Inelastic X-Ray Scattering & Lattice Dynamics

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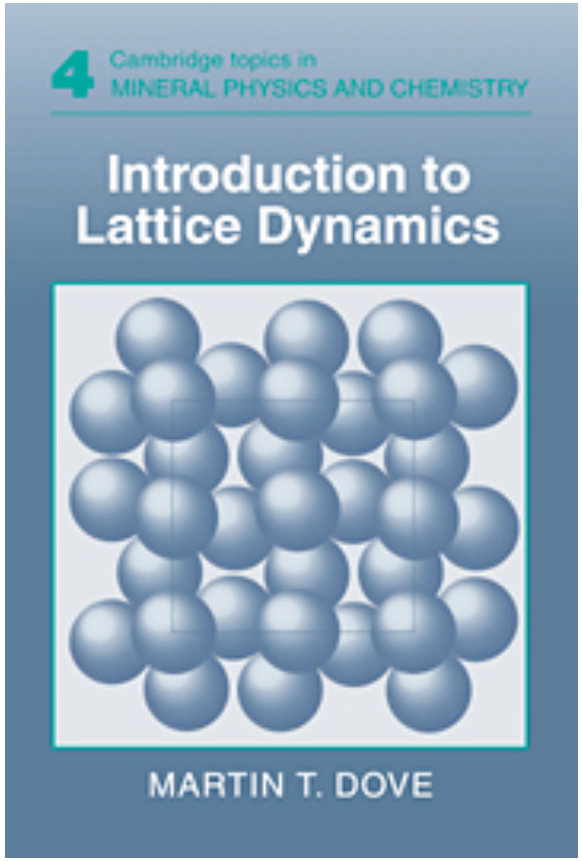
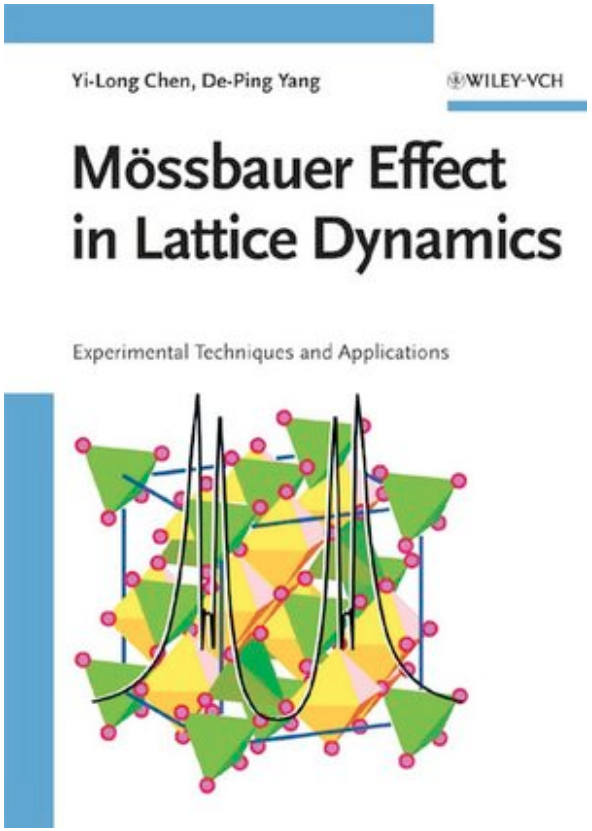
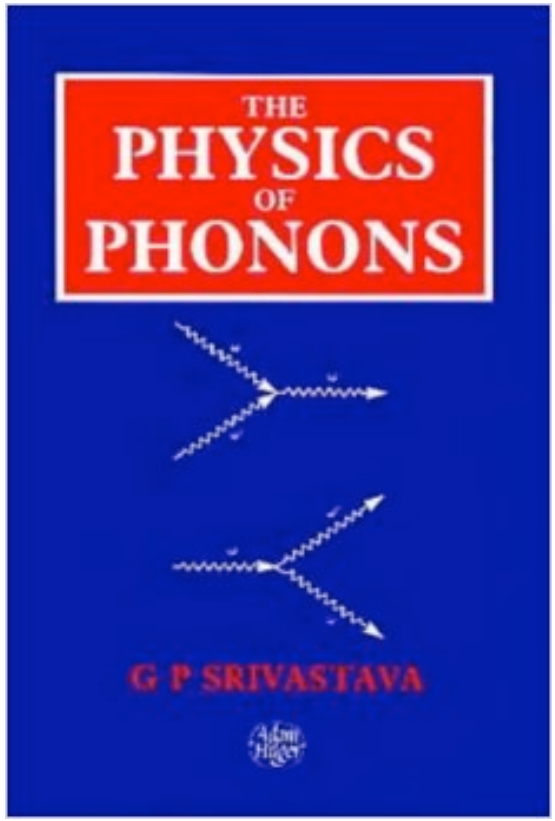
Thanks : T. S. Toellner, J. Zhao, M. Y. Hu, A. Alatas, W. Bi, A. Said, T. Gog

Lattice dynamics for beginners







Inelastic X-Ray Scattering & Spectroscopy @ APS

IXS

RIXS

MCS

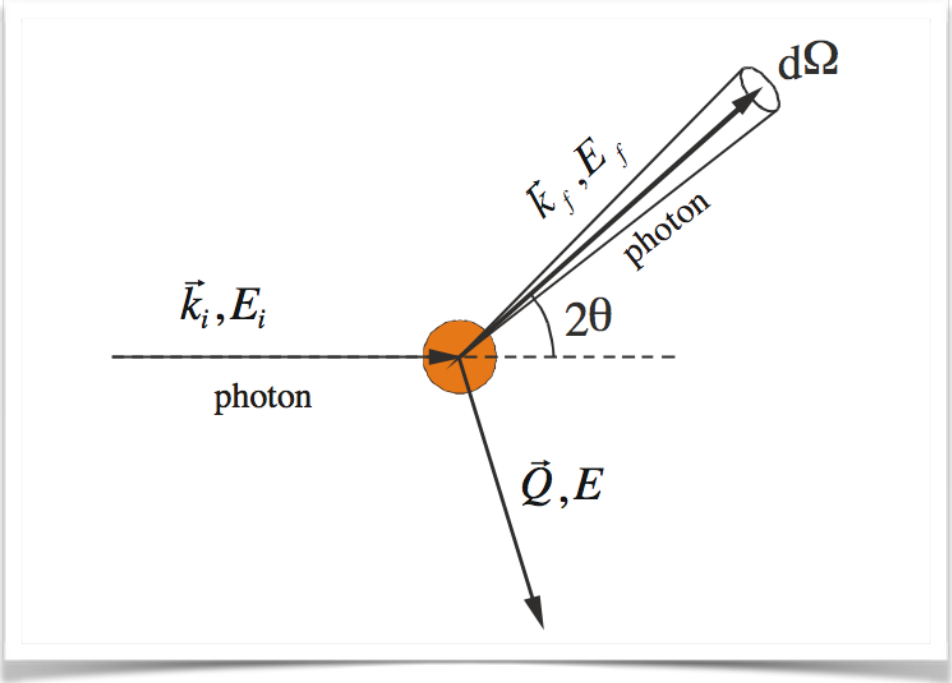
NRIXS

XRS

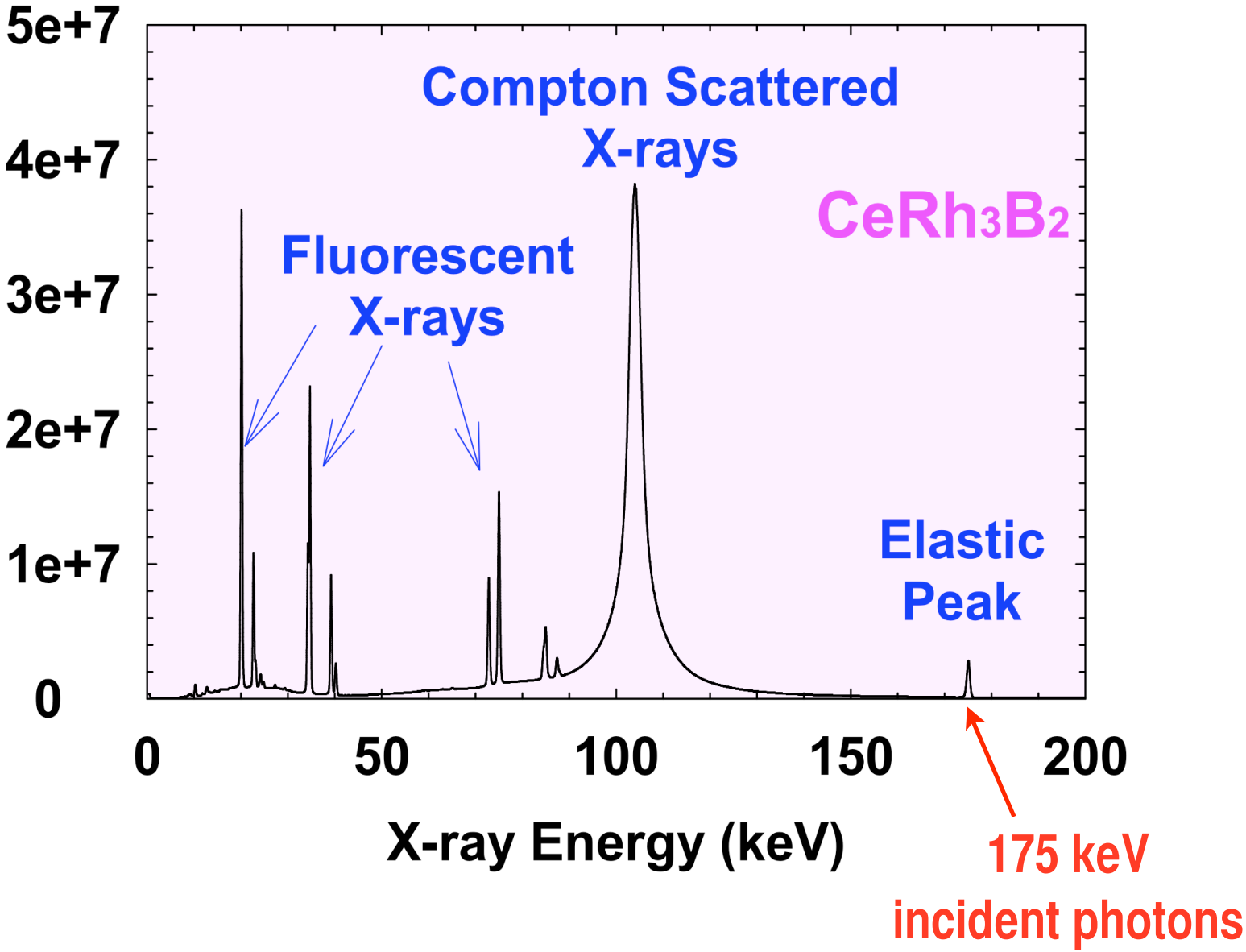
XES

NRIS

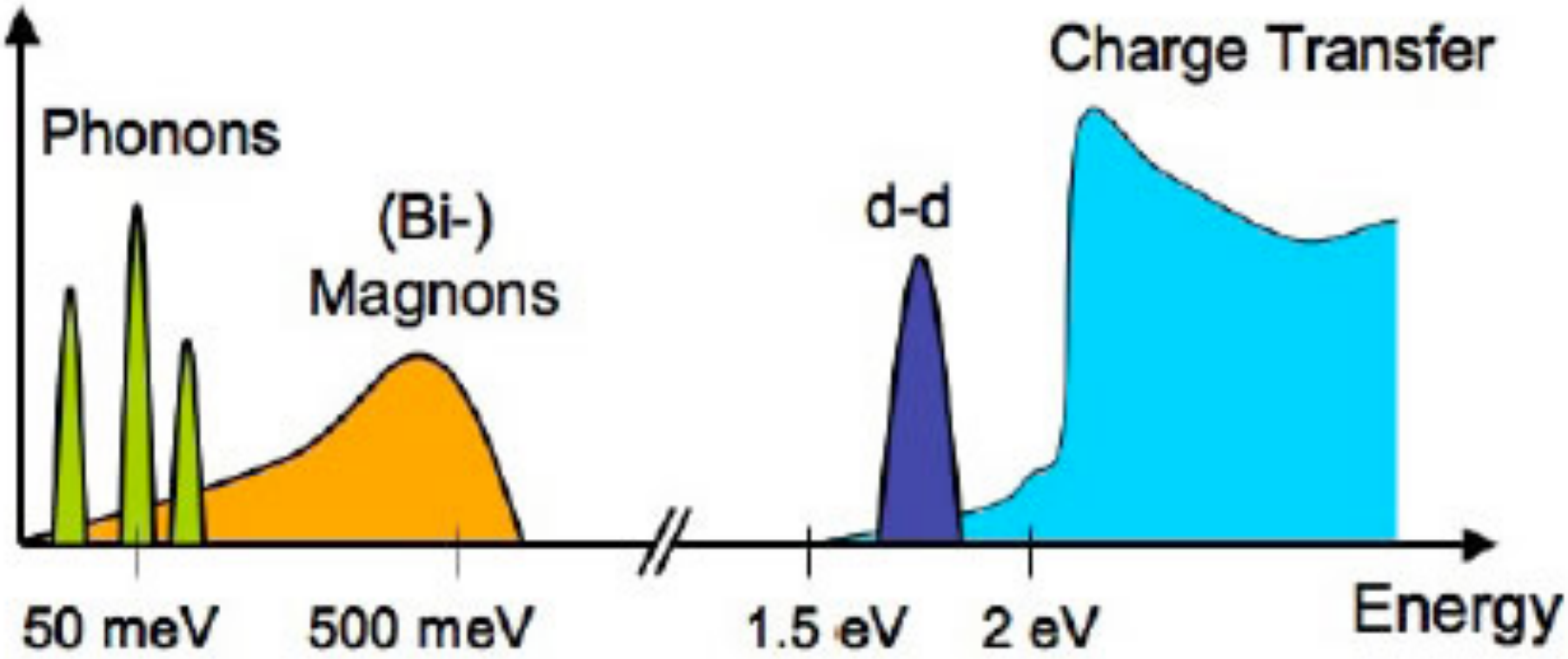
Compton

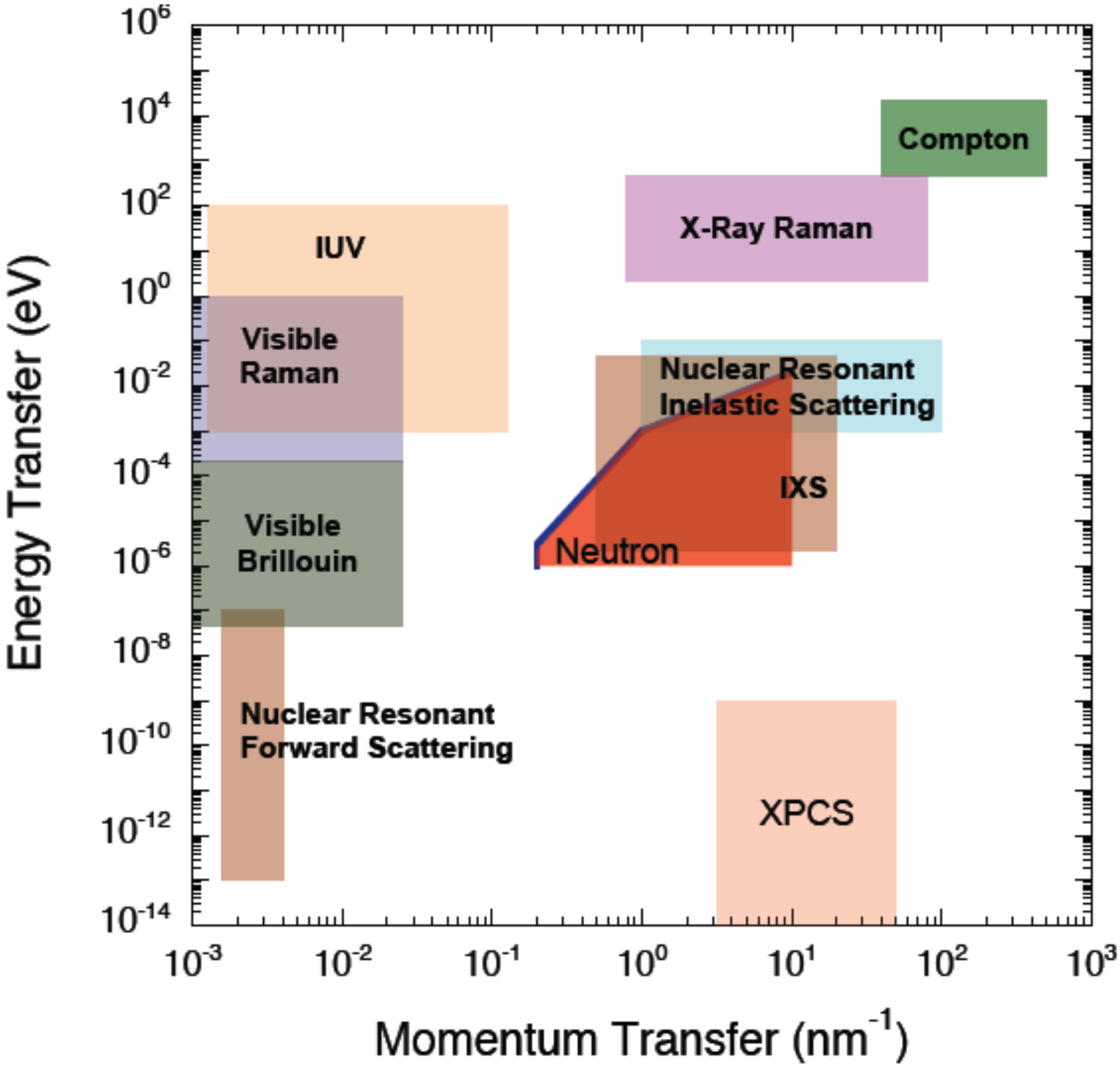


- Nuclear Resonant Inelastic X-Ray scattering, NRIXS: Sectors 3, 16, 30
- Momentum Resolved High Energy Resolution IXS (HERIX) Sectors 3, 30
- Resonant Inelastic X-Ray Scattering, RIXS : Sector 27
- X-Ray Raman Scattering, XRS : Sectors 13, 16, 20
- X-Ray Emission Spectroscopy, XES: Sectors 6, 13, 16, 17



Courtesy: Y. Sakurai, Spring-8


Charge Transfer
Phonons
(Bi-)
Magnons
d-d
50 meV
500 meV
1.5 eV
2 eV
Energy


Energy Transfer (eV)
Momentum Transfer (nm^{-1})
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Compton
X-Ray Raman
IUV
Visible Raman
Nuclear Resonant Inelastic Scattering
IXS
Neutron
Visible Brillouin
Nuclear Resonant Forward Scattering
XPCS

Lattice dynamics for beginners

Lattice dynamics describes vibrations of atoms in condensed matter:

- crystalline solids
- glasses, and
- liquids

However, some of the convenience gained by symmetry or periodic lattice is lost for glasses and liquids. Also, effect of surfaces and defects are glowing short-comings of the classical model.

Lattice dynamics is a reflection of forces acting upon atoms and leads to

- sound velocity
- vibrational entropy
- specific heat
- force constant
- compression tensor
- Young's modulus
- stiffness and resilience
- Gruneisen constant
- viscosity

Imagine that you can measure all that for a micron sized sample, at 3 Mbar at 4000 K, in a way that is element selective, or even better isotope selective.

Sectors 3, 16, 30

Many experimental techniques exist to study lattice dynamics

- sound velocity, deformation, thermal expansion,heat capacity….
- spectroscopic methods using light, x-rays and neutrons, and electrons
- point contact spectroscopy

Atomic motions are described as harmonic traveling waves, characterized by

- wavelength, λ
- angular frequency, ω
- momentum vector along the direction of propagation, $\vec{k} = \frac{\lambda}{2\pi}$

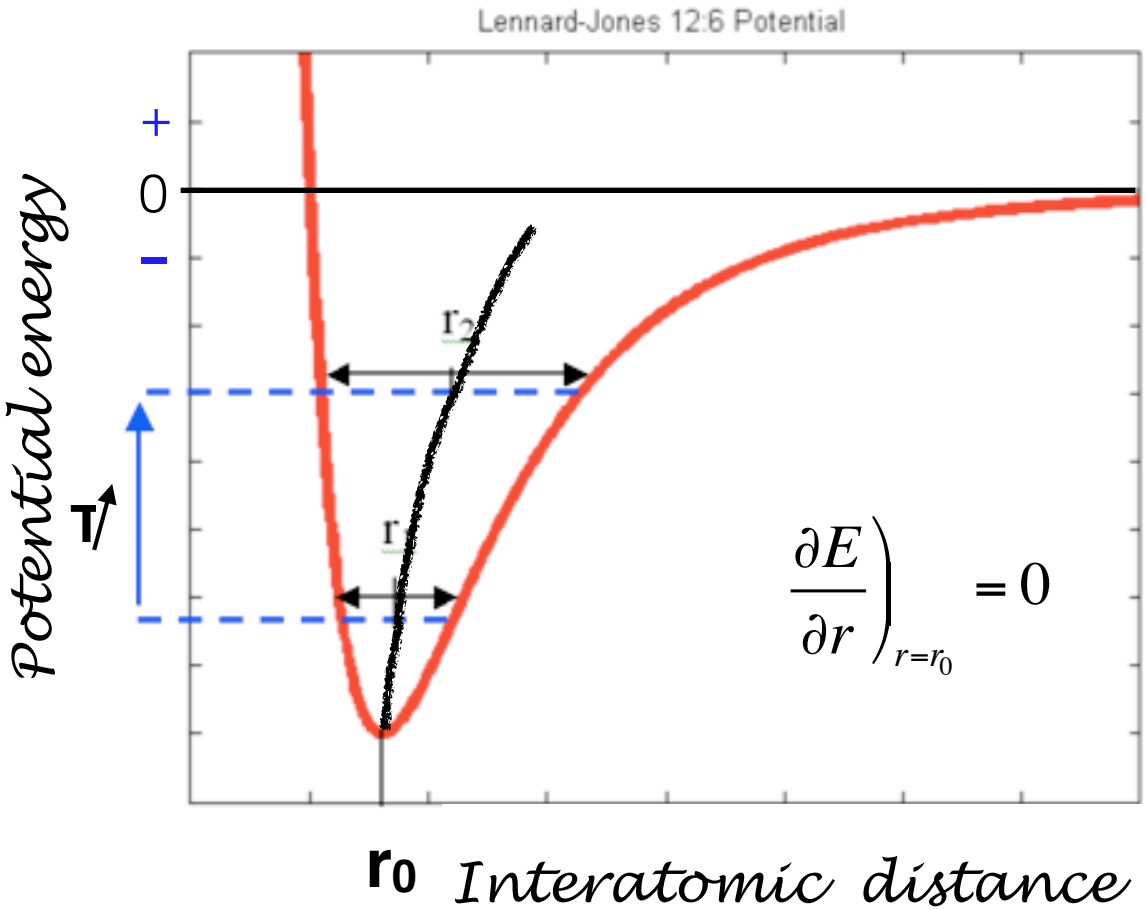
Two main approximations should be noticed:

- **Born-Oppenheimer (adiabiatic) approximation**

  - Motion of atoms are independent and decoupled from the electrons.
  - All electrons follow the nuclei. This can be justified by considering the time scales involved: 10^{-15} s (femto) for electrons, 10^{-12} s (pico) for nuclei

- **Harmonic approximation**

  - At equilibrium, attractive and repulsive forces are balanced.
  - When atoms move away from the equilibrium positions, they are forced to come back by restoring forces.
  - Magnitude of atomic displacements are small compared to interatomic distance.
  - All atoms in equivalent positions in every unit cell move together.



There should be no thermal expansion in the harmonic model.

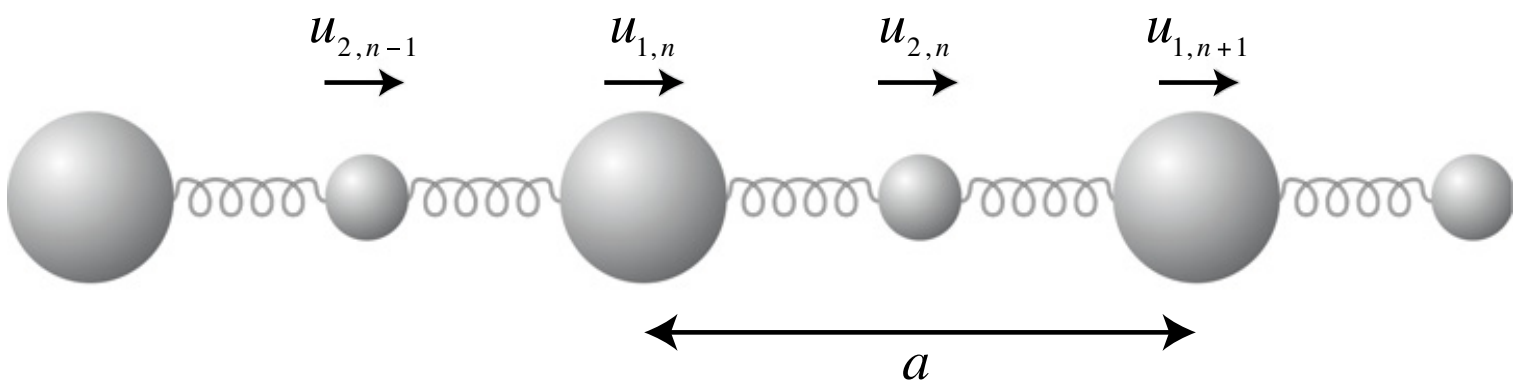
The fact that there is thermal expansion is an indication that the potential under which the atoms move is not harmonic.

