y^6\right)$$

Slide 5: Magnetic Vector Potential (2)

21Q40:

A regular quad in SNS ring arc section; Magnetic vector potential A_z & A_x @ $y=z=0$ vs. x (only linear term is plotted).