However, harmonic model has so many convenient features that we adopt it to explain many features of atomic vibrations.

$$E(r) = E_0 + \frac{1}{2}\left.\frac{\partial^2 E}{\partial r^2}\right|_{r_0}(r-r_0)^2 + \frac{1}{3!}\left.\frac{\partial^3 E}{\partial r^3}\right|_{r_0}(r-r_0)^3 + \frac{1}{4!}\left.\frac{\partial^4 E}{\partial r^4}\right|_{r_0}(r-r_0)^4 + \cdots$$

ignoring these terms is the harmonic approximation

Diatomic infinite 1-D chain



$$E = \frac{1}{2} J \sum_n (u_{1,n} - u_{2,n})^2 + \frac{1}{2} J \sum_n (u_{2,n} - u_{1,n+1})^2$$

$$J = \frac{\partial^2 E}{\partial u_{1,n} \partial u_{2,n}}$$

Force constant (spring constant)

$$u_{1,n}(t) = \tilde{u}_1 \exp\left(i(kna - \omega t)\right)$$

$$u_{2,n}(t) = \tilde{u}_2 \exp\left(i(kna - \omega t)\right)$$

Time dependent displacement of two atoms in terms of relative displacement of each atom

$$E_{1,n} = \frac{1}{2}J(u_{1,n} - u_{2,n})^2 + \frac{1}{2}J(u_{1,n} - u_{2,n-1})^2$$

$$E_{2,n} = \frac{1}{2}J(u_{2,n} - u_{1,n})^2 + \frac{1}{2}J(u_{2,n} - u_{1,n+1})^2$$

Energy

$$f_{1,n} = -\frac{\partial E_{1,n}}{\partial u_{1,n}} = -J(u_{1,n} - u_{2,n}) - J(u_{1,n} - u_{2,n-1})$$

$$f_{2,n} = -\frac{\partial E_{2,n}}{\partial u_{2,n}} = -J(u_{2,n} - u_{1,n}) - J(u_{2,n} - u_{1,n+1})$$

Force as derivative of energy

$$\ddot{u}_{1,n}(t) = -\omega^2 \tilde{u}_1 \exp i\,(kna - \omega t) = -\omega^2 u_{1,n}(t)$$

$$\ddot{u}_{2,n}(t) = -\omega^2 \tilde{u}_2 \exp i\,(kna - \omega t) = -\omega^2 u_{2,n}(t)$$

Acceleration

$$m_1 \ddot{u}_{1,n}(t) = -m_1 \omega^2 u_{1,n}(t) = -J(2u_{1,n}(t) - u_{2,n}(t) - u_{2,n-1}(t))$$

$$m_2 \ddot{u}_{2,n}(t) = -m_2 \omega^2 u_{2,n}(t) = -J(2u_{2,n}(t) - u_{1,n}(t) - u_{1,n+1}(t))$$

Newton's eq^n of motion

$$e_1 = m_1^{1/2}\tilde{u}_1; \quad e_2 = m_2^{1/2}\tilde{u}_2$$

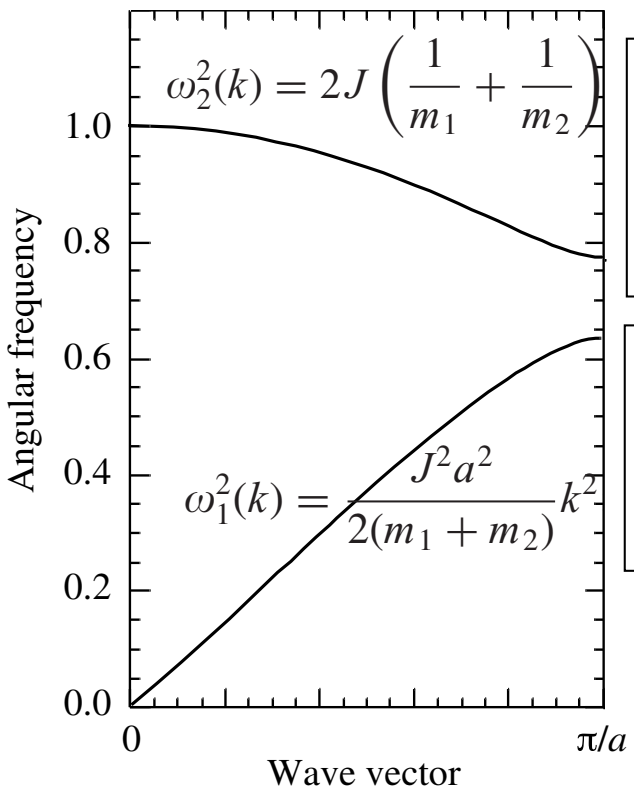
Mass normalized displacements (real)

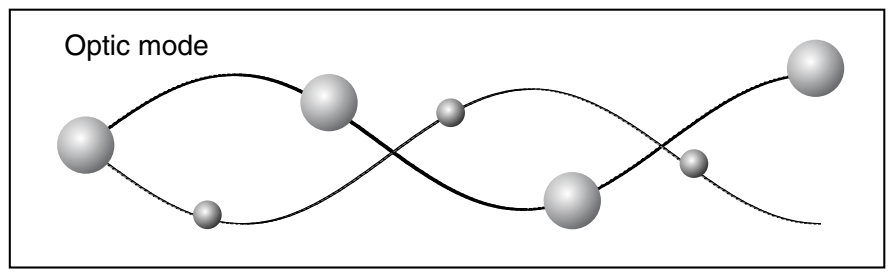
$$\omega^2 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \mathbf{D}(k) \cdot \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

Matrix form of Newton's eq^n of motion

$$\mathbf{D}(k) = \begin{pmatrix} 2J/m_1 & -J\left(1+\exp(-ika)\right)/\sqrt{m_1 m_2} \\ -J\left(1+\exp(+ika)\right)/\sqrt{m_1 m_2} & 2J/m_2 \end{pmatrix}$$

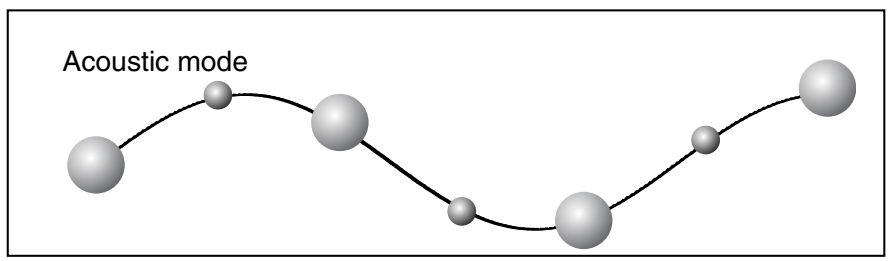
Eigen solutions





out-of-phase

$$m_1^{1/2} e_1 = -m_2^{1/2} e_2$$



$$m_1^{-1/2} e_1 = m_2^{-1/2} e_2$$

in-phase

Inelastic X-Ray Scattering: A plethora of different techniques

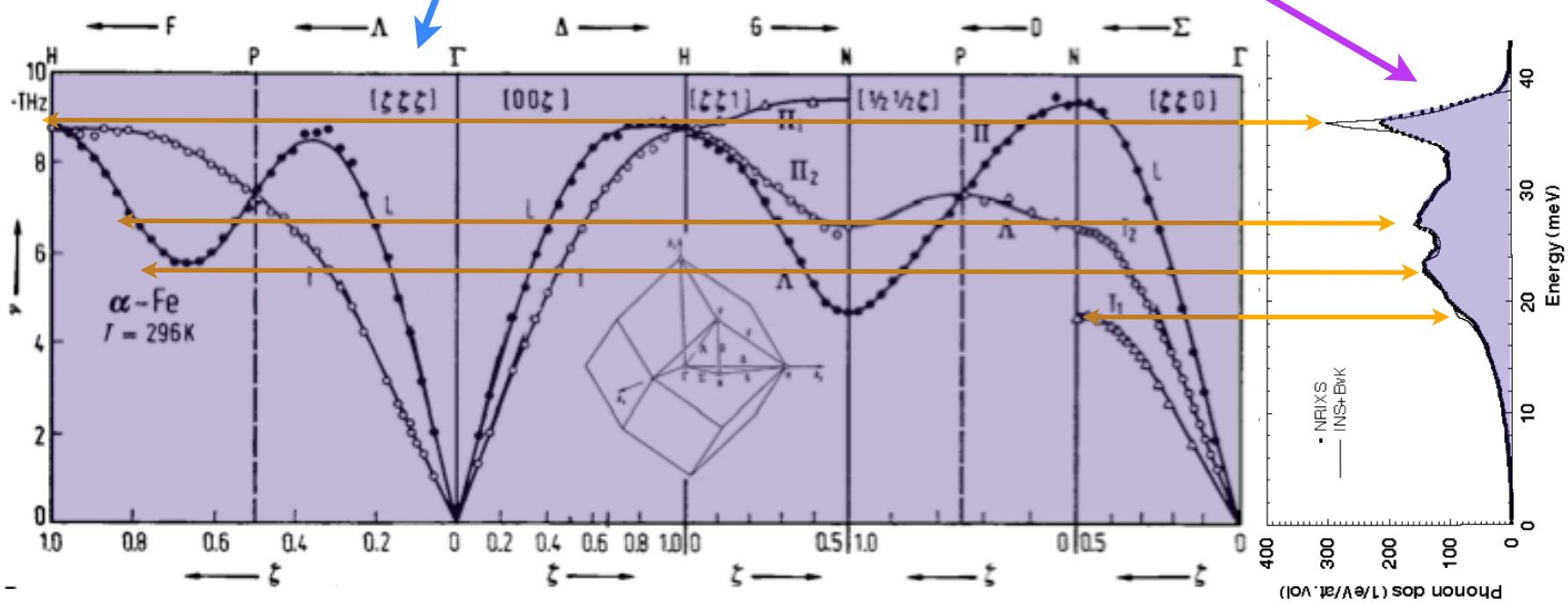
IXS

Non-Resonant

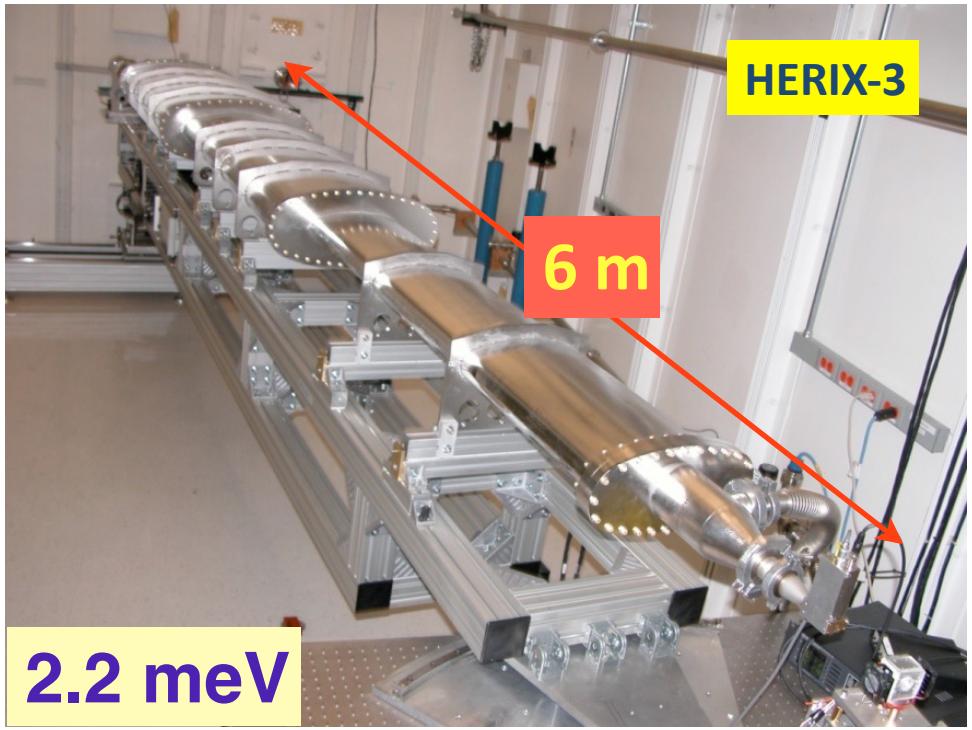
Resonant

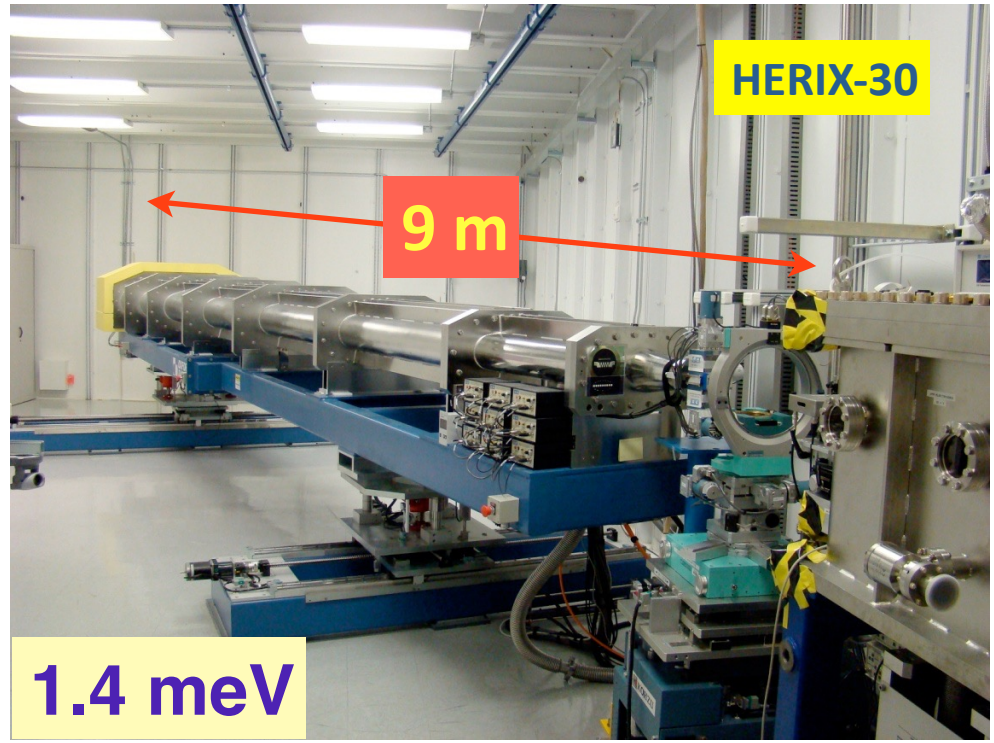
ΔE ~ meV
IXS

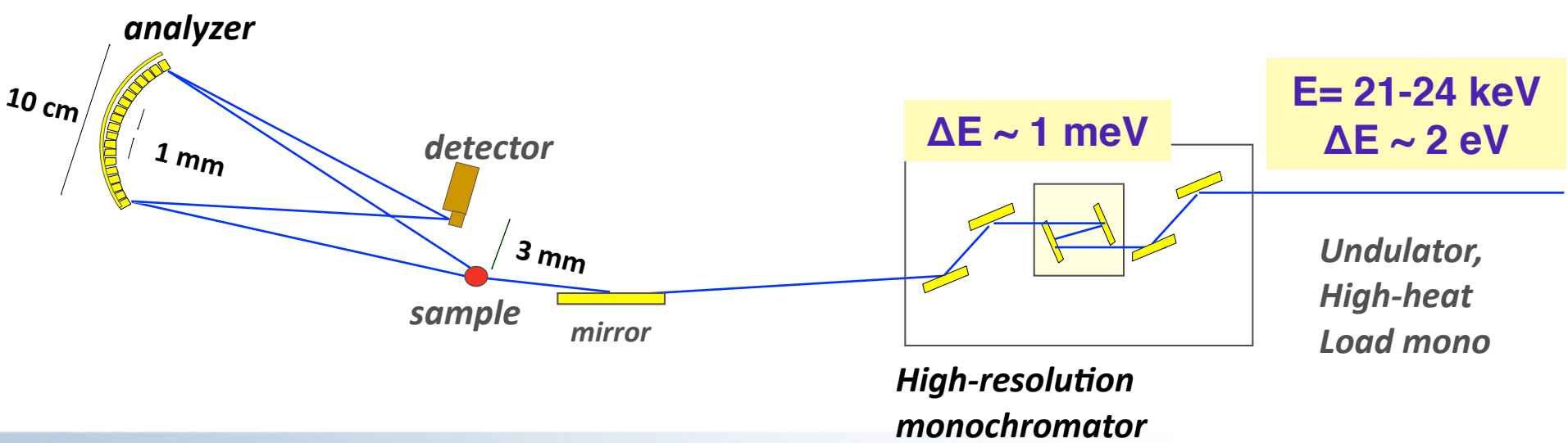
ΔE ~ 1 meV
Nuclear resonant

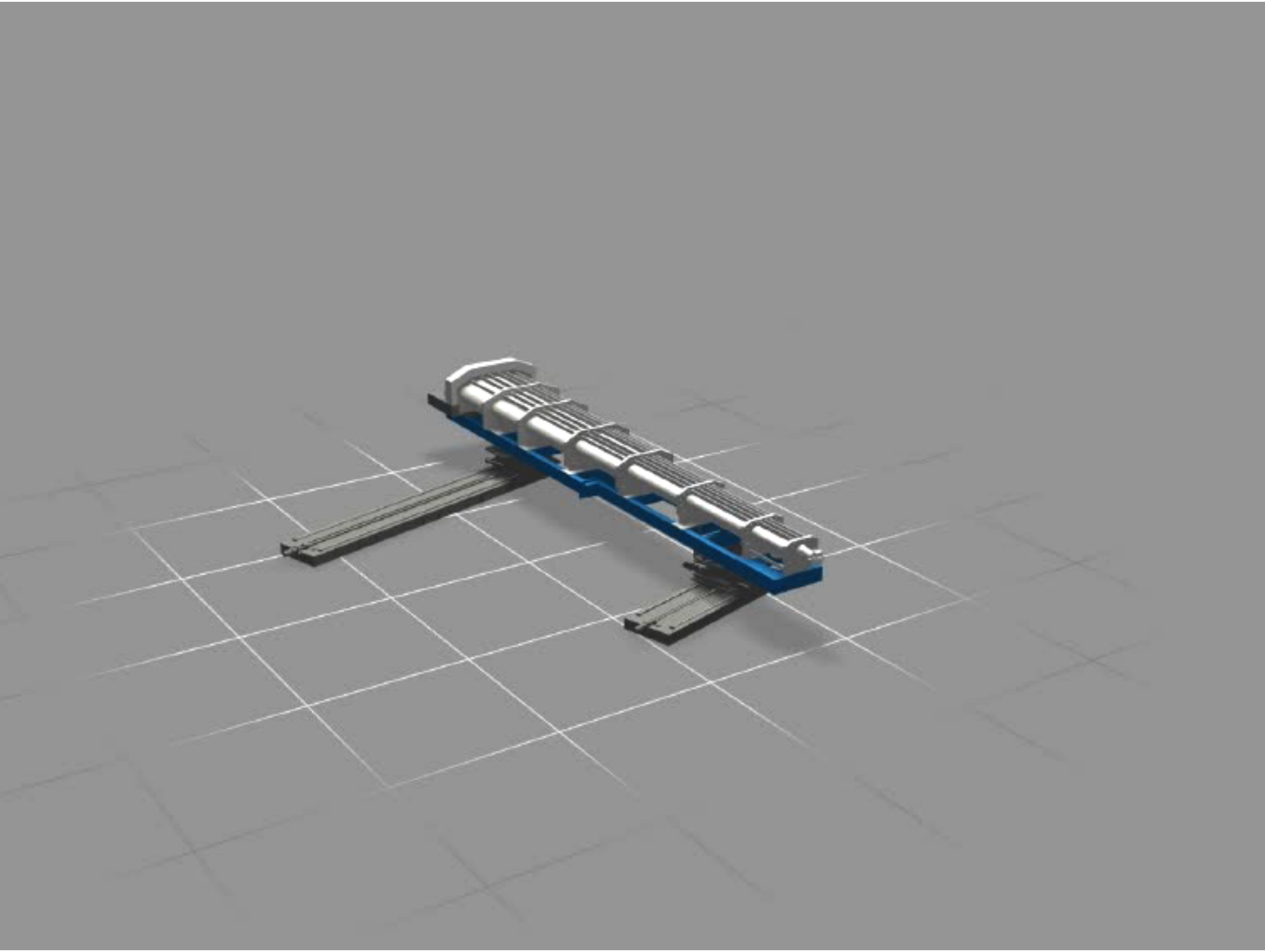


HERIX-3 and HERIX-30









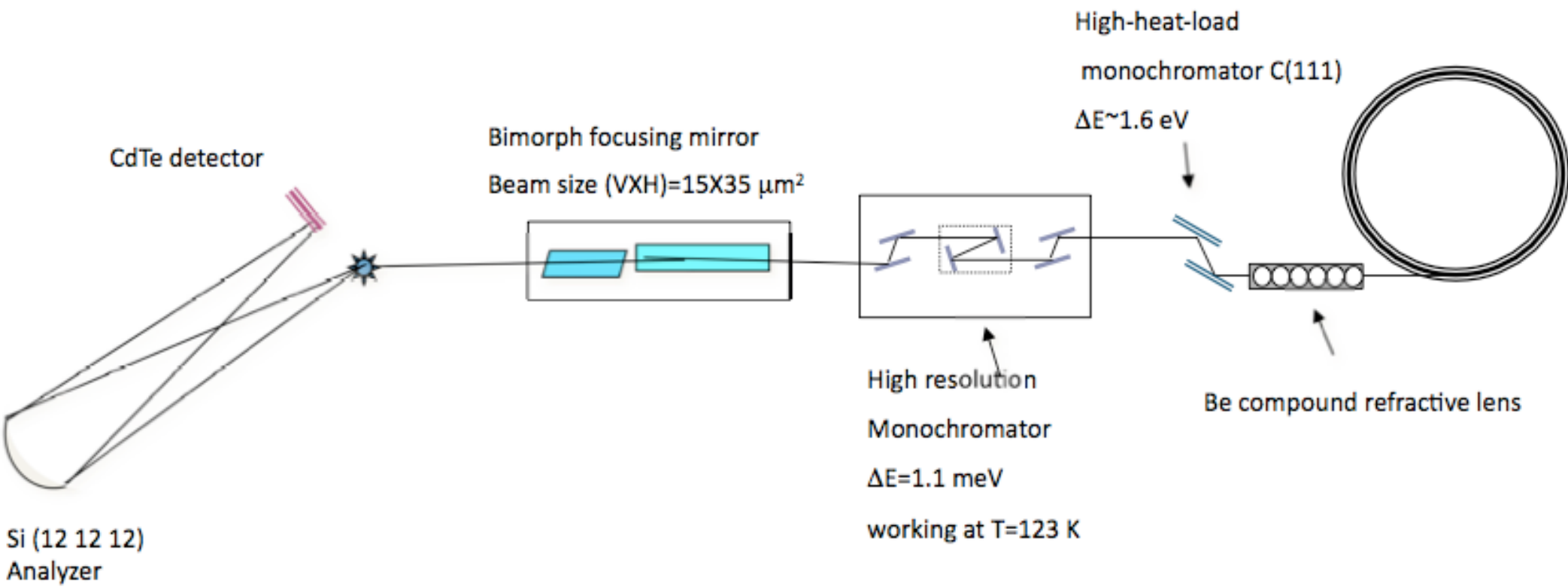

High-heat-load
monochromator C(111)
ΔE~1.6 eV
CdTe detector
Bimorph focusing mirror
Beam size (VXH)=15X35 μm²
High resolution
Monochromator
ΔE=1.1 meV
working at T=123 K
Be compound refractive lens
Si (12 12 12)
Analyzer

Table 1. Crystal contributions for different silicon reflections. The reflections are higher orders of the (1, 1, 1) and the (3, 1, 0) reflections, currently used at the existing spectrometers.

Reflection	E (keV)	τ_{ext} (μm)	τ_{abs} (μm)	$\delta E_{\text{crystal}}$ (meV)	Reflectivity (%)
(7, 7, 7)	13.839	40	300	5.13	81
(12, 4, 0)	14.438	30	380	6.21	87
(8, 8, 8)	15.816	50	500	4.42	85
(9, 9, 9)	17.793	100	700	1.99	76
(18, 6, 0)	21.657	200	1220	1.23	78
(11, 11, 11)	21.747	280	1240	0.85	70
(12, 12, 12)	23.724	270	1590	0.79	75
(13, 13, 13)	25.701	600	1990	0.37	61
(24, 8, 0)	28.876	840	2730	0.255	61
(15, 15, 15)	29.655	1400	2930	0.153	46

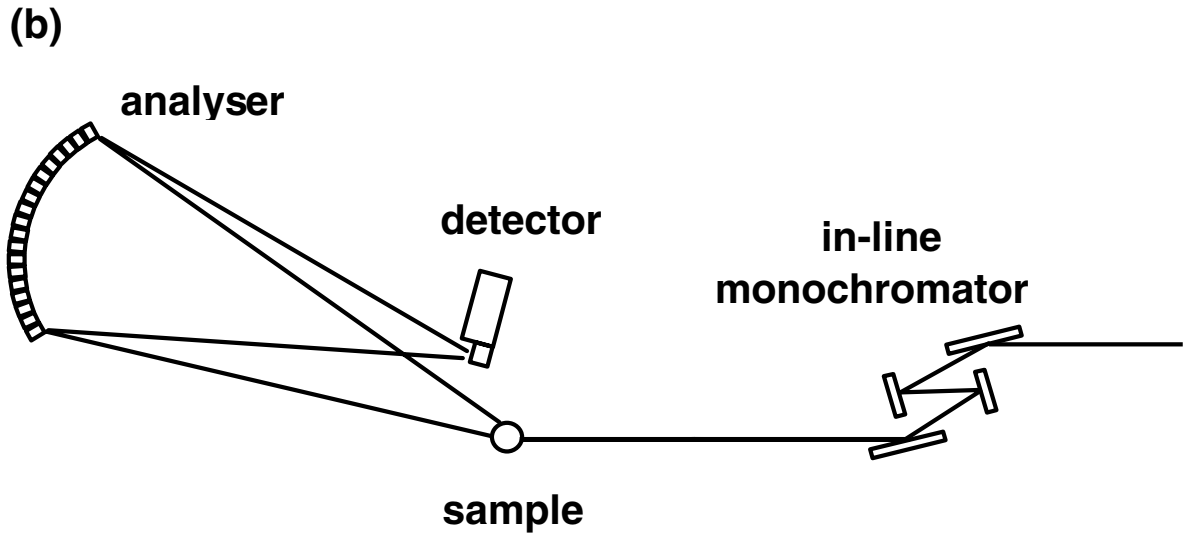


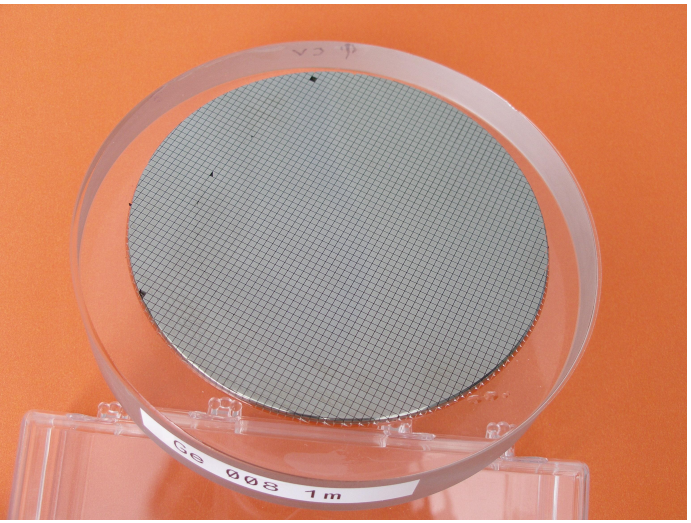
Table 2. Settings for the analyser for different energy resolutions. The total energy resolution is here twice the value of the geometrical contribution of the analyser.

δE_{total} (meV)	δE_{geom} (meV)	Reflection	E (keV)	θ_{Bragg} (deg)	L (m)
12.42	6.21	(12, 4, 0)	14.438	89.95	2.22
10.26	5.13	(7, 7, 7)	13.839	89.96	2.39
8.84	4.42	(8, 8, 8)	15.816	89.97	2.76
3.98	1.99	(9, 9, 9)	17.793	89.976	4.35
2.46	1.23	(18, 6, 0)	21.657	89.983	6.11
1.70	0.85	(11, 11, 11)	21.747	89.986	7.36
1.58	0.79	(12, 12, 12)	23.724	89.989	7.98
0.74	0.37	(13, 13, 13)	25.701	89.991	12.1
0.51	0.255	(24, 8, 0)	28.876	89.993	15.5
0.30	0.153	(15, 15, 15)	29.655	89.995	20.2

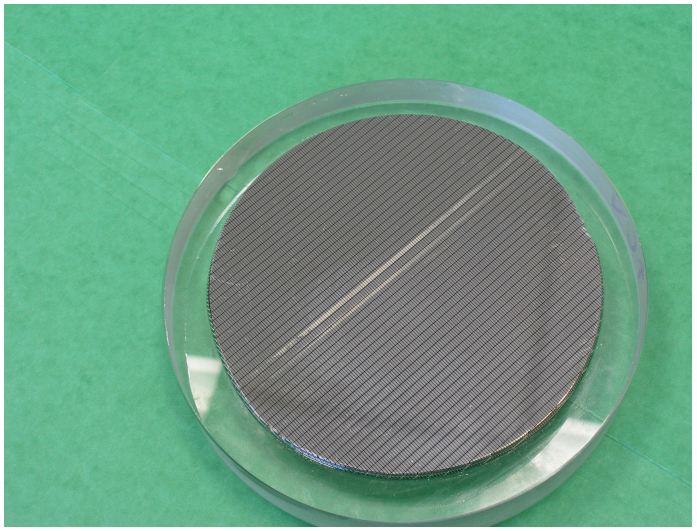
Choice of energy

Si Reflection at 90 °	Energy (keV)	Resolution (meV)	Reflectivity (%)
18 6 0	21.657	1.23	78
11 11 11	21.747	0.83	70
13 11 9	21.985	0.81	69
15 11 7	22.685	0.70	68
20 4 0	23.280	0.87	76
12 12 12	23.724	0.80	75
14 14 8	24.374	0.69	74
22 2 0	25.215	0.576	71
13 13 13	25.701	0.37	60

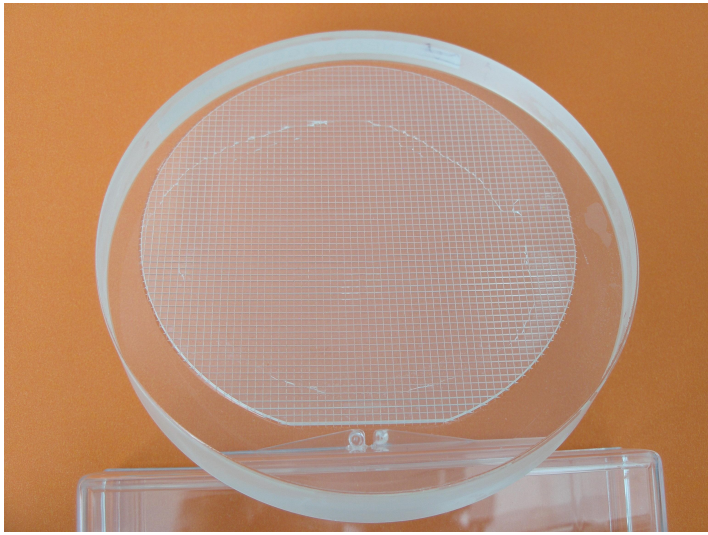
Bent-diced IXS analyzers



Si



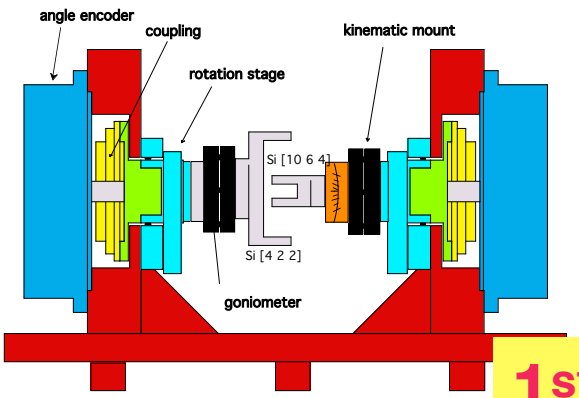
Ge



$LiNbO_3$

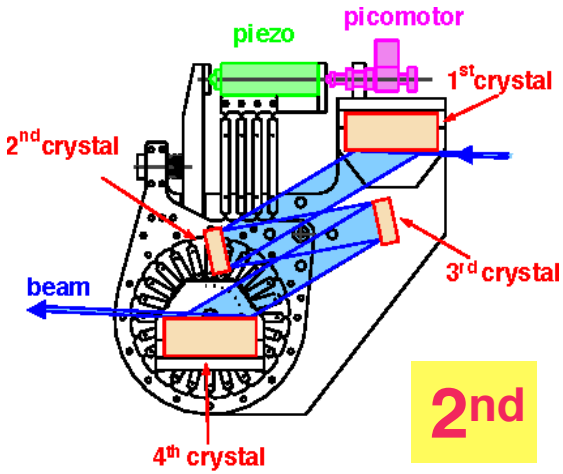
Generations of high-resolution monochromators

1992



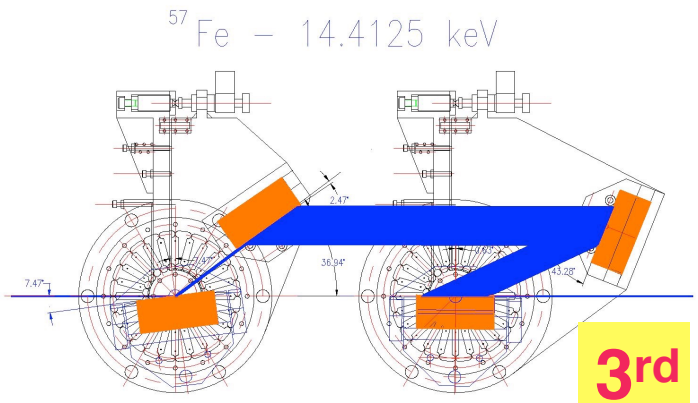
1st

1999



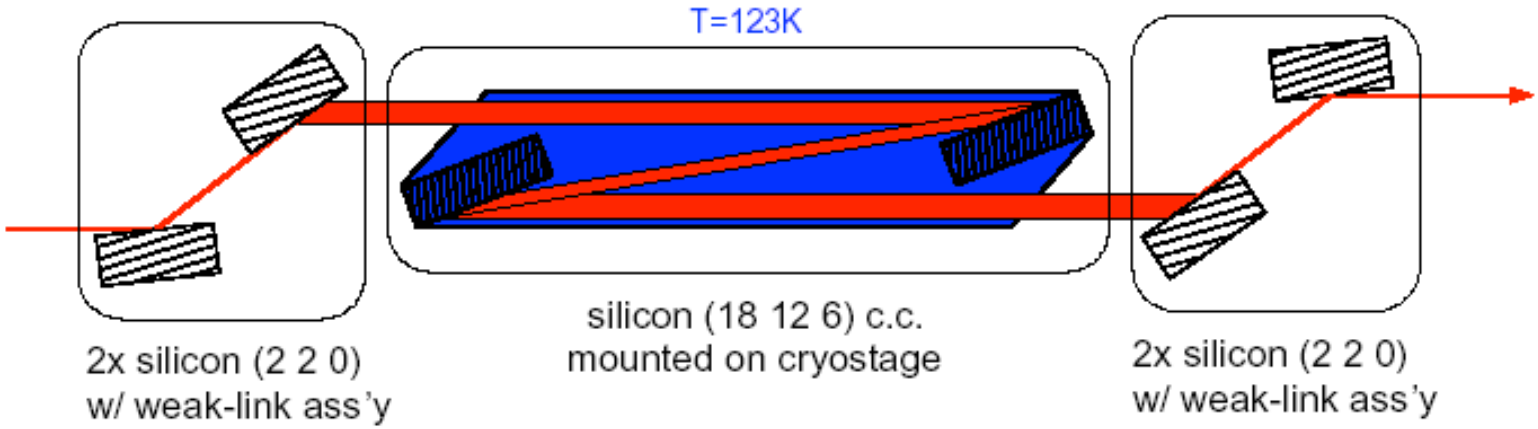
2nd

2002



3rd

2004



4th

T. Toellner, D. Shu

What is being measured ?

$$\frac{d^2\sigma}{d\Omega\, \mathrm{d}\omega} = r_0^2 \frac{\omega_f}{\omega_i} \left|\mathbf{e}_i \cdot \mathbf{e}_f\right| N \sum_{i,f} \left| \left\langle i \middle| \sum e^{i\mathbf{Q}\mathbf{r}_\mathbf{j}} \middle| f \right\rangle \right|^2 \delta(E_f - E_i - \mathbf{h}\omega)$$

Thomson cross section **Dynamical structure factor S(Q,w)**

$$S(\mathbf{Q},\omega) = \frac{1}{2\pi} \int dt\, e^{-i\omega t} \left\langle \phi_i \middle| \sum_{ll'} f_l(\mathbf{Q}) e^{-i\mathbf{Q}\cdot\mathbf{r}_l(t)} f_{l'}(\mathbf{Q}) e^{i\mathbf{Q}\cdot\mathbf{r}_{l'}(0)} \middle| \phi_i \right\rangle$$

Density-density correlations

$$f(Q) = f_{ion}(Q) + f_{valence}(Q)$$

Atomic form factor

Dynamic structure factor

Sum over phonon branch j at reduced momentum transfer, q

Sum over different atoms in the unit cell

Atomic form factor for each atom

scaling with square root of mass

Debye-Waller factor to account for bond strength

phonon occupation probability

$$S(Q,\omega) = \sum_{q,j}\left|\sum_{s} f_s(Q)\frac{\hbar}{\sqrt{2m_s}} e^{-W_s} e^{i\vec{Q}\cdot\vec{R}_s}\left[\vec{Q}\cdot\vec{e}(q,s,j)\right]\right|^2 \frac{\left(\frac{1}{e^{\hbar\omega/kT}-1}+\frac{1}{2}\pm\frac{1}{2}\right)}{\omega_{q,j}}\delta\left(\omega\pm\omega_{q,j}\right)$$

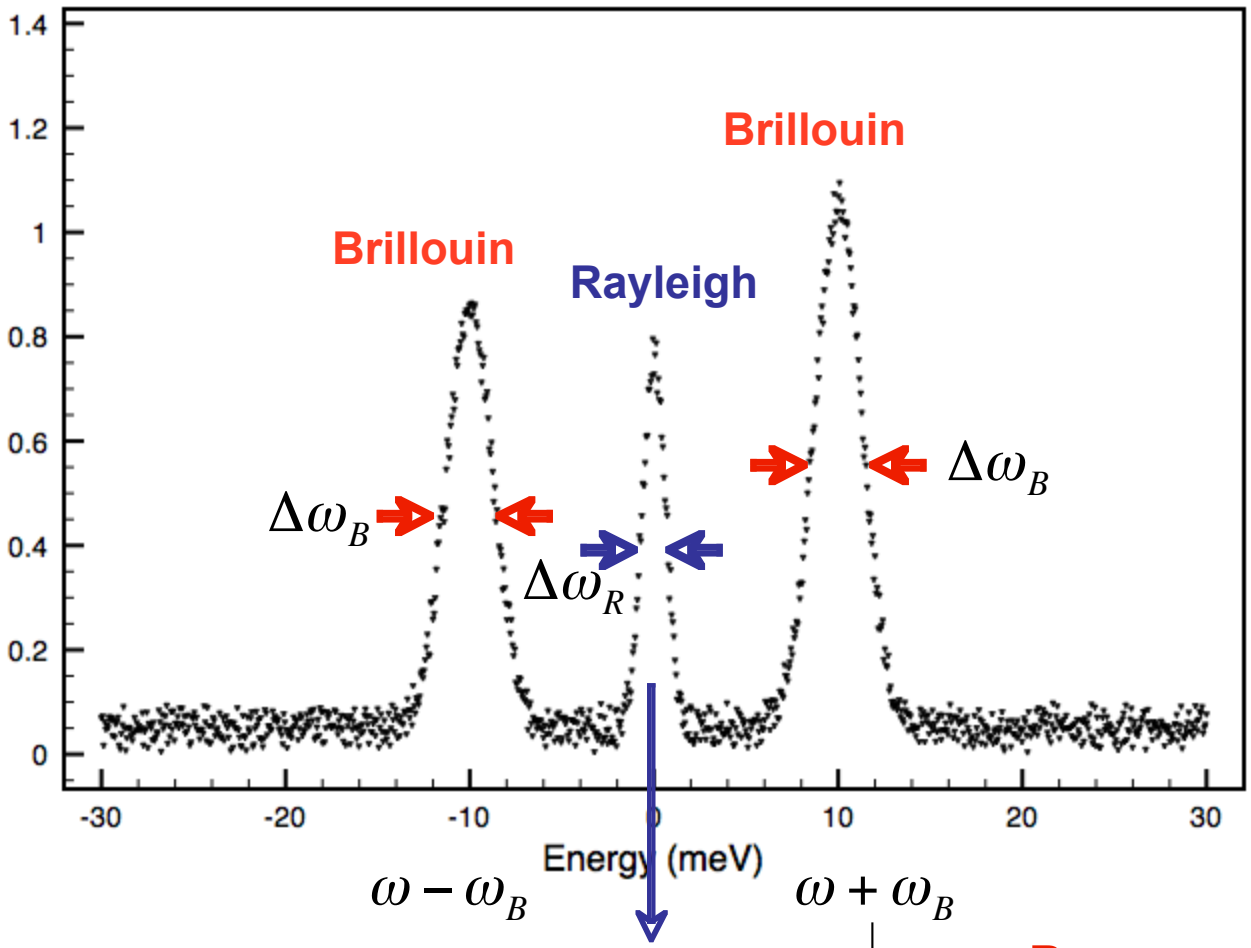
Phase of the scattering amplitude

phonon frequency

Polarization factor between momentum transfer and phonon's polarization vector

delta-function in ω

External probe-photon



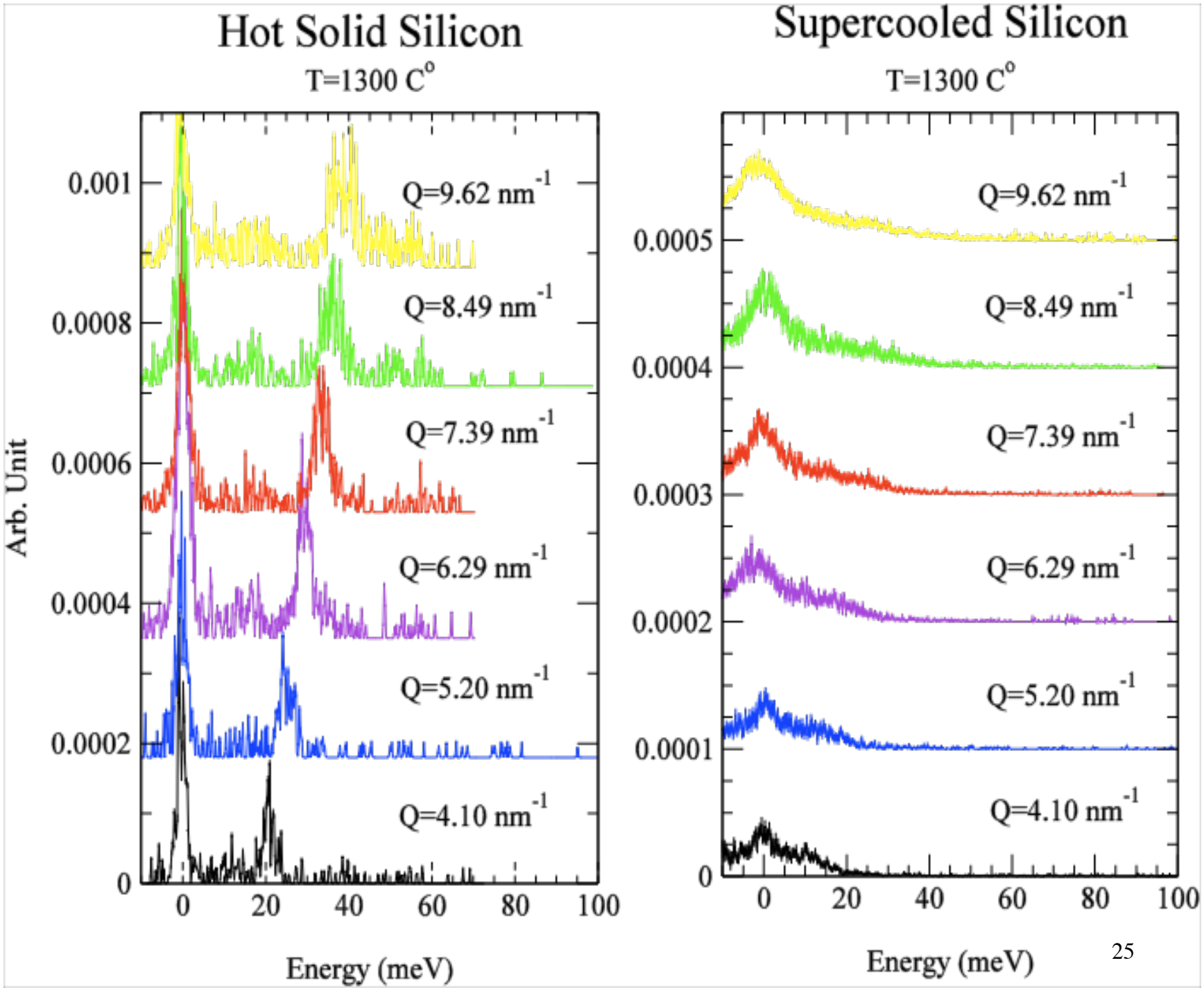
Entropy fluctuations, $\Delta\omega_R \sim \alpha q^2$

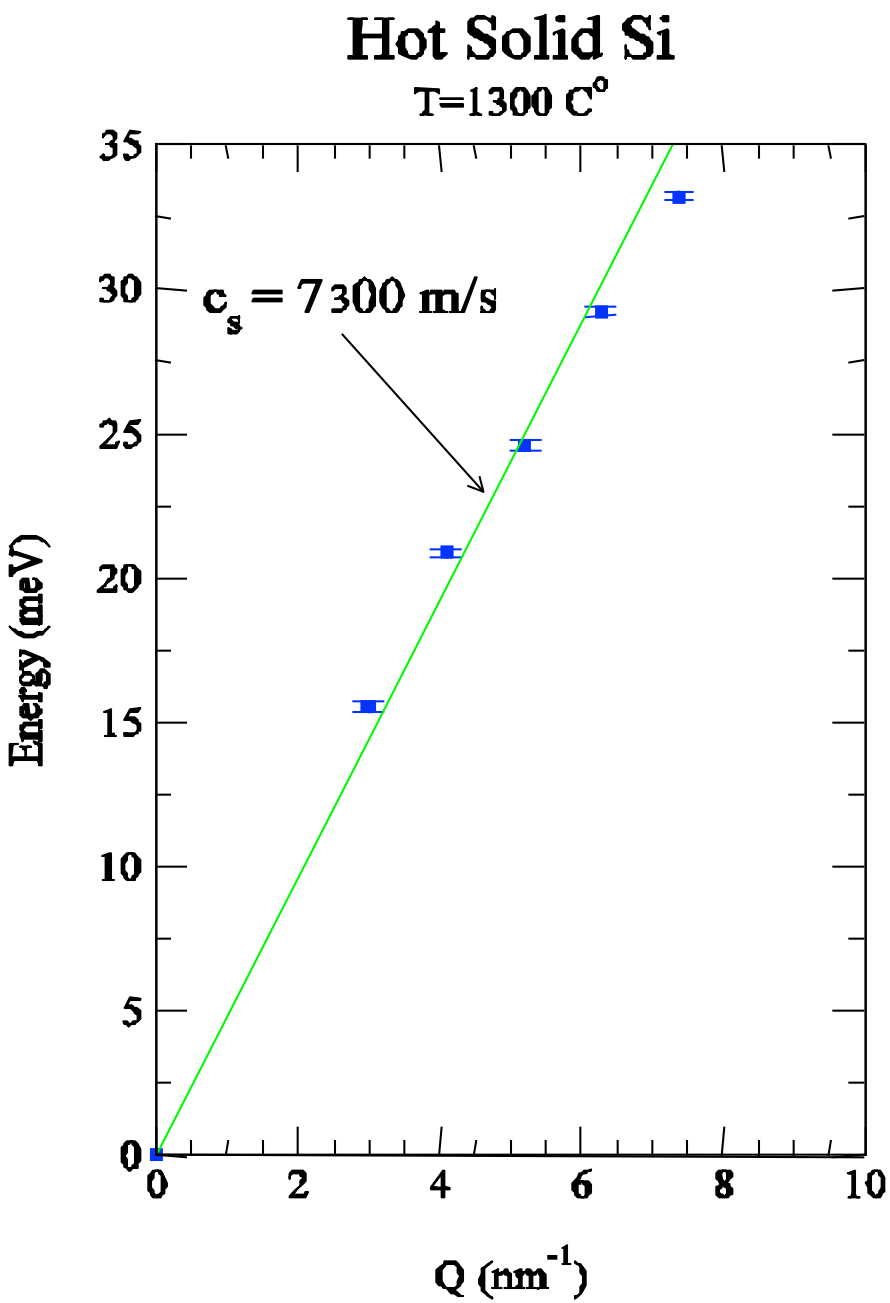
Concentration fluctuations $\Delta\omega_R \sim D q^2$

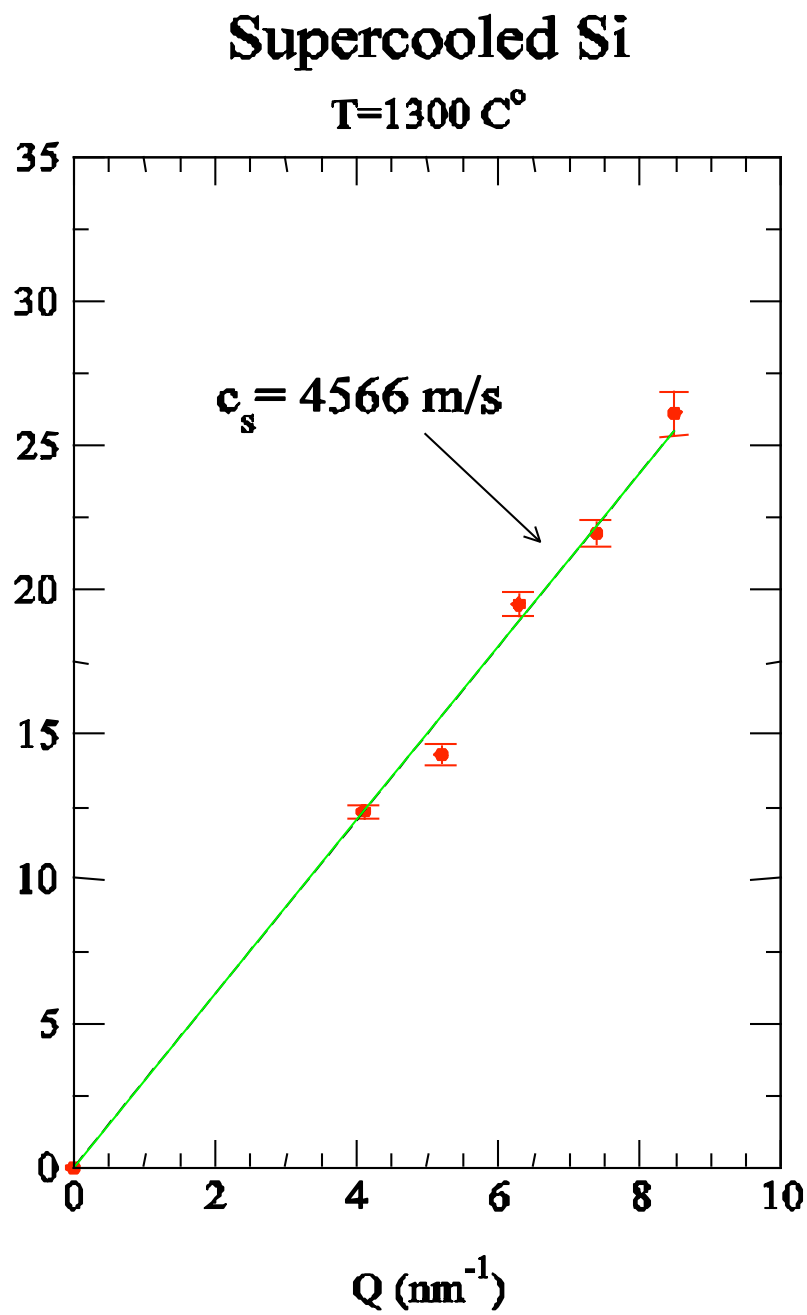
Pressure fluctuations

$$\omega_B(q) = V \cdot q$$

$$\Delta\omega_B \sim V q^2$$


Hot Solid Silicon
T=1300 C°
Supercooled Silicon
T=1300 C°
Arb. Unit
Q=9.62 nm-1
Q=8.49 nm-1
Q=7.39 nm-1
Q=6.29 nm-1
Q=5.20 nm-1
Q=4.10 nm-1
0.001
0.0008
0.0006
0.0004
0.0002
0
0.0005
0.0004
0.0003
0.0002
0.0001
0
20
40
60
80
100
Energy (meV)


Hot Solid Si
T=1300 C°
c_s = 7300 m/s
Energy (meV)
Q (nm^{-1})
0
5
10
15
20
25
30
35
0
2
4
6
8
10


Supercooled Si
T=1300 C°
c_s = 4566 m/s
Q (nm^{-1})
0
5
10
15
20
25
30
35
0
2
4
6
8
10

Where is quantum mechanics in all of this?

$$E_{1,n} = \frac{1}{2}J(u_{1,n} - u_{2,n})^2 + \frac{1}{2}J(u_{1,n} - u_{2,n-1})^2$$

$$E_{2,n} = \frac{1}{2}J(u_{2,n} - u_{1,n})^2 + \frac{1}{2}J(u_{2,n} - u_{1,n+1})^2$$

Diatomic model

$$E = \frac{1}{4}\sum_{n,n'}\sum_{j,j'}\phi_{n,n'}^{j,j'}\left(u_{j,n} - u_{j',n'}\right)^2 = \frac{1}{2}\sum_{n,n'}\sum_{j,j'}u_{j,n}\Phi_{n,n'}^{j,j'}u_{j',n'}$$

Generalized model

j, j': atoms in the unit cell

n, n': unit cells in the crystal

$\phi_{j,j'}^{n,n'}$: differential of individual bond energy with respect to displacement

$\Phi_{j,j'}^{n,n'}$: differential of overall bond energy of all lattice

PHONON's: $\phi\omega\nu\acute{\eta}$ (phonē), *sound*

- Phonons are periodic oscillations in condensed systems.

- They are inherently involved in thermal and electrical conductivity.

- They can show anomalous (non-linear) behavior near a phase transition.

- They can carry sound (acoustic modes) or couple to electromagnetic radiation or neutrons (acoustical and optical).

- Have energy of $\hbar\omega$ as quanta of excitation of the lattice vibration mode of angular frequency ω. Since momentum, $\hbar k$, is exact, they are delocalized, collective excitations.

- Phonons are bosons, and they are not conserved. They can be created or annihilated during interactions with neutrons or photons.

- They can be detected by Brillouin scattering (acoustic), Raman scattering, FTIR (optical).

- Their dispersion throughout the BZ can ONLY be monitored with x-rays (IXS), or neutrons (INS).

- Accurate prediction of phonon dispersion require correct knowledge about the force constants: COMPUTATIONAL TECHNIQUES ARE ESSENTIAL.

$$u_{j\ell}(t) = \frac{1}{\sqrt{Nm_j}} \sum_{\mathbf{k},\lambda} \mathbf{e}_{\mathbf{k},\lambda} \exp(i\mathbf{k}\cdot\mathbf{r}_{j\ell})\, Q(\mathbf{k},\lambda,t)$$

Fourier relationship between real space and time and reciprocal space and time

$\mathbf{e}_{\mathbf{k},\lambda}$: mode eigenvector

$Q(\mathbf{k},\lambda,t)$: normal mode coordinate

$$\dot{u}_{j\ell}(t) = \frac{-i}{\sqrt{Nm_j}} \sum_{\mathbf{k},\lambda} \omega_{\mathbf{k},\lambda}\mathbf{e}_{\mathbf{k},\lambda} \exp(i\mathbf{k}\cdot\mathbf{r}_{j\ell})\, Q(\mathbf{k},\lambda,t)$$

Velocity

$$\frac{1}{2}\sum_{j,\ell} m_j \left|\dot{\mathbf{u}}_{j\ell}\right|^2 = \frac{1}{2}\sum_{\mathbf{k},\lambda} \omega^2_{\mathbf{k},\lambda} \left|Q(\mathbf{k},\lambda)\right|^2$$

Kinetic energy

$$\frac{1}{2}\sum_{\substack{j,j'\\ \ell,\ell'}} \mathbf{u}^T_{j\ell}\cdot\Phi^{j,j'}_{\ell,\ell'}\cdot\mathbf{u}_{j'\ell'} = \frac{1}{2}\sum_{\mathbf{k},\lambda} \omega^2_{\mathbf{k},\lambda} \left|Q(\mathbf{k},\lambda)\right|^2$$

Potential energy (via Virial theorem)

$$\frac{1}{2}\sum_{j,\ell} m_j \left|\dot{\mathbf{u}}_{j\ell}\right|^2 + \frac{1}{2}\sum_{\substack{j,j'\\ \ell,\ell'}} \mathbf{u}^T_{j\ell}\cdot\Phi^{j,j'}_{\ell,\ell'}\cdot\mathbf{u}_{j'\ell'} = \sum_{\mathbf{k},\lambda} \omega^2_{\mathbf{k},\lambda} \left|Q(\mathbf{k},\lambda)\right|^2.$$

Total energy, in terms of normal mode coordinates

$$\omega^2 \mathbf{e} = \mathbf{D}(\mathbf{k}) \cdot \mathbf{e} \quad \Rightarrow \quad \omega^2 = \mathbf{e}^{\mathrm{T}} \cdot \mathbf{D}(\mathbf{k}) \cdot \mathbf{e}$$

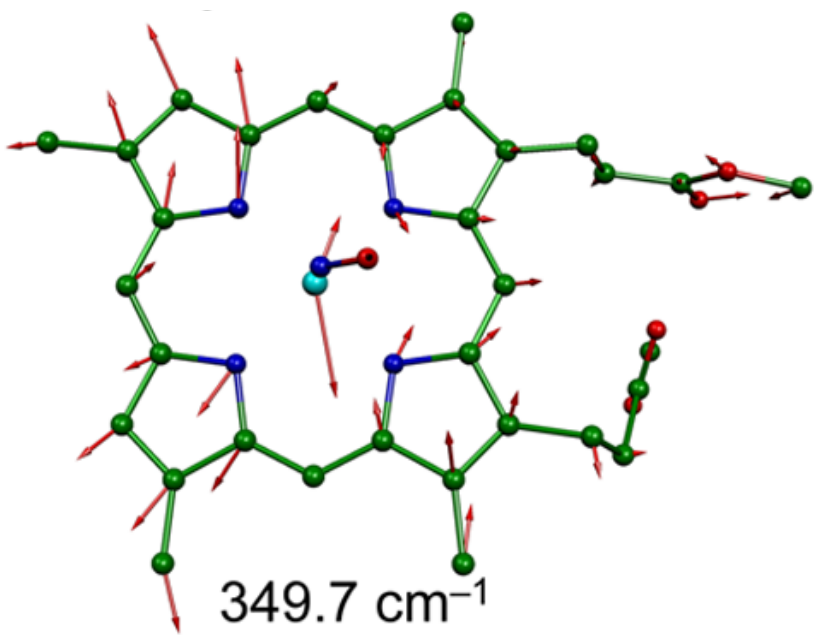
Eigenvalue eq^{n}.

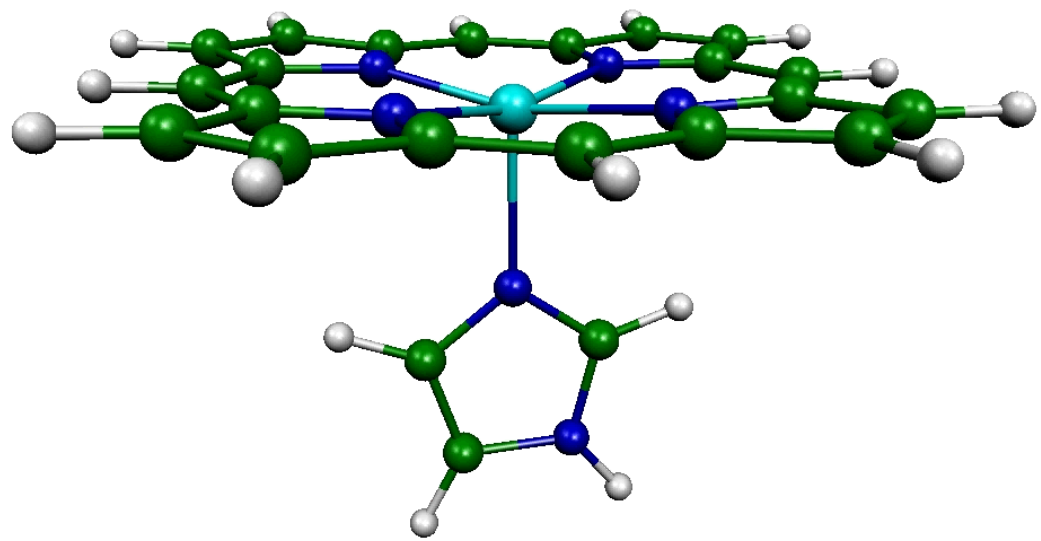
$$D_{j,j'}(\mathbf{k}) = \frac{1}{\sqrt{m_j m_{j'}}} \sum_{n'} \Phi_{0,n'}^{j,j'} \exp\left(i\mathbf{k} \cdot (\mathbf{r}_{j,0} - \mathbf{r}_{j',n'})\right)$$

Dynamical matrix

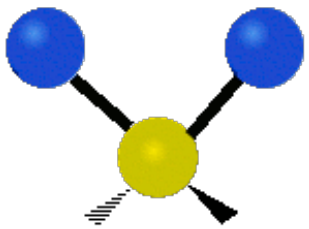
$$\mathbf{e}_\lambda^{\mathrm{T}} \cdot \mathbf{e}_\lambda = 1; \quad \mathbf{e}_{\lambda'}^{\mathrm{T}} \cdot \mathbf{e}_\lambda = \delta_{\lambda',\lambda}$$

Eigenvalues are orthonormal..

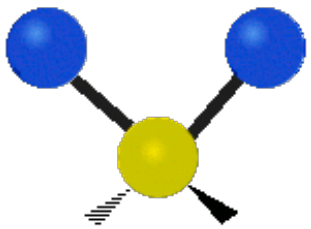




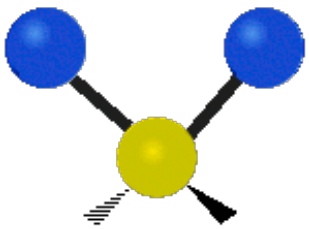
symmetrical stretching



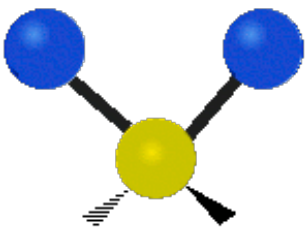
asymmetrical stretching



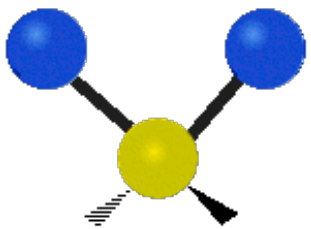
scissoring



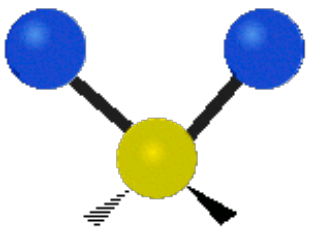
rocking

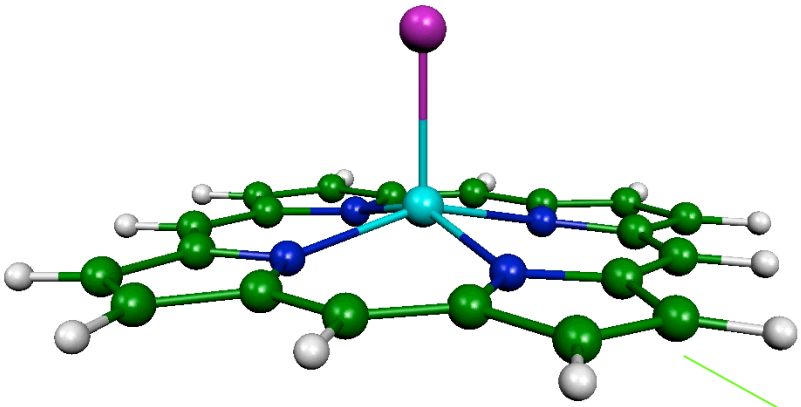


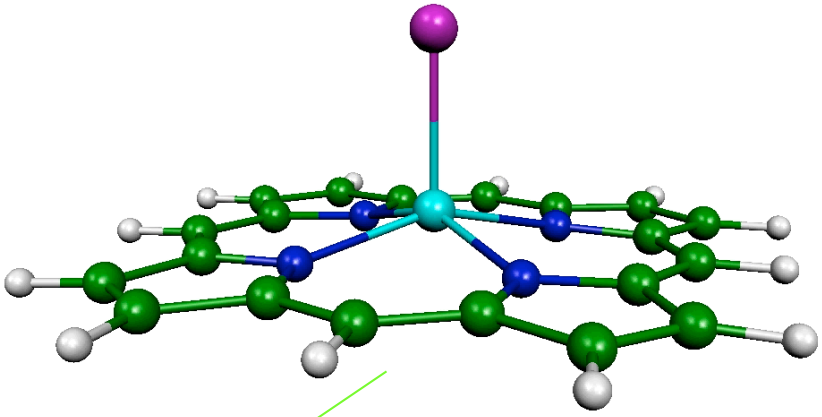
wagging

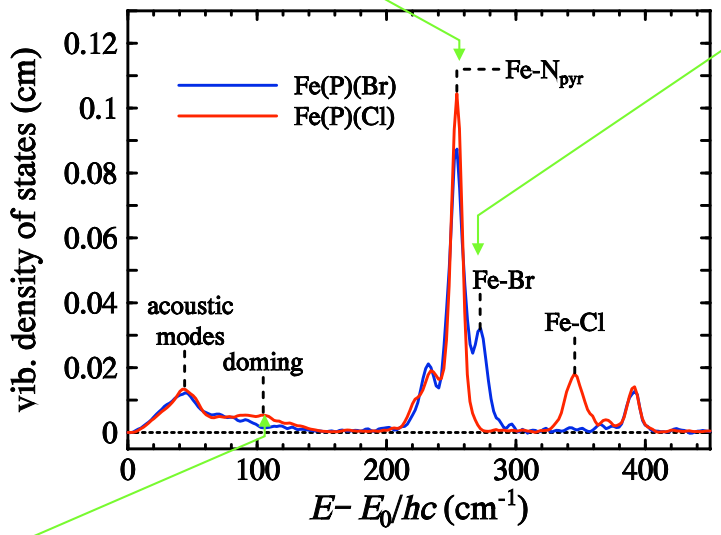


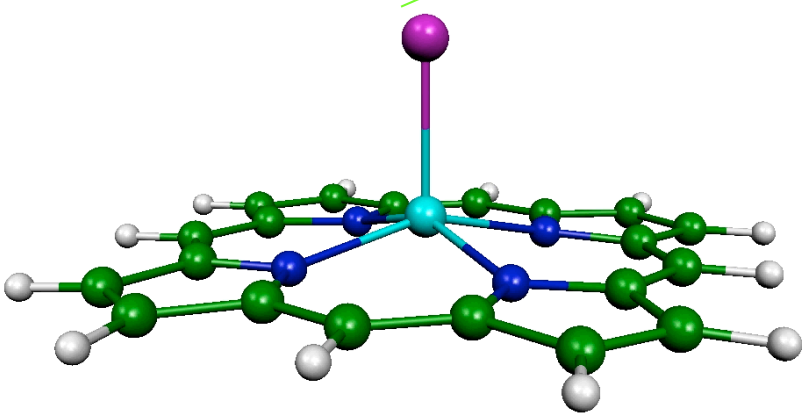
twisting

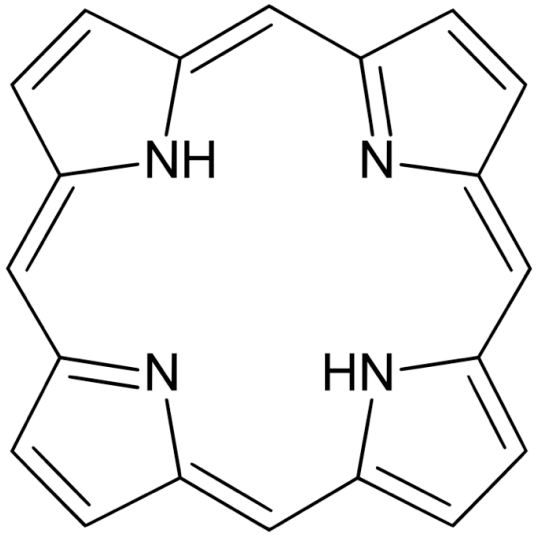







vib. density of states (cm)
0.12
0.1
0.08
0.06
0.04
0.02
0
Fe(P)(Br)
Fe(P)(Cl)
Fe-N$_{pyr}$
Fe-Br
Fe-Cl
acoustic
modes
doming
0
100
200
300
400
$E - E_0/hc$ (cm^{-1})




NH
N
N
HN

PHONONS (cont'd)

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

Energy of a single oscillation as a function of number of phonons. The second term +1/2 is the "zero-point" energy.

$$E = \sum_{\mathbf{k},\lambda} \omega^2_{\mathbf{k},\lambda} \left|Q\left(\mathbf{k},\lambda\right)\right|^2 = \sum_{\mathbf{k},\lambda} \left(n_{\mathbf{k},\lambda} + \frac{1}{2}\right)\hbar\omega_{\mathbf{k},\lambda}.$$

Total energy, in terms of normal mode coordinates

$$\left\langle n(\omega_{\mathbf{k},\lambda})\right\rangle = \frac{1}{\exp(\hbar\omega_{\mathbf{k},\lambda}/k_{\mathrm{B}}T) - 1}$$

Bose-Einstein statistics for average number of modes at a given temperature

$$\mathcal{H} = \frac{1}{2}\sum_{j,\ell} m_j \left|\dot{\mathbf{u}}_{j\ell}\right|^2 + \frac{1}{2}\sum_{\substack{j,j'\\ \ell,\ell'}} \mathbf{u}^T_{j\ell} \cdot \Phi^{j,j'}_{\ell,\ell'} \cdot \mathbf{u}_{j'\ell'}$$

Hamiltonian of the system:

$\mathcal{H}$=Kin. En. + Pot. En

Phonon density of states

Many thermodynamic functions like free energy, specific heat, and entropy are additive functions of phonon density of states.

This stems from the notion that the normal modes do not interact in the harmonic approximation.

Phonon density of states is the number of modes in a unit energy interval.

$$c_v(T) = 3Nk \int \frac{\hbar^2 \omega^2 e^{\hbar\omega/kT}}{(kT)^2 (1 - e^{\hbar\omega/kT})^2} \cdot g(\omega) \cdot d\omega$$

Vibrational specific heat

Phonon density of states is a key ingredient for many thermodynamic properties

If we choose to write in terms of energy, $E = \hbar\omega, \quad \beta = 1/k_B T$

$$c_v(T) = 3k_B \int (\beta E/2)^2 \operatorname{csc}h(\beta E) \cdot g(E) \cdot dE$$

Vibrational specific heat

$$S_v(T) = 3k_B \int_0^\infty \{\beta E/2 \bullet \cot h(\beta E) - \ln[2\sin h(\beta E)]\} \cdot g(E) \cdot dE$$

Vibrational entropy

$$f_{LM} = e^{-E_R \int \{g(E)/2\}.\coth(\beta E/2)} dE$$

Lamb-Mössbauer factor

$$g(E) = \frac{3m}{2\pi^2 \hbar^3 \rho v_D^3} E^2$$

Debye Sound velocity

$$\langle F \rangle = \frac{M}{\hbar^2} \int_0^\infty E^2 g(E) dE$$

Average restoring force constant

And, some thermodynamics

$$\mathcal{Z} = \frac{1}{1 - \exp(-\beta\hbar\omega)}$$

Partition function

$$F = -k_{\mathrm{B}} T \ln \mathcal{Z}$$

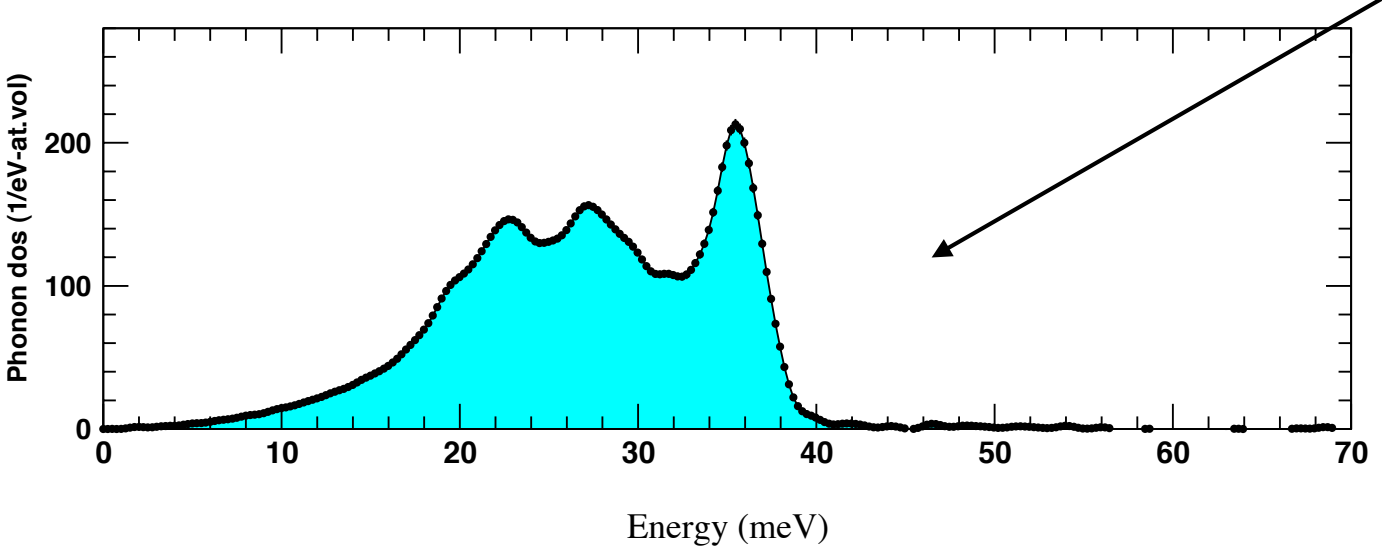
Free energy

$$C = -T \frac{\partial^2 F}{\partial T^2}$$

Heat capacity

$$E = \sum_{\mathbf{k},\lambda} \left(\langle n(\omega_{\mathbf{k},\lambda}) \rangle + \frac{1}{2} \right) \hbar\omega_{\mathbf{k},\lambda} \equiv \int \left(\langle n(\omega) \rangle + \frac{1}{2} \right) \hbar\omega \, g(\omega) \, \mathrm{d}\omega.$$

Energy in terms of
phonon density of states

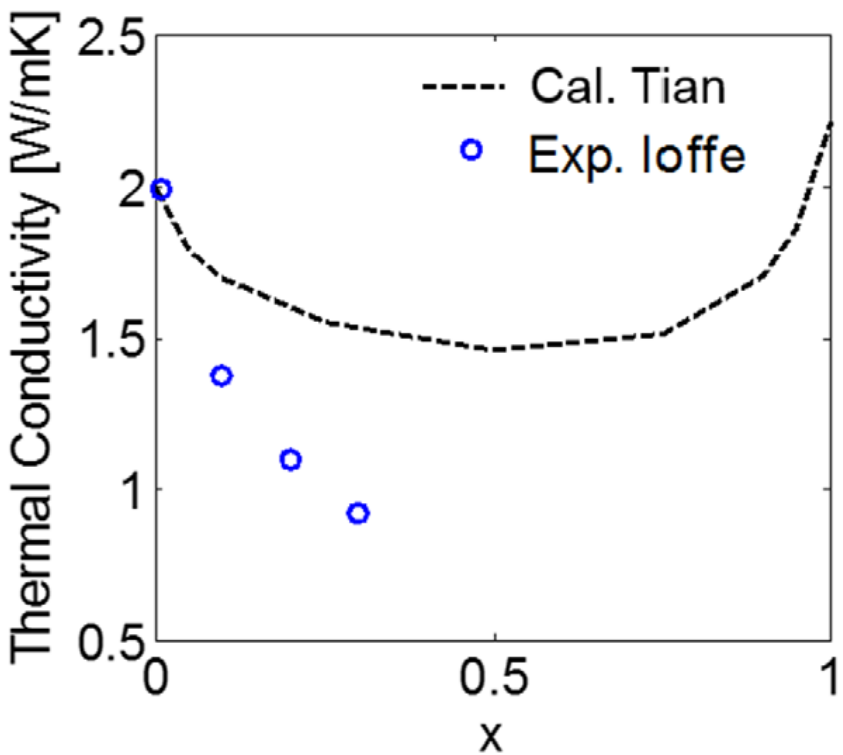


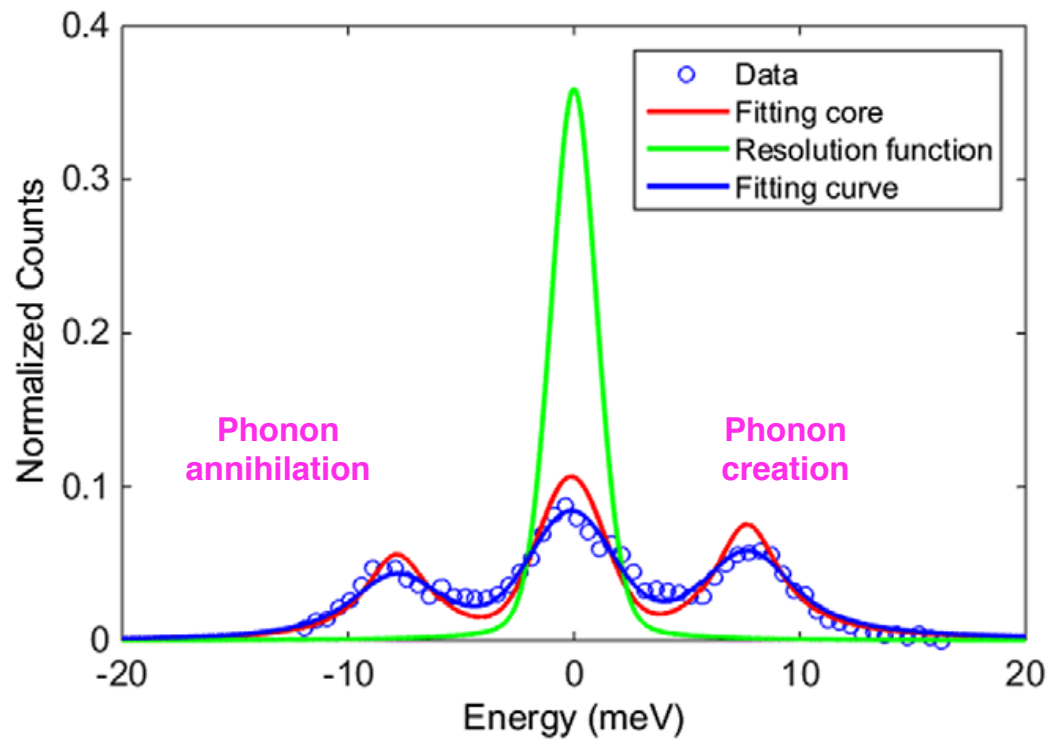
Inelastic x-ray scattering measurements of phonon dispersion and lifetimes in $PbTe_{1-x}Se_x$ alloys:

Do phonons remain a good representation of the vibrational modes in $PbTe_{1-x}Se_x$ alloys when alloying seems to decrease thermal conductivity?

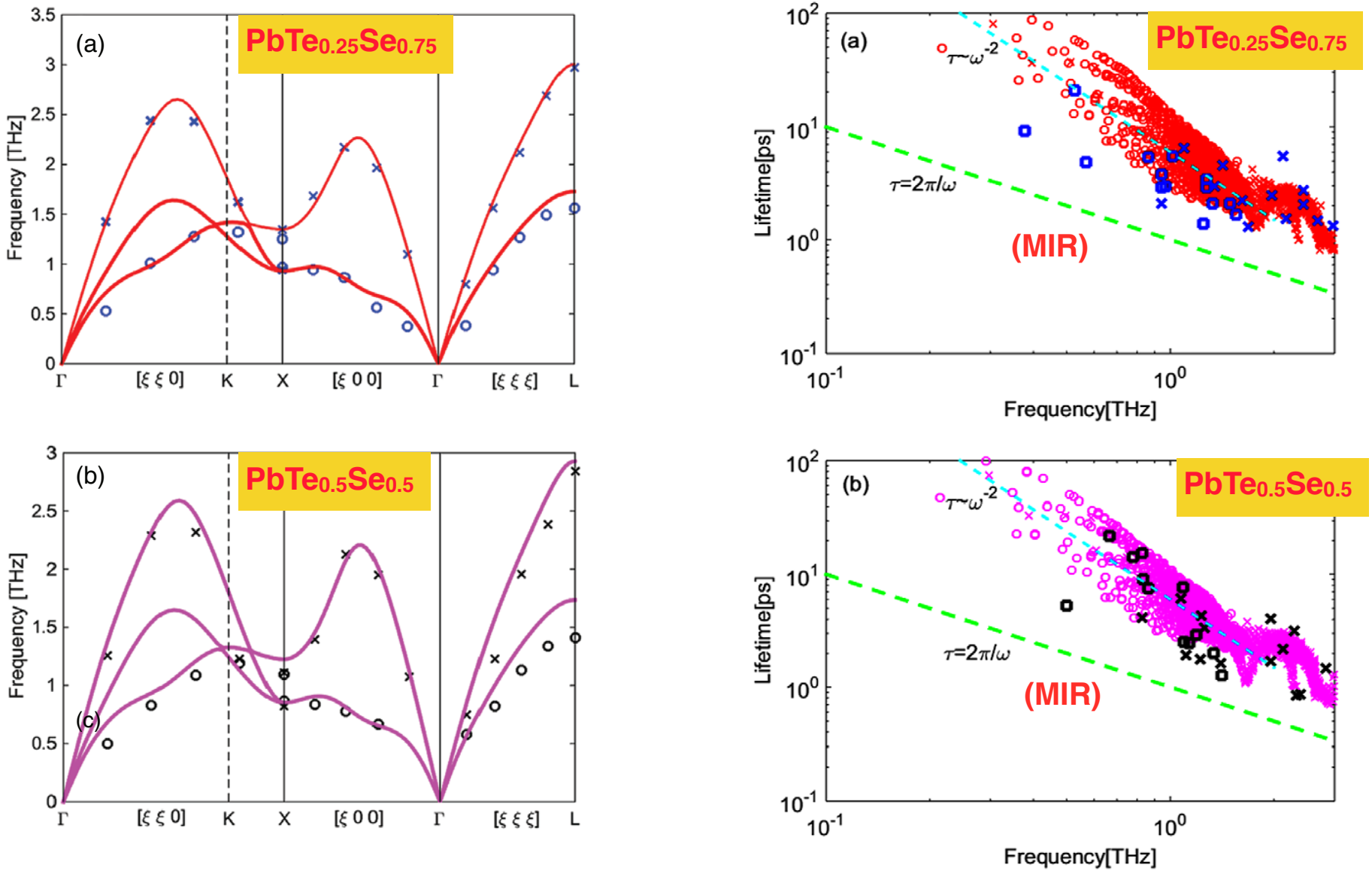
$ZT = S^2\sigma T/\kappa$ where S, σ, κ, T represent the Seebeck coefficient, electrical conductivity, thermal conductivity and absolute temperature. For thermoelectric materials, higher ZT is desirable, hence lowering κ while keeping σ is required.

Phonon lifetime measurements is one way to learn why thermal conductivity is lowered upon alloying.





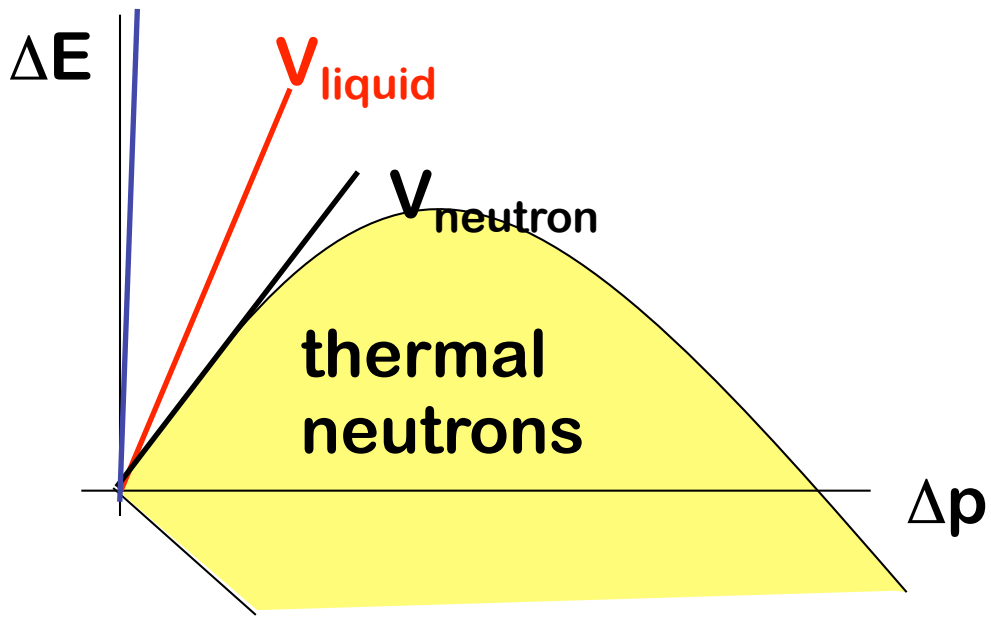
Z. Tia, Mingda Li, Z. Ren, H. Ma, A. Alatas, S. D Wilson and J. Li, **J. Phys.: Condens. Matter 27 (2015) 375403**



Mott-Ioffe-Regel limit (MIR) : When electron scattering rate is comparable to Fermi energy, E_F/h, thermal resistivity reaches its maximum. The discrepancy between virtual crystal approximation and measured thermal conductivity can be attributed to mass disorder as well as force constant variations.

The experimental data on phonon lifetimes provide a bench mark for theoretical work to directly compare lifetimes and advance our understanding of the thermal transport in alloys.

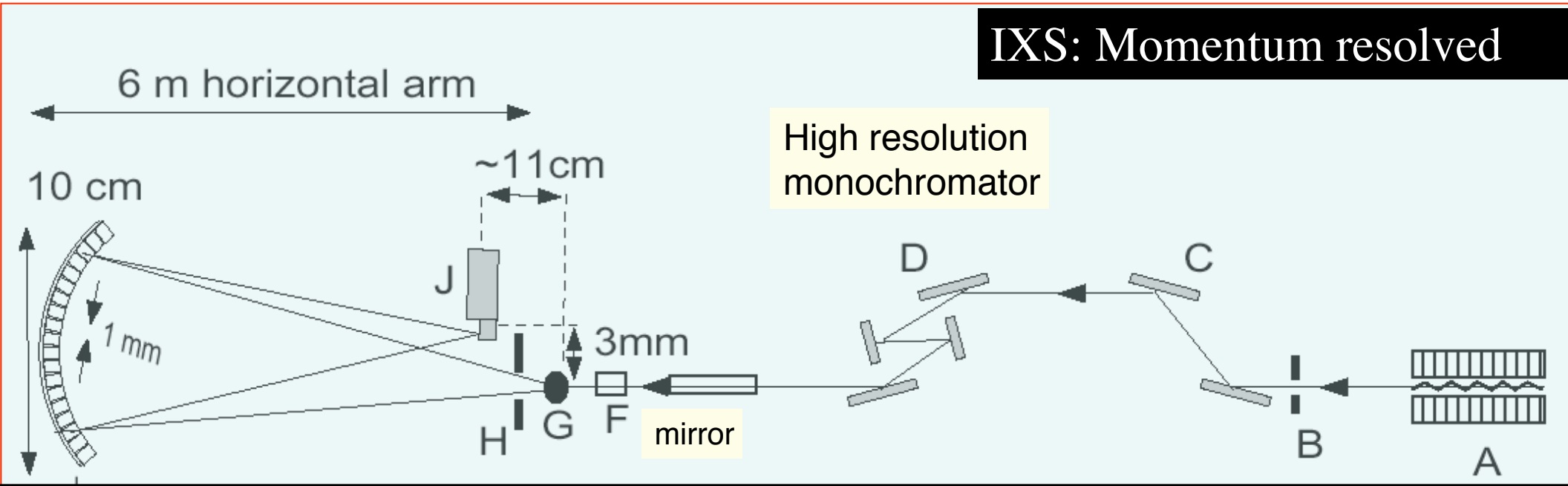
Why x-rays instead of neutrons or visible light ?

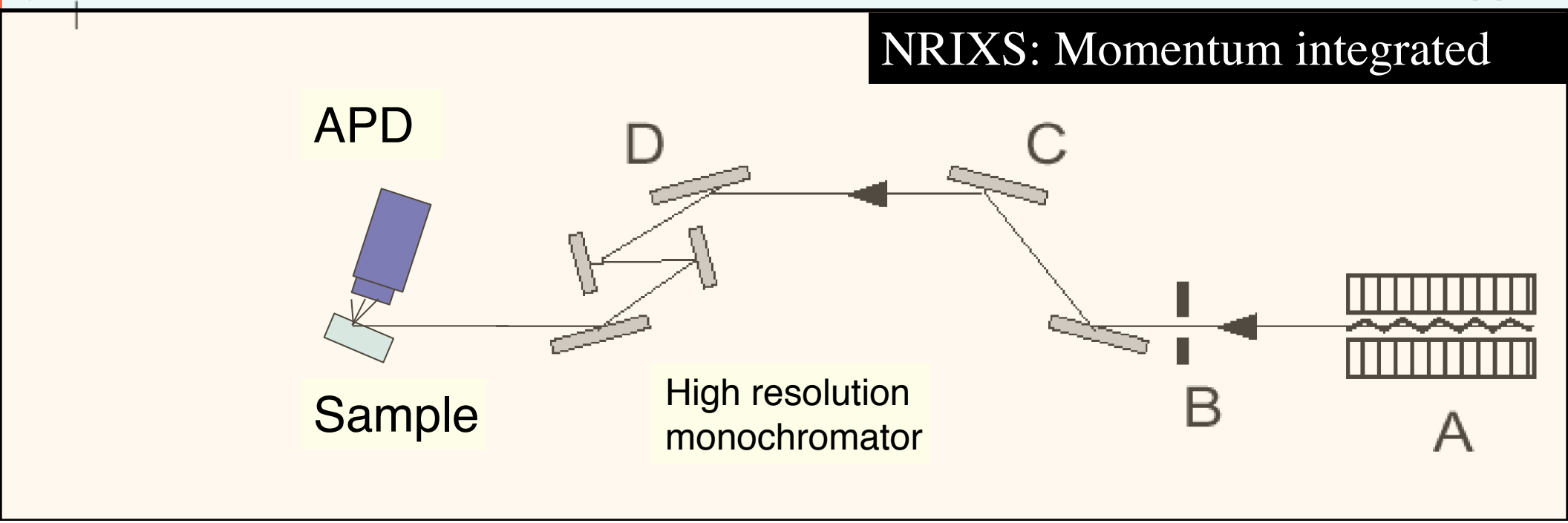


Limited momentum transfer capability of neutrons at low energies favor x-rays to study collective excitations with large dispersion, like sound modes.

When the sound velocity exceeds that of neutrons in the liquid, x-rays become unique. The low-momentum/high-energy transfer region is only accessible by x-rays.

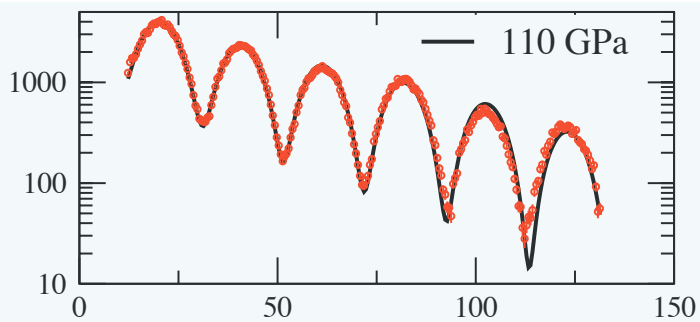
Inelastic X-Ray Scattering: two approaches



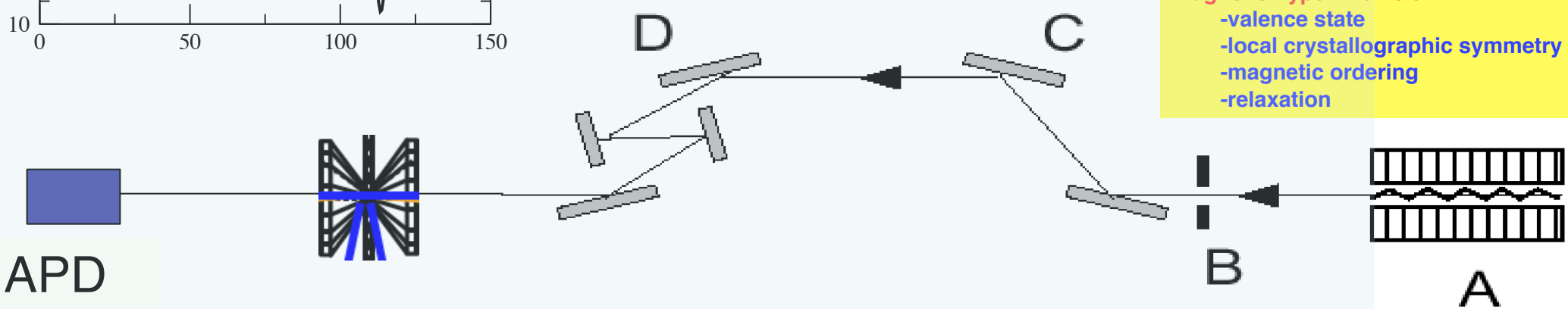


Nuclear Resonant Scattering

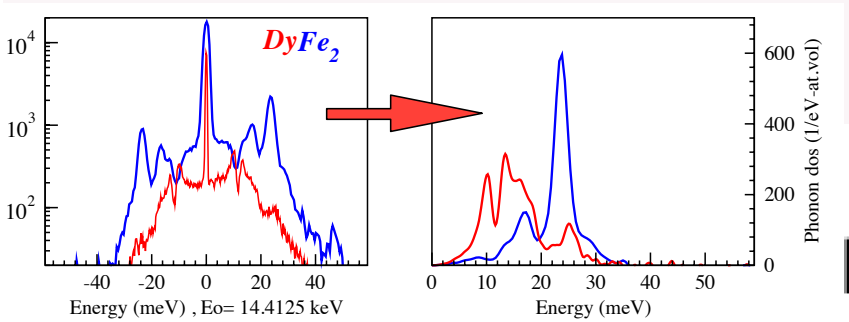
SMS: Synchrotron Mössbauer Spectroscopy
NFS : Nuclear Forward Scattering



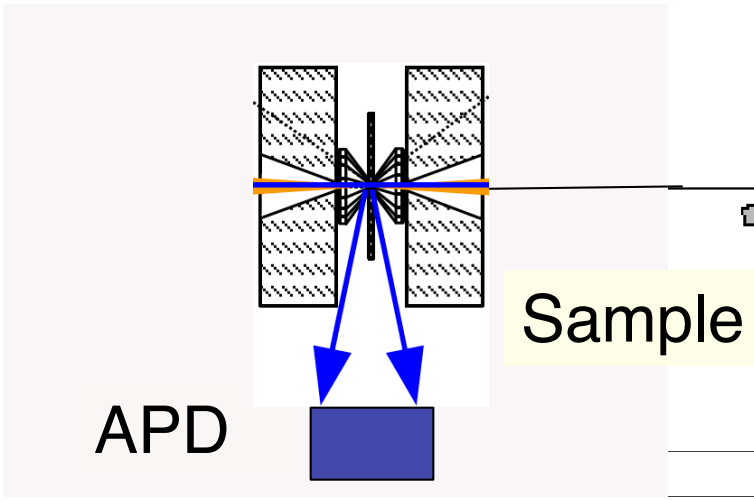
Isomer shift
Quadrupole splitting
Magnetic hyperfine field
- -valence state
- -local crystallographic symmetry
- -magnetic ordering
- -relaxation

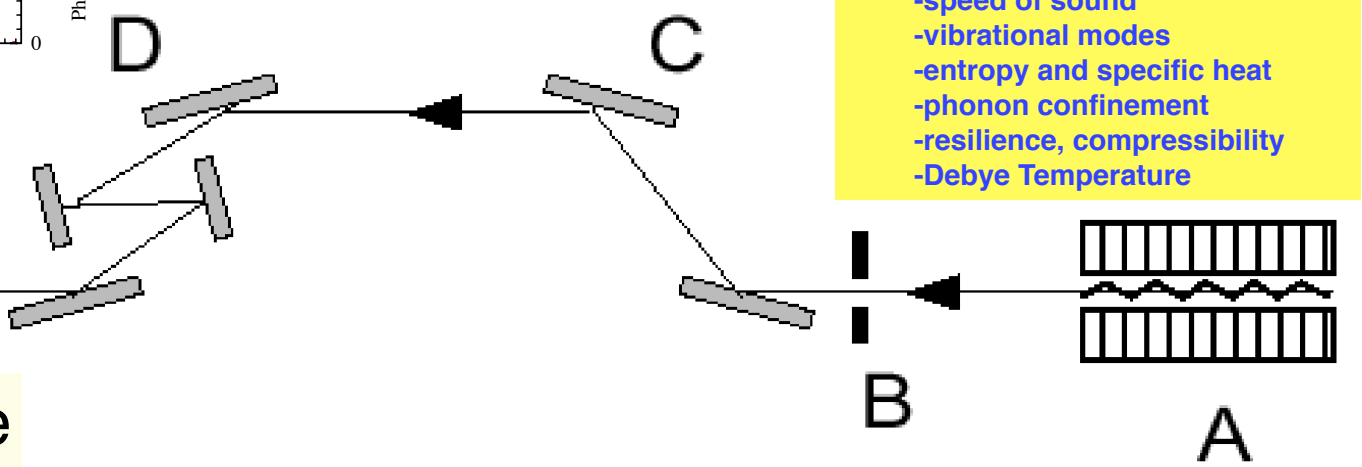


NRIXS: Nuclear Resonant Inelastic X-ray Scattering
NRVS: Nuclear Resonant Vibrational Spectroscopy



Partial phonon density of states
Recoil-free fraction
- -speed of sound
- -vibrational modes
- -entropy and specific heat
- -phonon confinement
- -resilience, compressibility
- -Debye Temperature





Kernresonanzabsorption von γ-Strahlung in Ir^{191}

Von Rudolf L. Mössbauer

Aus dem Laboratorium für technische Physik der Technischen Hochschule in München und dem Institut für Physik im Max-Planck-Institut für medizinische Forschung in Heidelberg

(Z. Naturforschg. 14 a, 211—216 [1959]; eingegangen am 5. November 1958)

Bei der Emission und Selbstabsorption von weicher γ-Strahlung in Kernen treten bei tiefen Temperaturen in Festkörpern sehr starke Linien mit der natürlichen Linienbreite auf. Diese Linien erscheinen als Folge davon, daß bei tiefen Temperaturen bei einem Teil der Quantenübergänge der γ-Rückstoßimpuls nicht mehr vom einzelnen Kern aufgenommen wird, sondern von dem Kristall als Ganzes. Da die scharfen Emissions- und Absorptionslinien energetisch an der gleichen Stelle liegen, tritt ein sehr starker Resonanzfluoreszenzeffekt auf. Durch eine „Zentrifugen"-Methode, bei der die Emissions- und Absorptionslinien gegeneinander verschoben werden, läßt sich der Fluoreszenzeffekt unterdrücken und so eine unmittelbare Bestimmung der natürlichen Linienbreite von Resonanzlinien vornehmen. Erste Messungen nach dieser Methode ergeben für die Lebenszeit τ des 129 keV-Niveaus in Ir^{191}: $\tau = \left(1{,}4\ ^{+0{,}2}_{-0{,}1}\right) \cdot 10^{-10}$ sec.

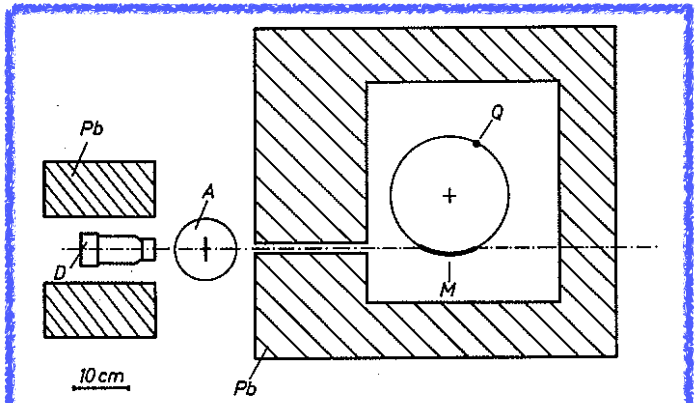
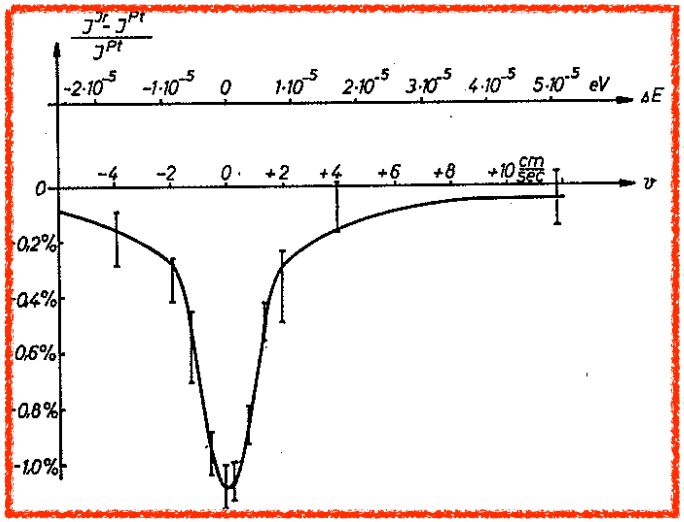
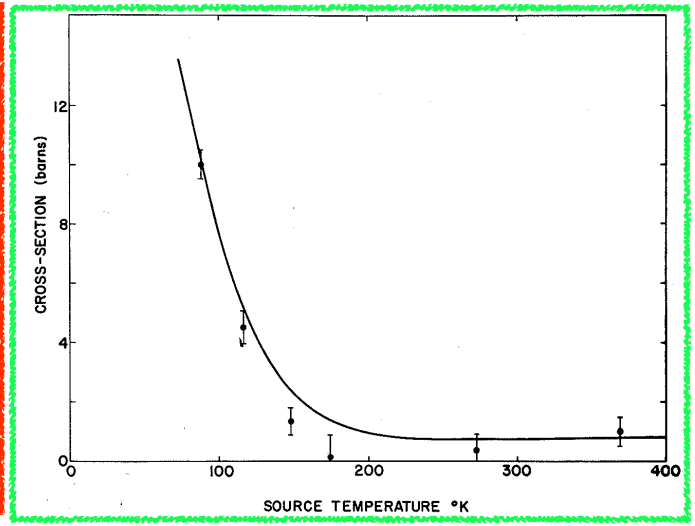
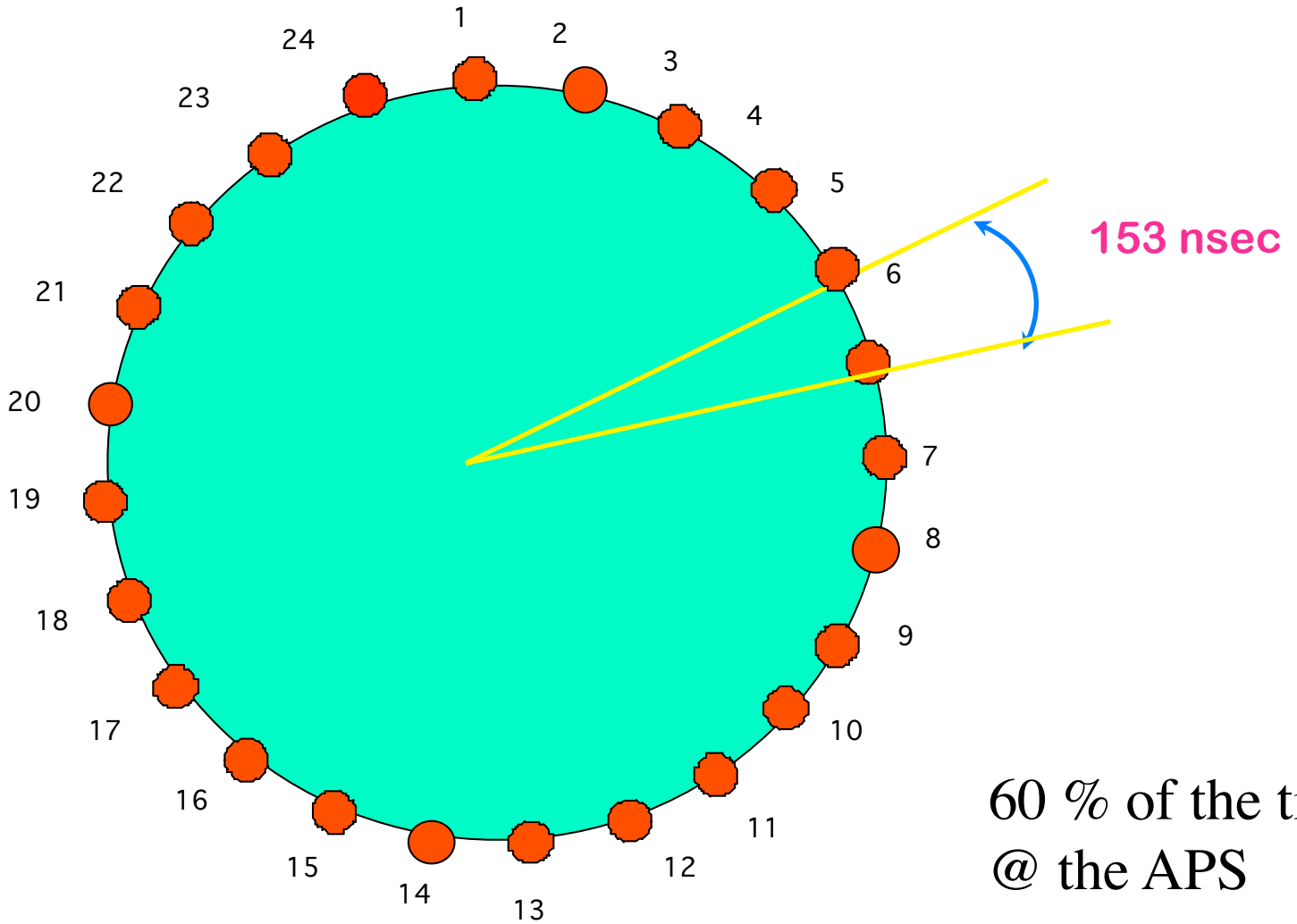


Abb. 2. Versuchsgeometrie. A Absorber-Kryostat; Q rotierender Kryostat mit Quelle; D Szintillationsdetektor. M ist der bei der Messung ausgenützte Teil des Rotationskreises der Quelle.





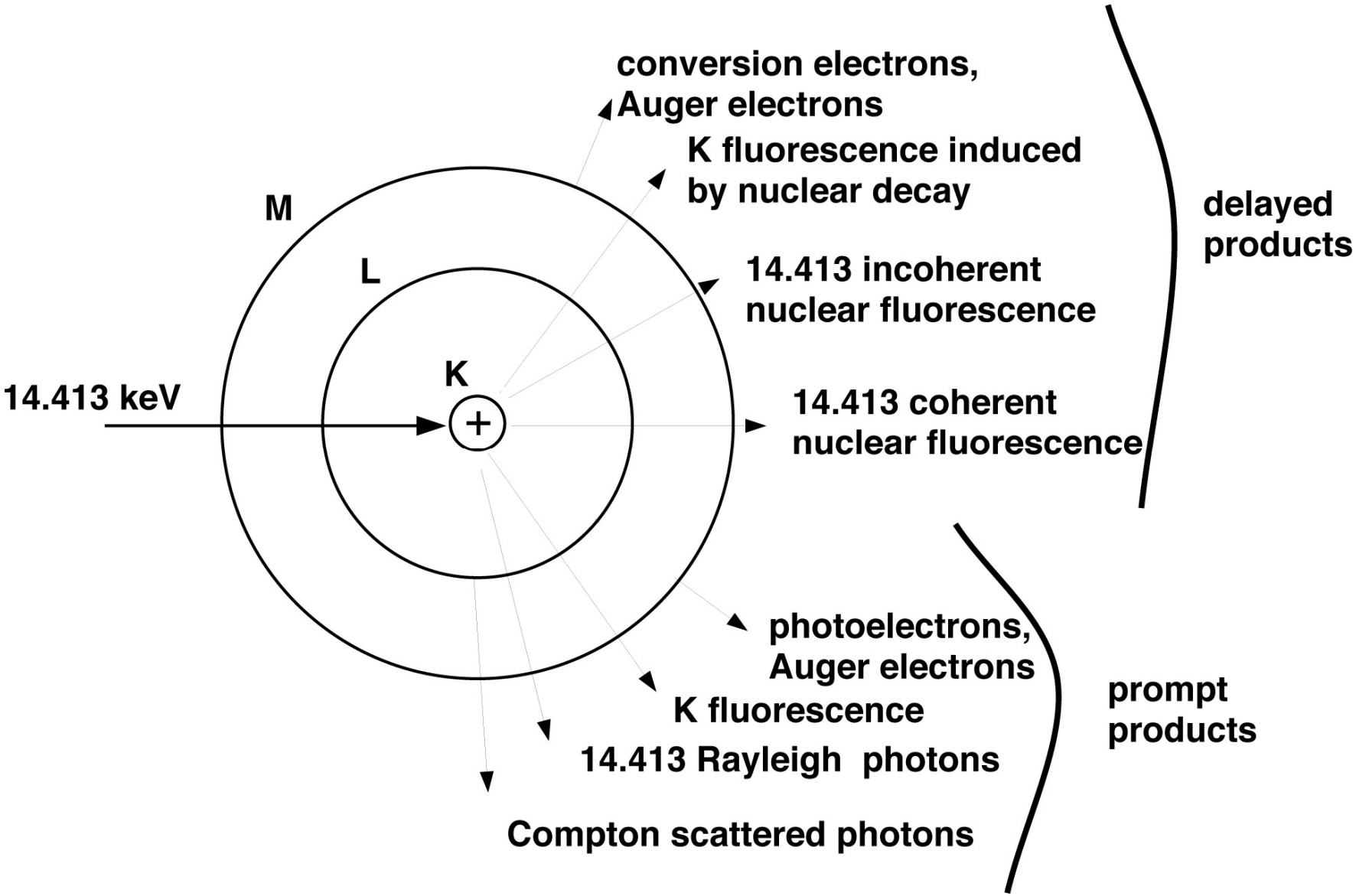
Standard Time structure @ APS



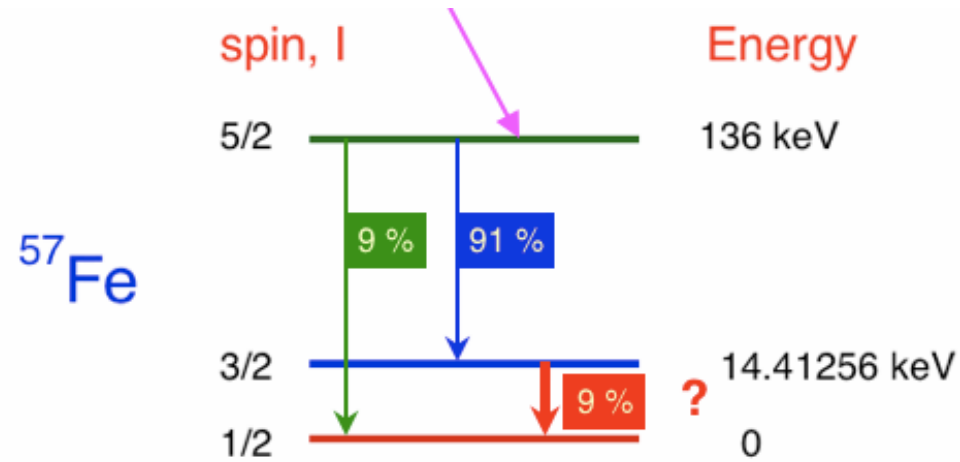
60 % of the time
@ the APS

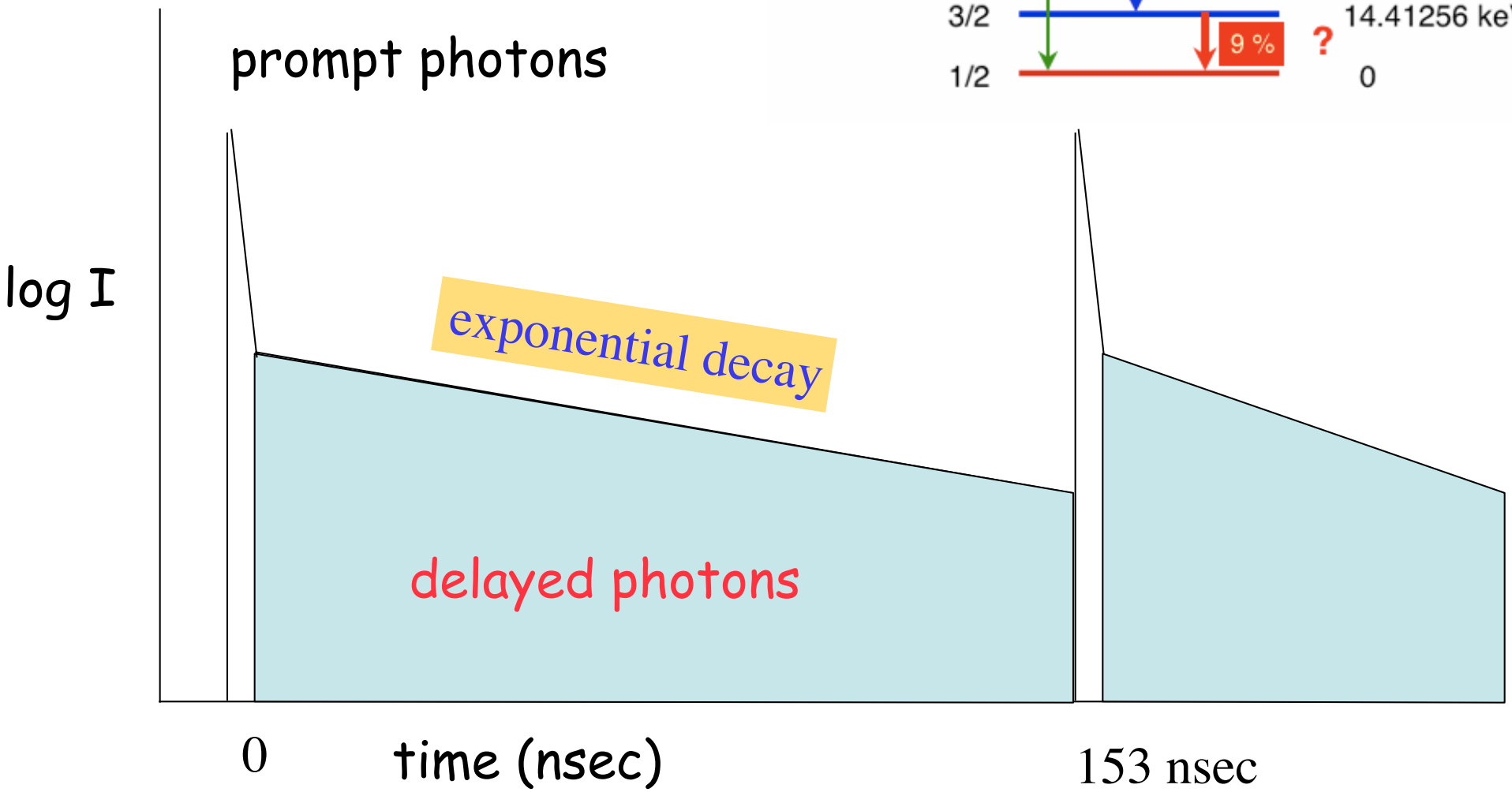
1 revolution=3.68 μsec =>1296 buckets

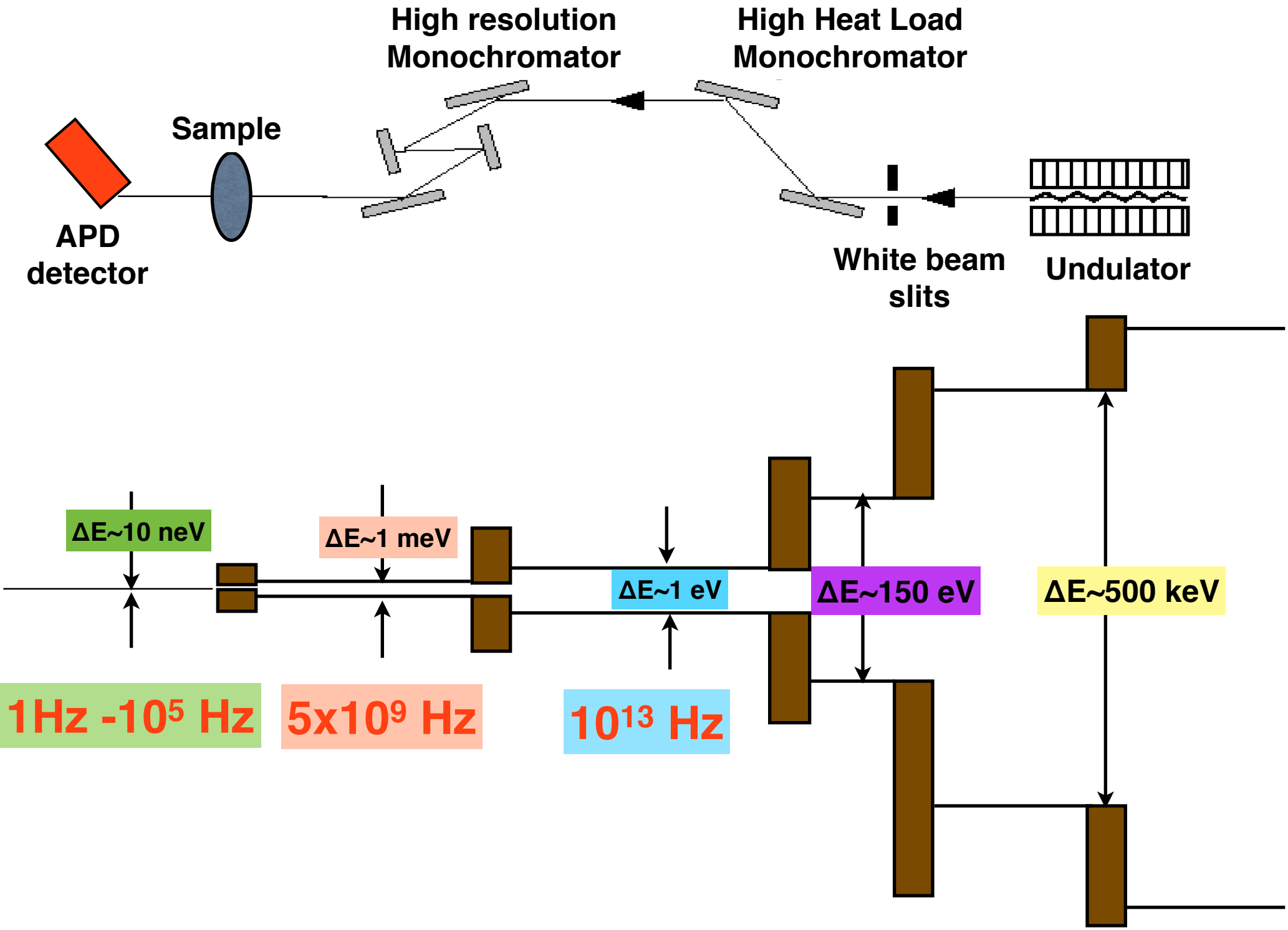
Nuclear Resonance and Fallout in ^{57}Fe-decay



Detection of nuclear decay



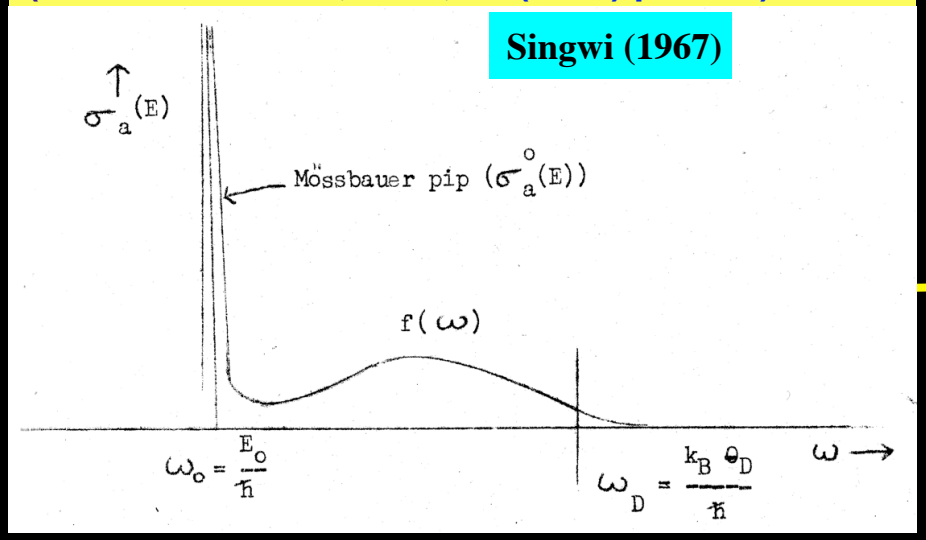



High resolution Monochromator
High Heat Load Monochromator
Sample
APD detector
White beam slits
Undulator
ΔE~10 neV
ΔE~1 meV
ΔE~1 eV
ΔE~150 eV
ΔE~500 keV
1Hz -10^5 Hz
5x10^9 Hz
10^13 Hz

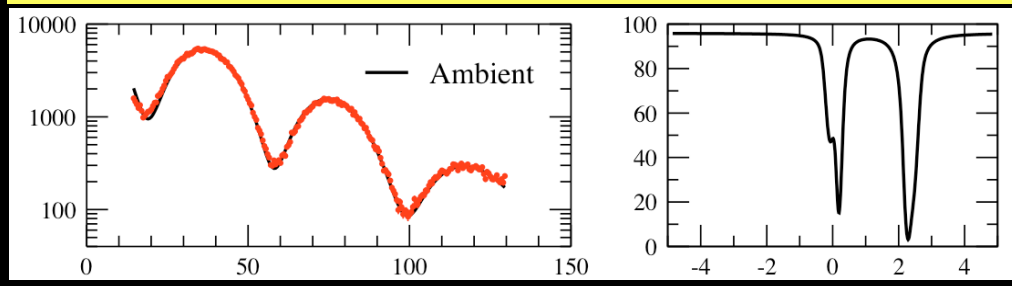
Early dreamers..

Zero phonon "Mössbauer pip"

Visualized by W. Vischer at Los Alamos & K. S. Singwi at Argonne (1960), but only properly observed after synchrotron radiation based tunable monochromators are realized.
(W. Sturhahn et al, PRL , 74 (1995) p. 3832)



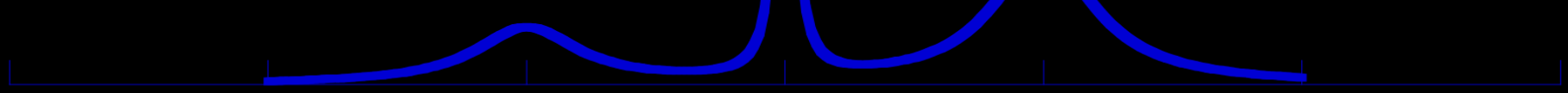
Visualized by Stan Ruby (1974) at Argonne and properly observed by E. Gerdau (Hamburg).
(S. L Ruby. J. de Physique 35 (1974) C6-209



5 neV + mono. resolution

Phonon annihilation

Phonon creation



- 0 +

Energy (~ meV)

VOLUME 74, NUMBER 19 PHYSICAL REVIEW LETTERS 8 MAY 1995

Phonon Density of States Measured by Inelastic Nuclear Resonant Scattering

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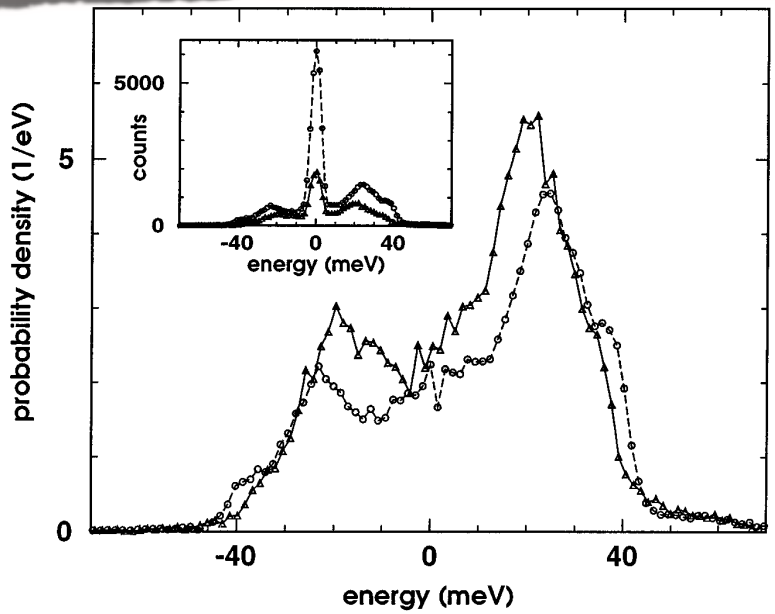
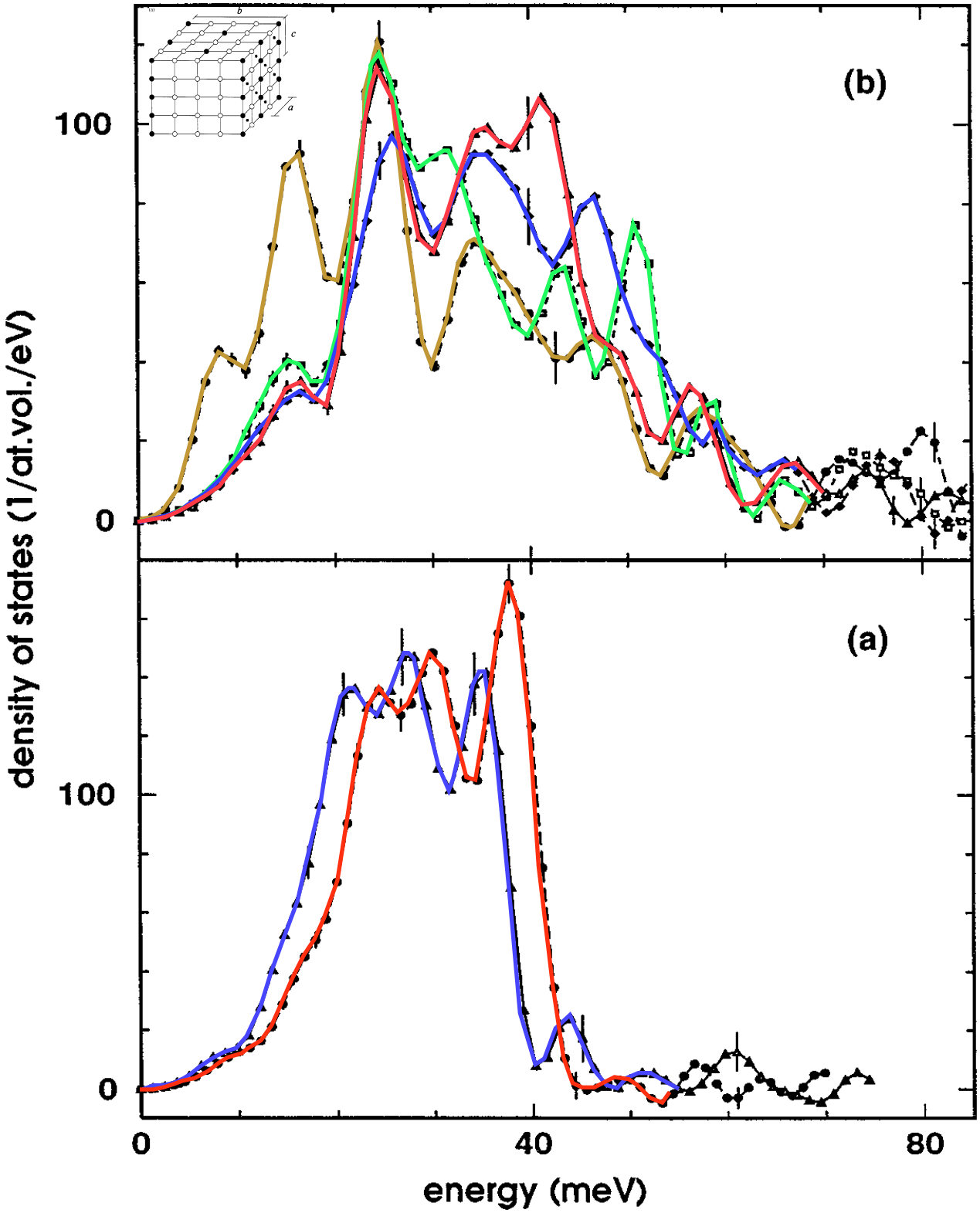
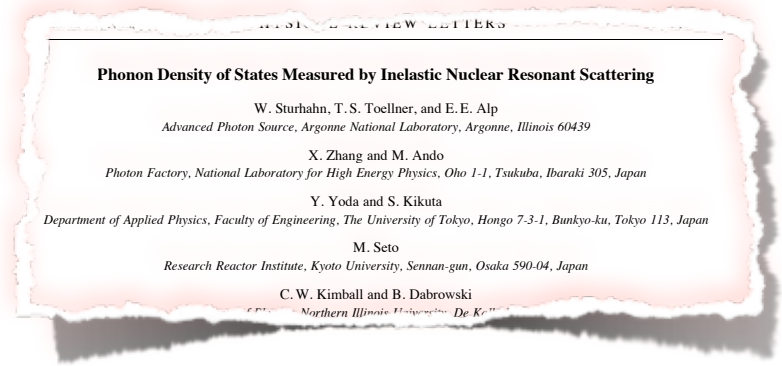
C. W. Kimball and B. Dabrowski
Department of Physics, Northern Illinois University, De Kalb, Illinois 60115
(Received 16 December 1994)



> 400 citations





Phonon Density of States Measured by Inelastic Nuclear Resonant Scattering

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C. W. Kimball and B. Dabrowski

$Sr_2Fe_2O_5$ **Brownmillerite**

$Sr_2Fe_2O_{5.5}$ **Orthorombic perovskite**

$Sr_2Fe_2O_{5.75}$ **Tetragonal perovskite**

$Sr_2Fe_2O_6$ **Cubic perovskite**

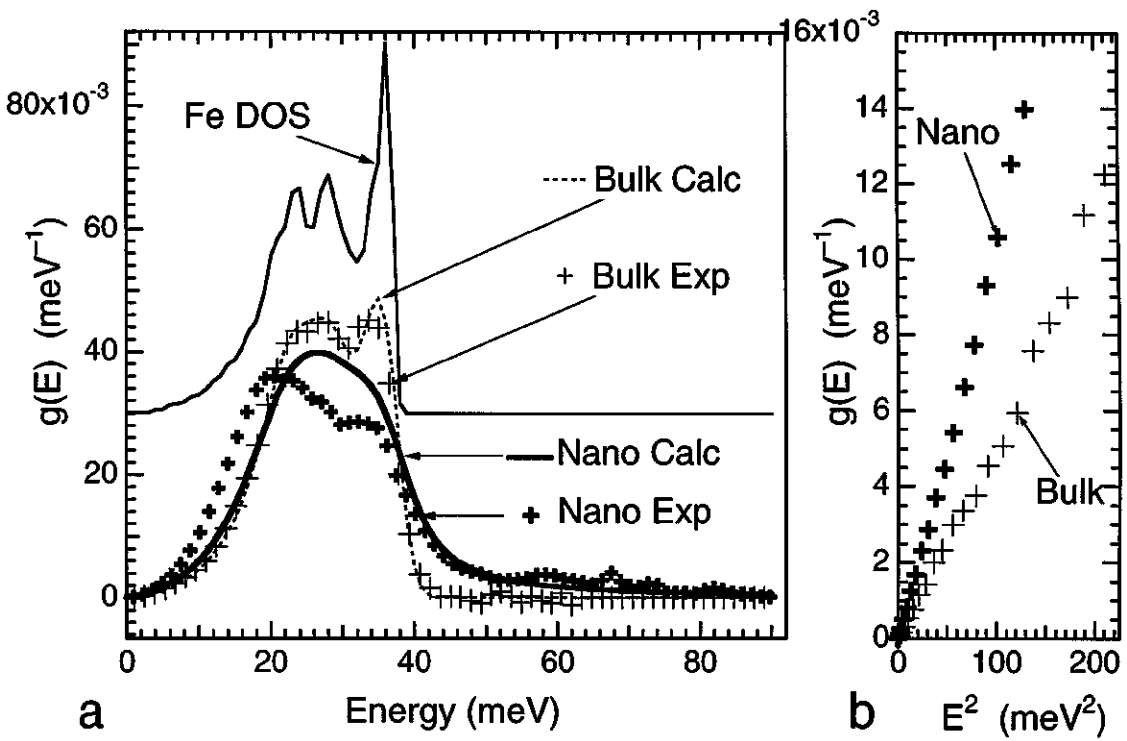
bcc-iron

stainless steel, 304

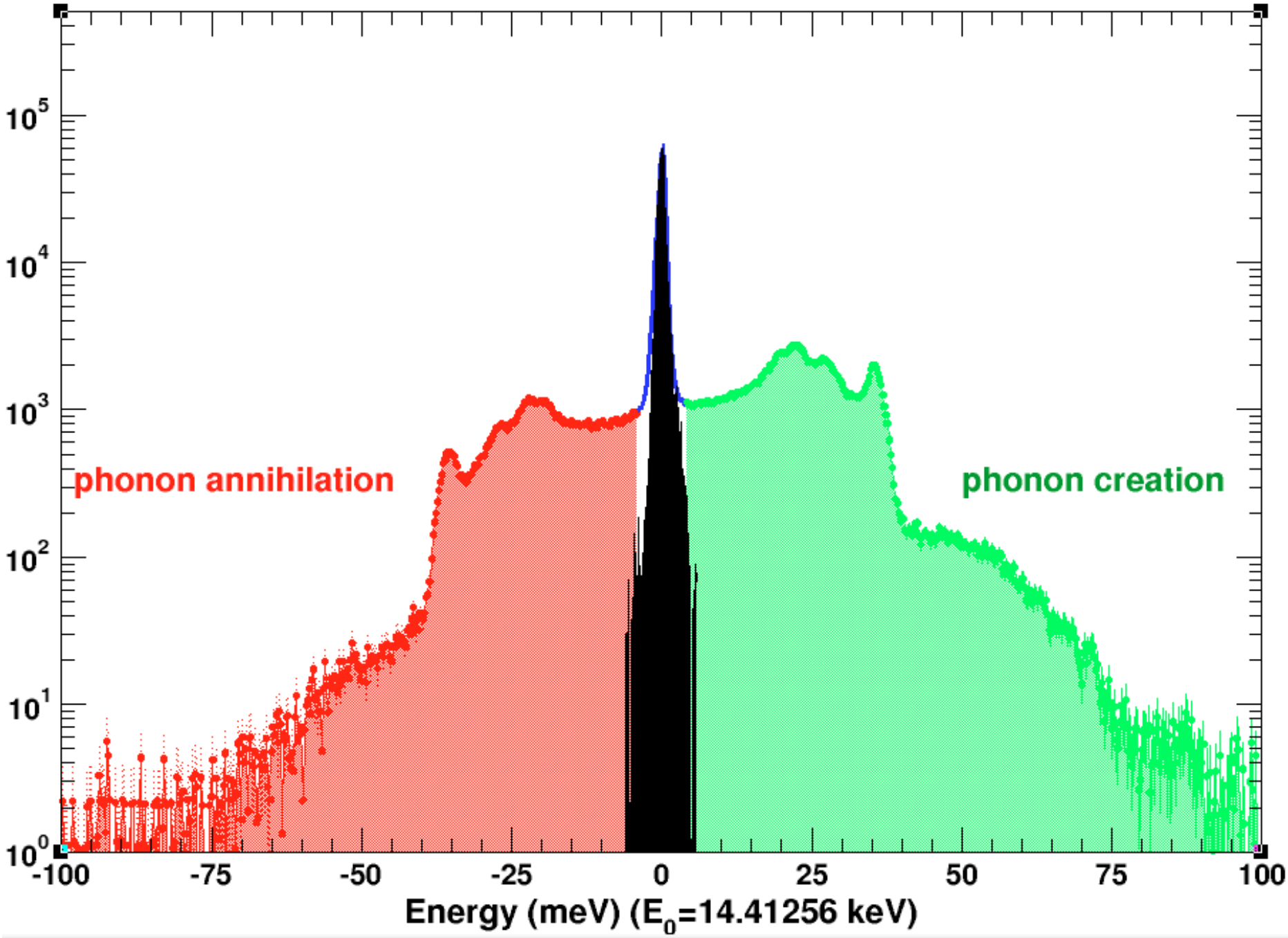
Phonons in Nanocrystalline ^{57}Fe

B. Fultz,[1] C.C. Ahn,[1] E.E. Alp,[2] W. Sturhahn,[2] and T.S. Toellner[2]
[1]*Division of Engineering and Applied Science, 138-78, California Institute of Technology, Pasadena, California 91125*
[2]*Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439*
(Received 13 March 1997)

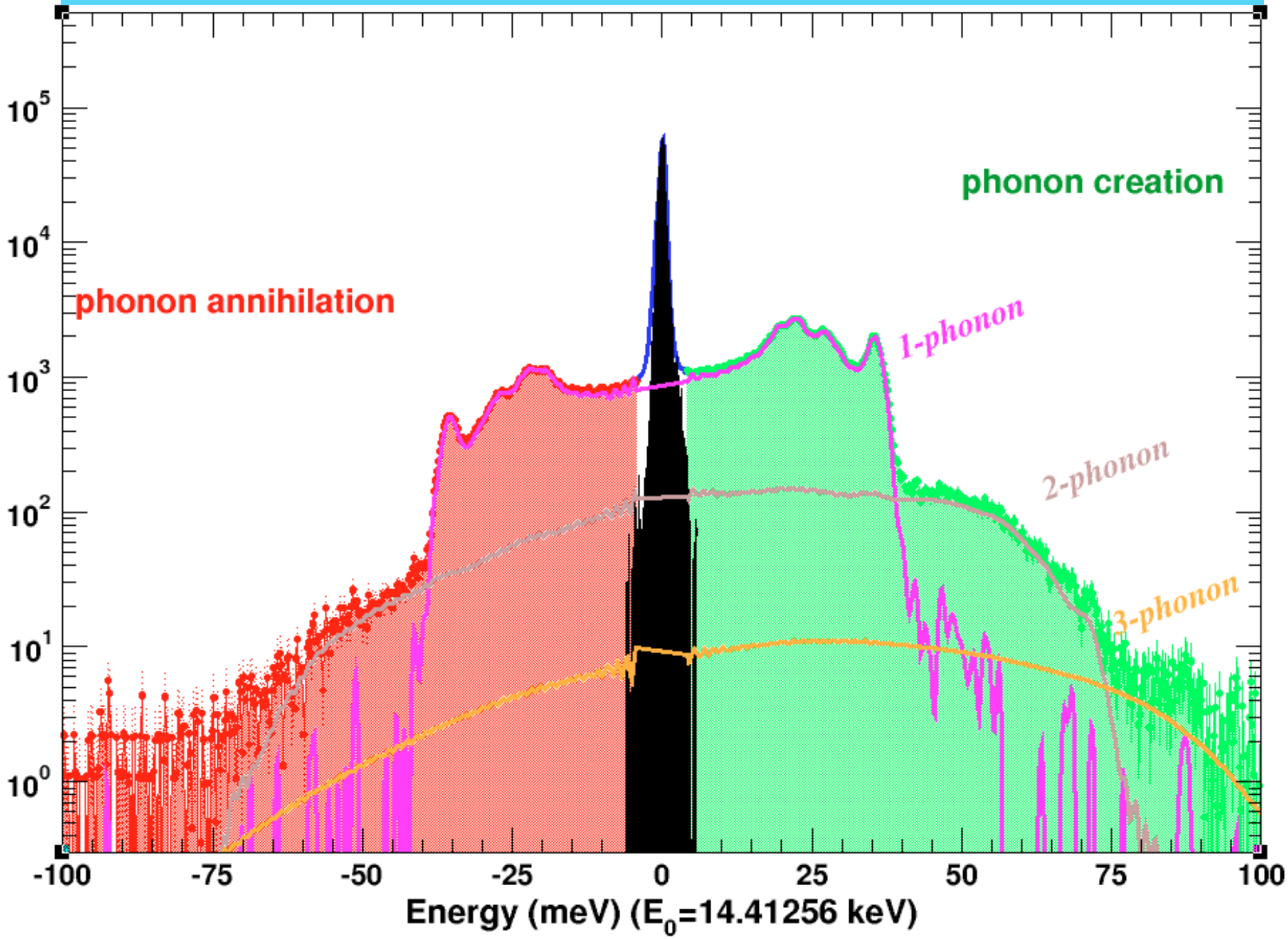
We measured the phonon density of states (DOS) of nanocrystalline Fe by resonant inelastic nuclear γ-ray scattering. The nanophase material shows large distortions in its phonon DOS. We attribute the high energy distortion to lifetime broadening. A damped harmonic oscillator model for the phonons provides a low quality factor, Q_u, averaging about 5, but the longitudinal modes may have been broadened most. The nanocrystalline Fe also shows an enhancement in its phonon DOS at energies below 15 meV. The difference in vibrational entropy of the bulk and nanocrystalline Fe was small, owing to competing changes in the nanocrystalline phonon DOS at low and high energies.



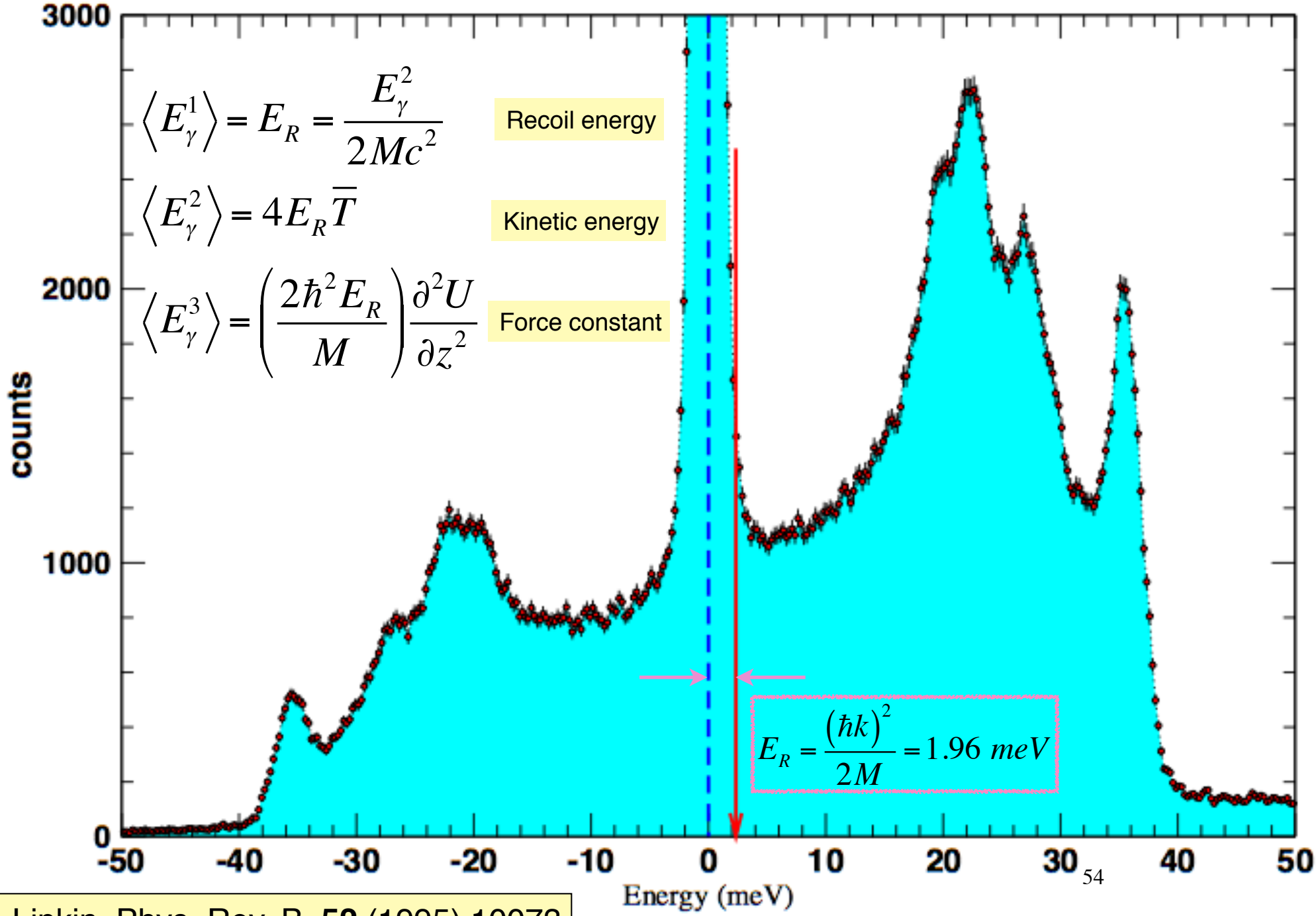
Phonon excitation probability



Multi-phonon decomposition



Lipkin's sum rules related to phonon excitation probability

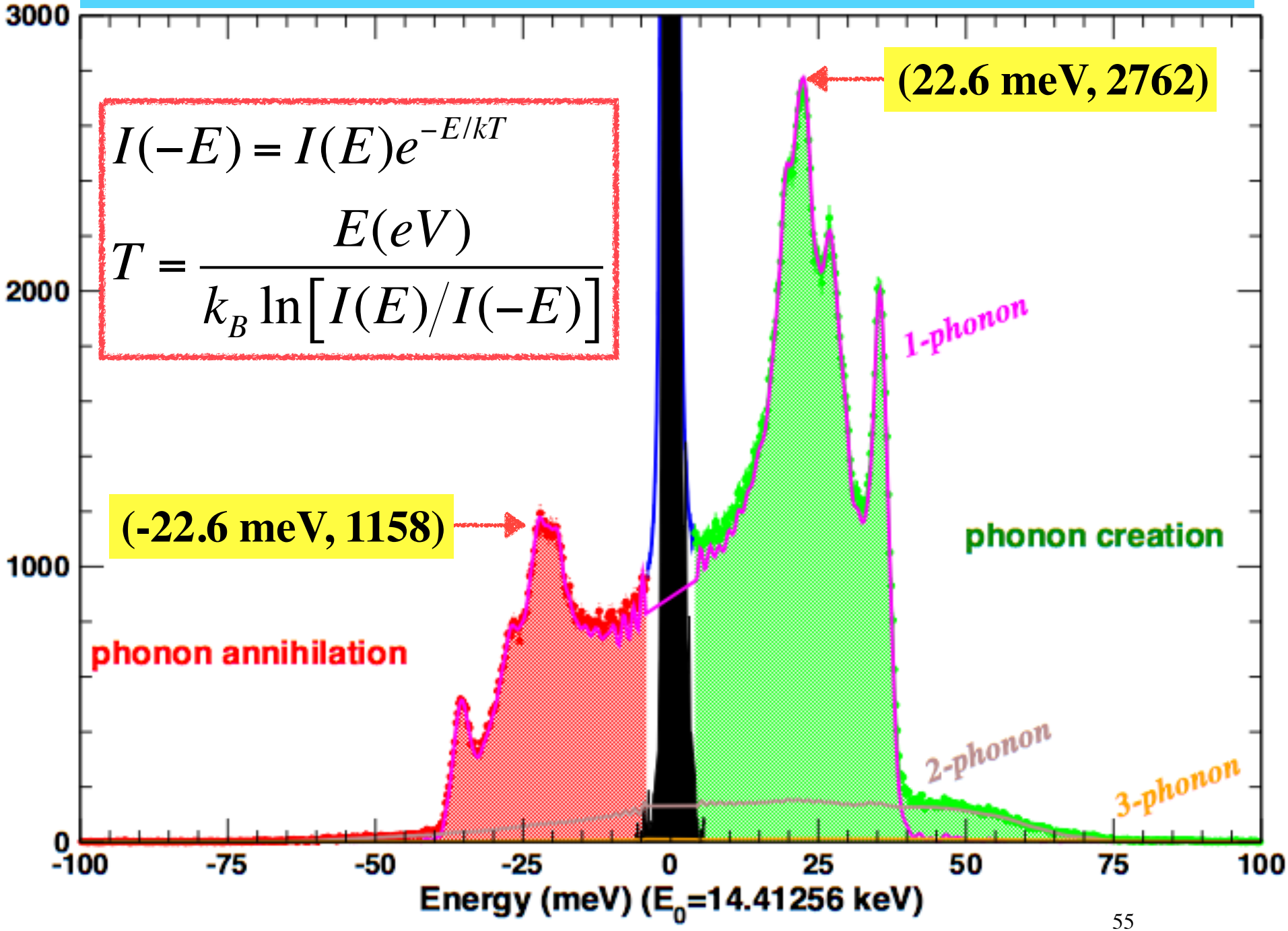


H. Lipkin, Phys. Rev. B, **52** (1995) 10073

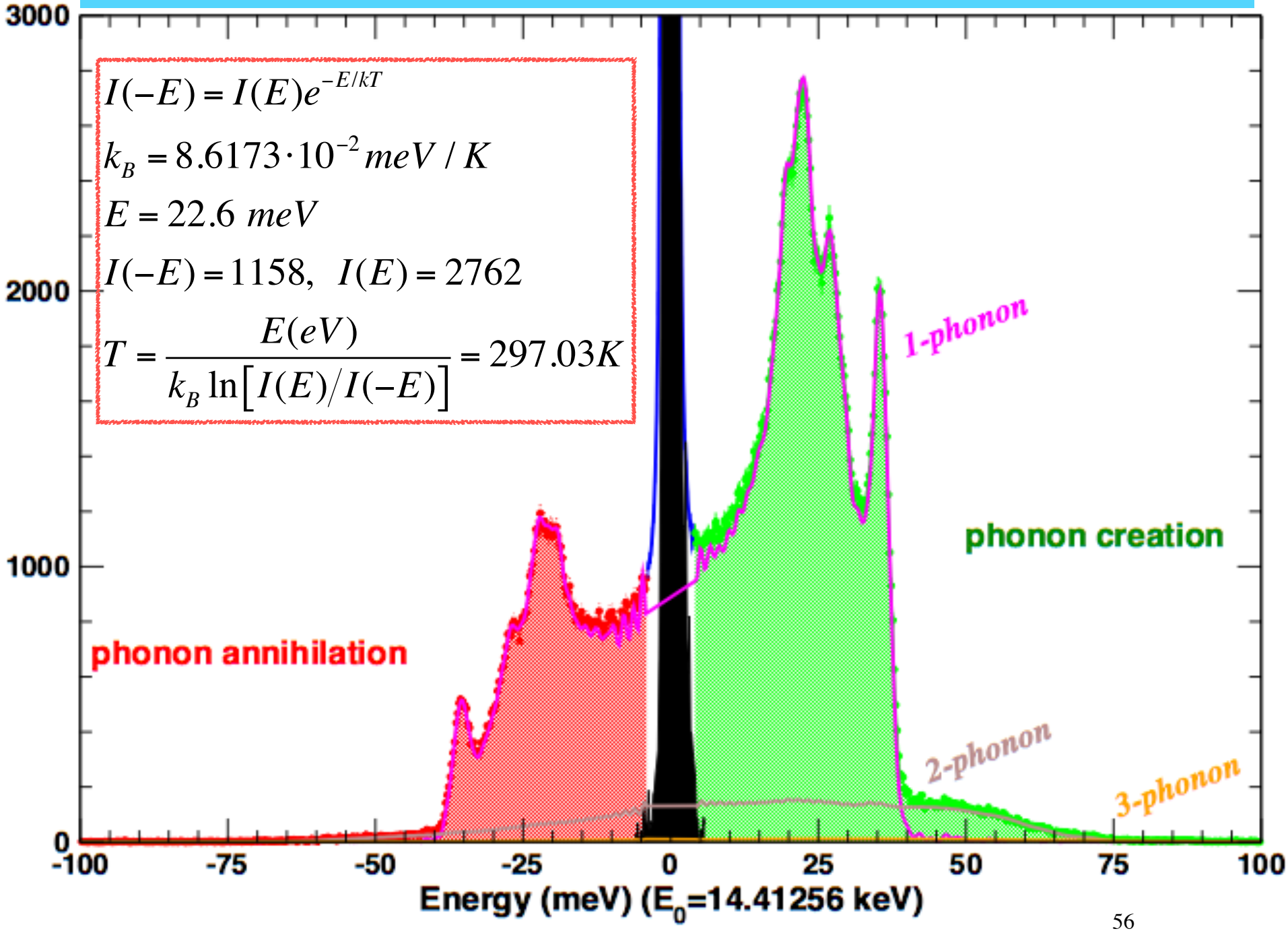
Detailed Balance

$$I(-E) = I(E)e^{-E/kT}$$

$$T = \frac{E(eV)}{k_B \ln\left[I(E)/I(-E)\right]}$$



Detailed Balance



$$I(-E) = I(E)e^{-E/kT}$$

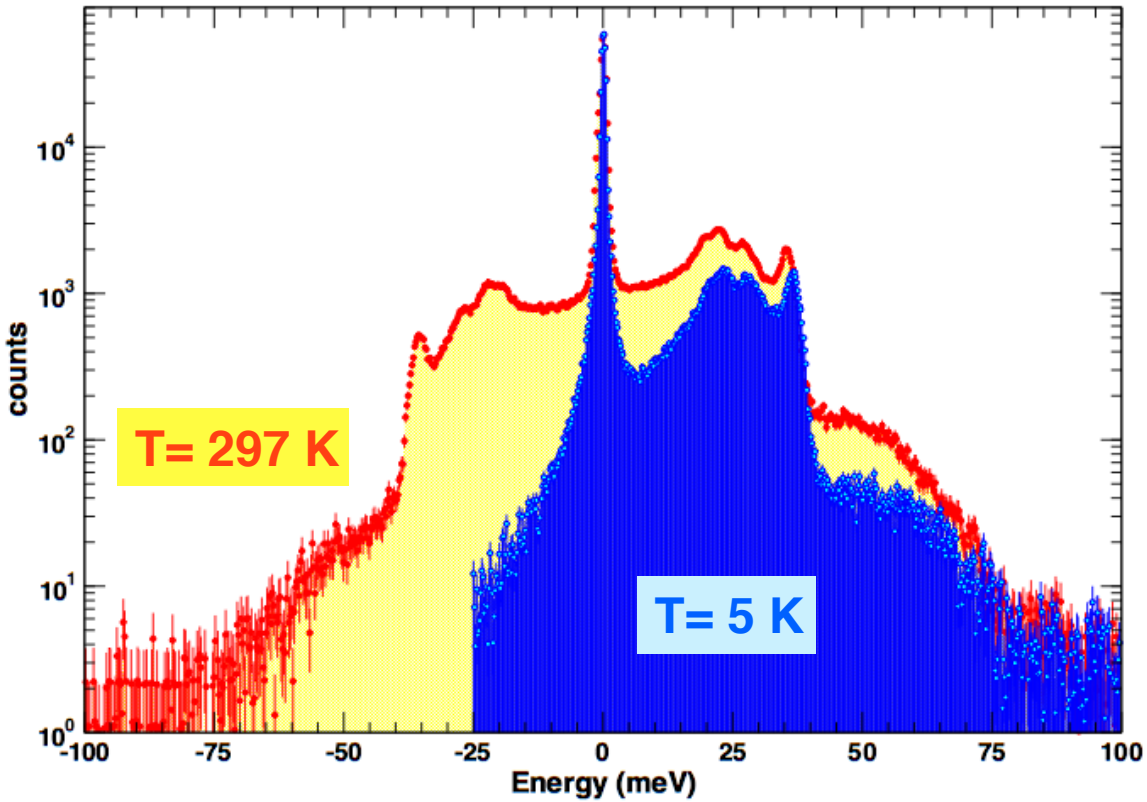
$$k_B = 8.6173 \cdot 10^{-2}\, meV / K$$

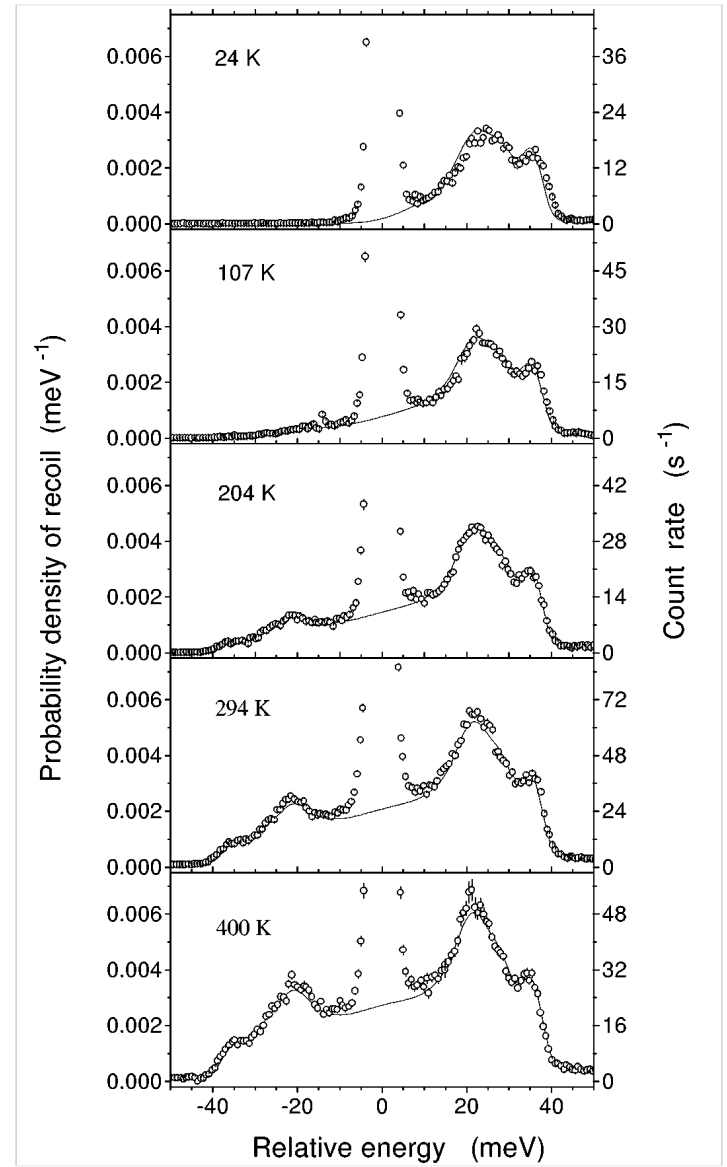
$$E = 22.6\ meV$$

$$I(-E) = 1158, \quad I(E) = 2762$$

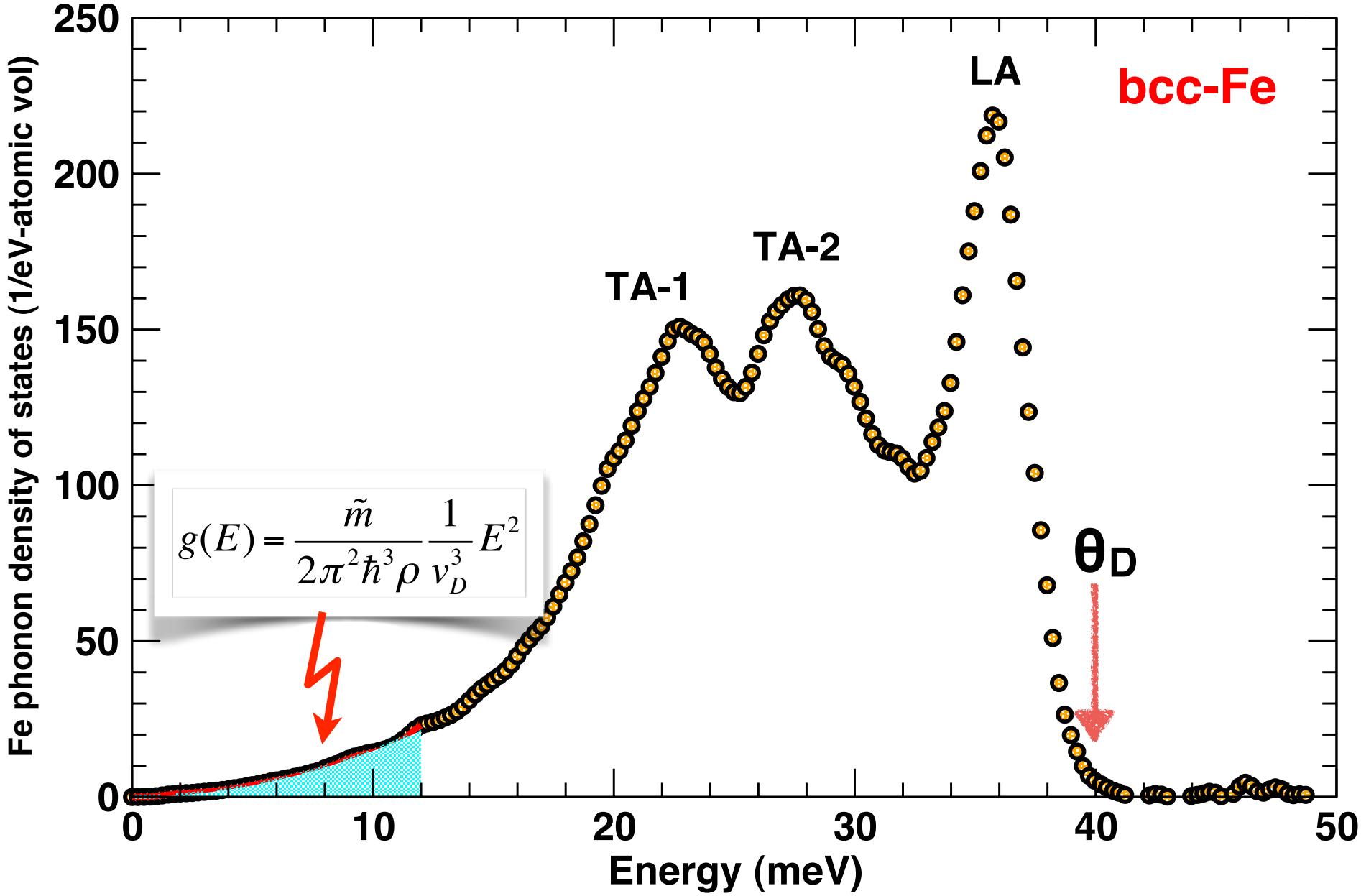
$$T = \frac{E(eV)}{k_B \ln\left[I(E)/I(-E)\right]} = 297.03K$$

Temperature dependence of phonon excitation probability

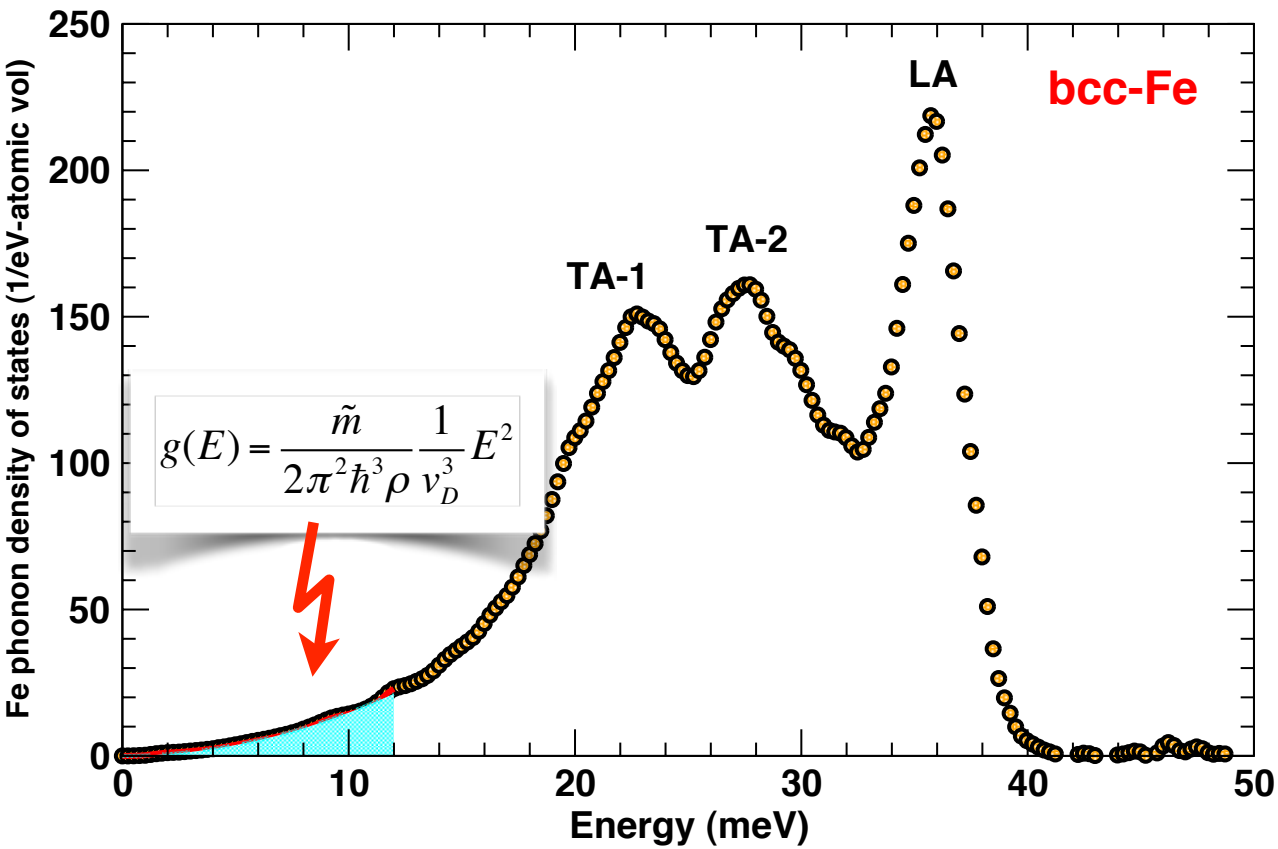




Chumakov, et al, Phys. Rev. B 54 (1996) 9596.


bcc-Fe
LA
TA-2
TA-1
θ_D
$g(E) = \frac{\tilde{m}}{2\pi^2\hbar^3\rho}\frac{1}{v_D^3}E^2$
Fe phonon density of states (1/eV-atomic vol)
Energy (meV)
0
50
100
150
200
250
0
10
20
30
40
50

Measurement of v_D, Debye sound velocity allows to resolve longitudinal and shear sound velocity, provided that bulk modulus and density, is independently and simultaneously measured by x-ray diffraction.



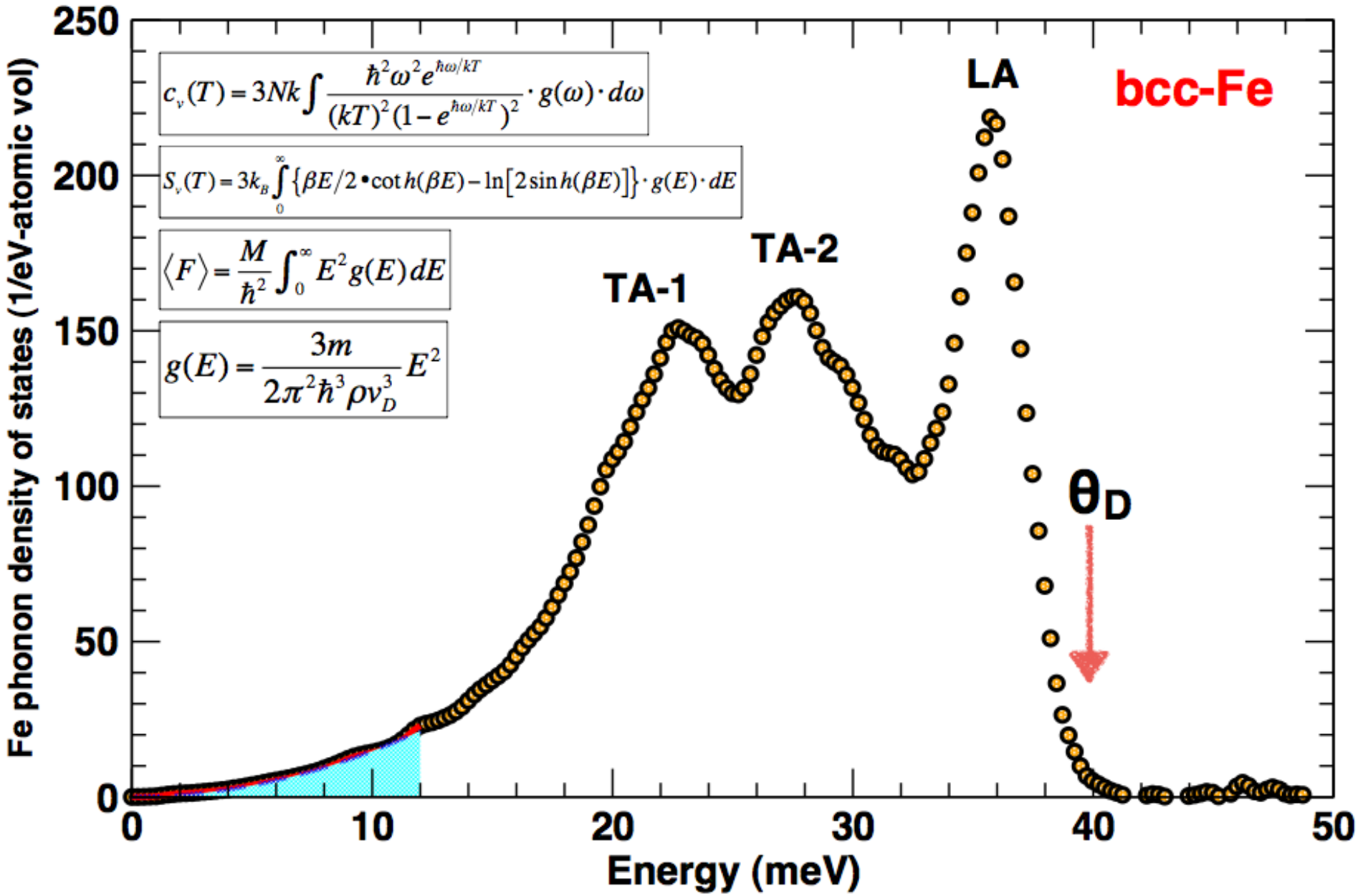
K (GPa)	ρ (g/cc)	V_D (m/s)	V_P (m/s)	V_S (m/s)	G (GPa)
165 ± 1	8.01	3510 ± 12	5813 ± 13	3146 ± 11	79.3 ± 0.6

$$\frac{K_S}{\rho} = V_P^2 - \frac{4}{3}V_S^2$$

$$\frac{G}{\rho} = V_S^2$$

$$\frac{3}{V_D^3} = \frac{1}{V_P^3} + \frac{2}{V_S^3}$$

K_S : adiabatic bulk modulus
G : shear modulus
V_P : compression wave velocity
V_S : shear wave velocity
V_D : Debye sound velocity
P : density


$c_v(T) = 3Nk \int \frac{\hbar^2 \omega^2 e^{\hbar\omega/kT}}{(kT)^2 (1 - e^{\hbar\omega/kT})^2} \cdot g(\omega) \cdot d\omega$
$S_v(T) = 3k_B \int_0^\infty \{\beta E/2 \bullet \coth(\beta E) - \ln[2\sinh(\beta E)]\} \cdot g(E) \cdot dE$
$\langle F \rangle = \frac{M}{\hbar^2} \int_0^\infty E^2 g(E) dE$
$g(E) = \frac{3m}{2\pi^2 \hbar^3 \rho v_D^3} E^2$
bcc-Fe
LA
TA-2
TA-1
θ_D
Fe phonon density of states (1/eV-atomic vol)
Energy (meV)
0
50
100
150
200
250
0
10
20
30
40
50

Phonon density of states

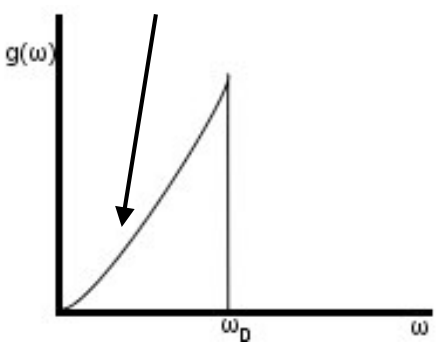
$$g(k)\,\mathrm{d}k = \frac{V}{(2\pi)^3}\,4\pi k^2\,\mathrm{d}k.$$

Number of wave vectors in a spherical shell of radius k per unit volume of reciprocal space.

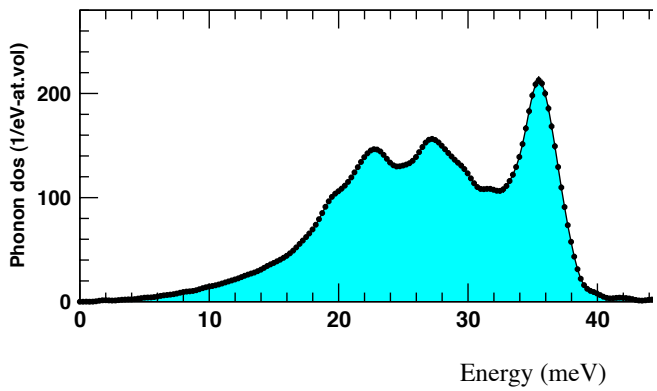
$$g(\omega) = \frac{3V}{2\pi^2 c^3}\omega^2$$

Phonon density of states has a quadratic dependence on frequency, and inversely proportional to the cube of sound velocity.

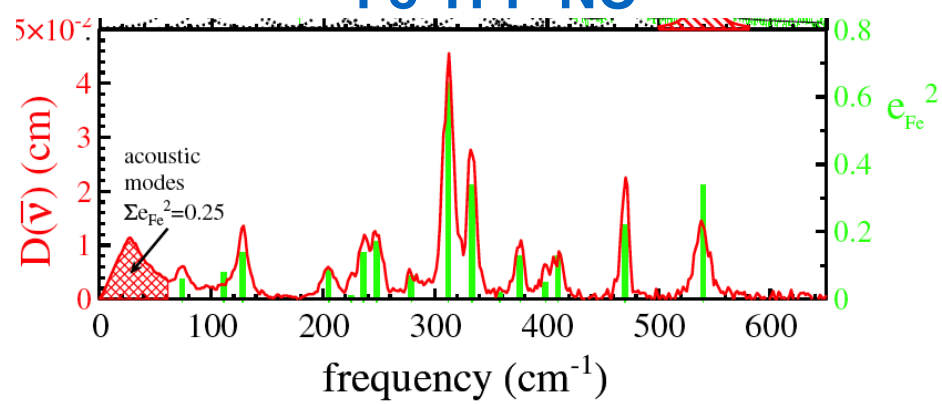
Debye model



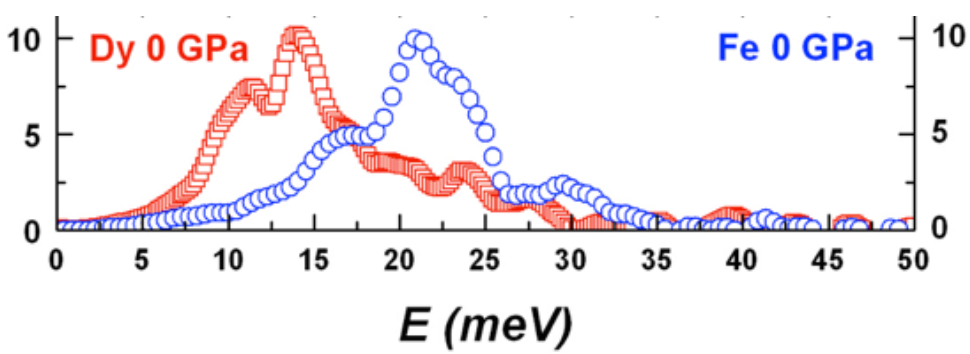
pure iron

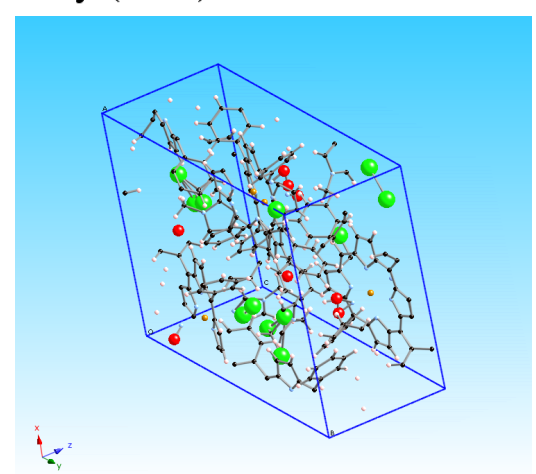


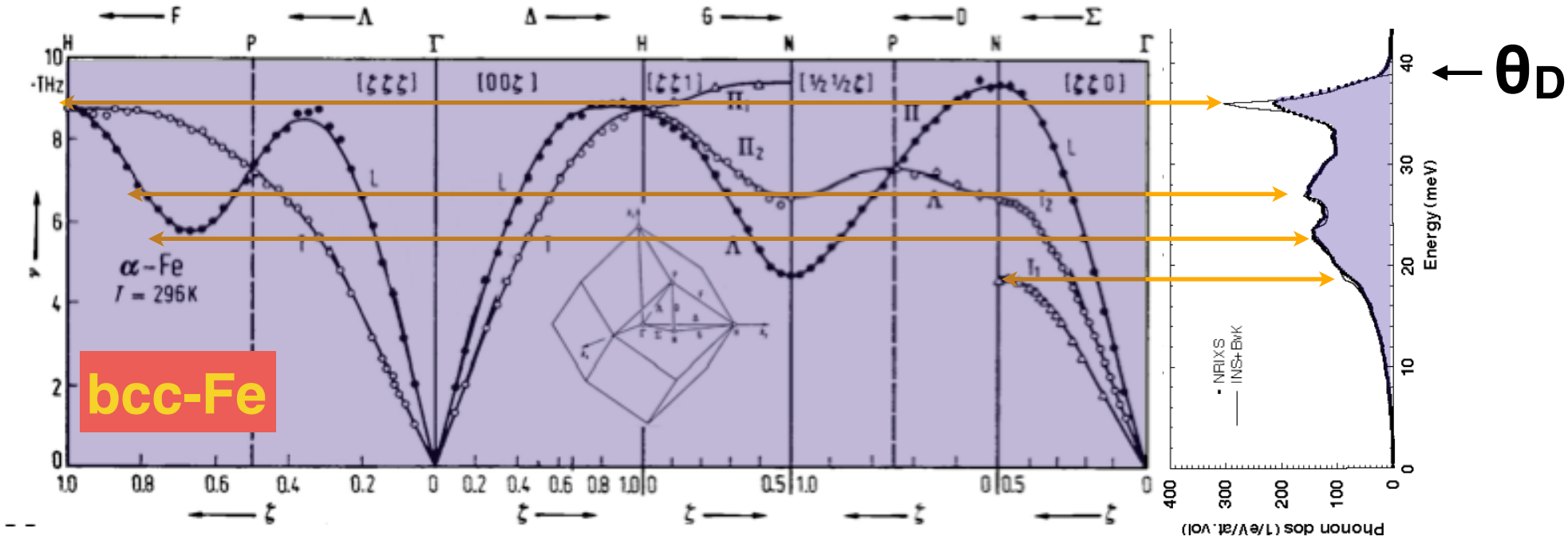
Fe-TPP-NO

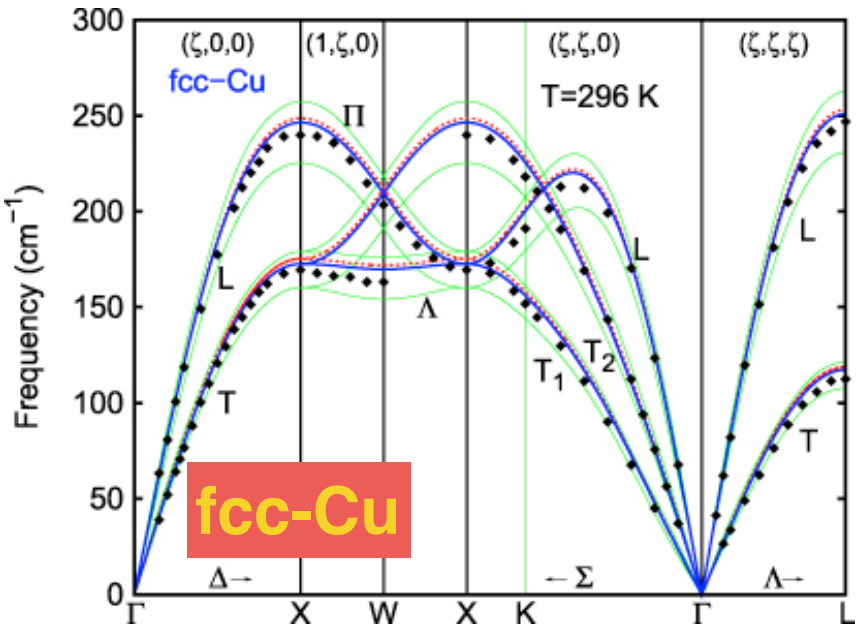


$DyFe_3$









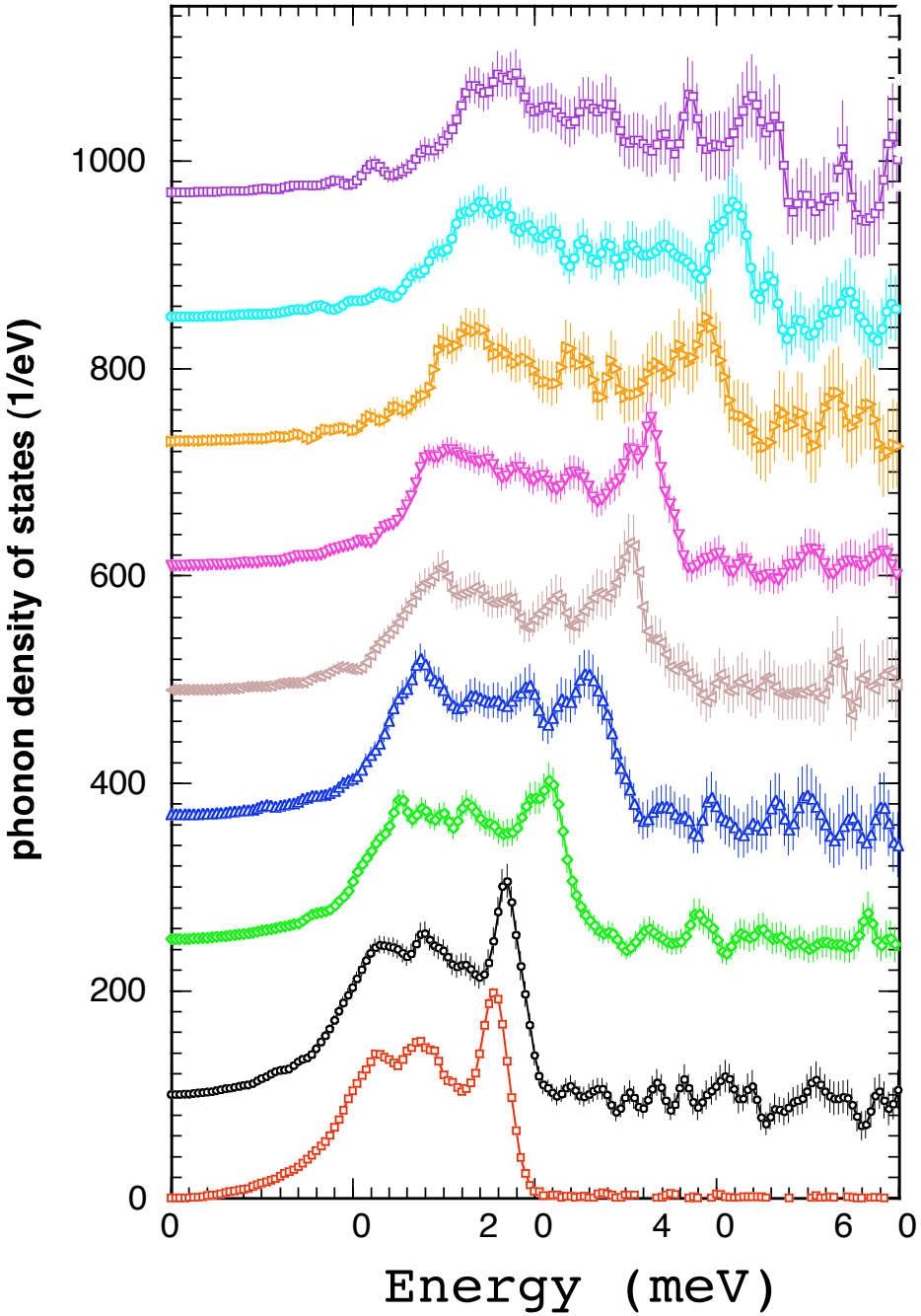
Let's assume that the acoustic modes have a linear relationship between frequency and wave vector:

$\omega = ck$, where c is average sound velocity

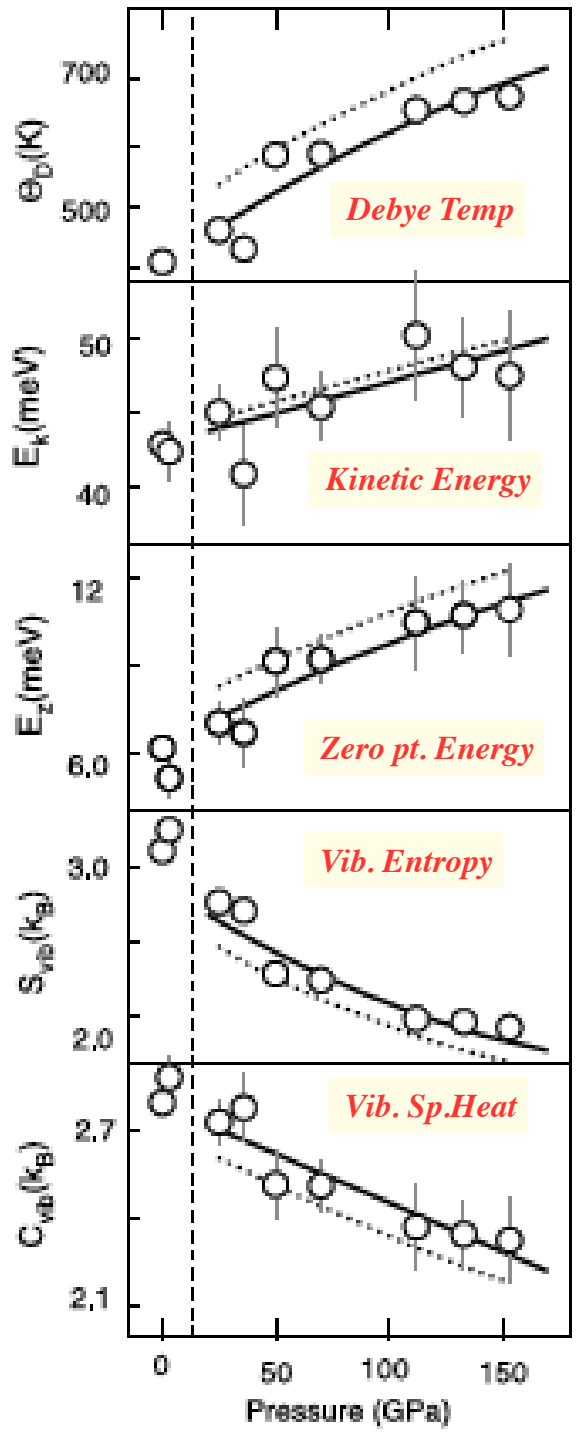
Maximum frequency cut off is at Debye energy: e.g. for Cu, this frequency is 240 cm^{-1} (~ 30 meV). Considering 1 meV = 11.605 K=8.065 cm^{-1}, this corresponds to 348 K, which is close to 344 K.

For Fe, the measured cut-off value is ~ 39.5 meV, which corresponds to 458 K, very close to reported 460 K.

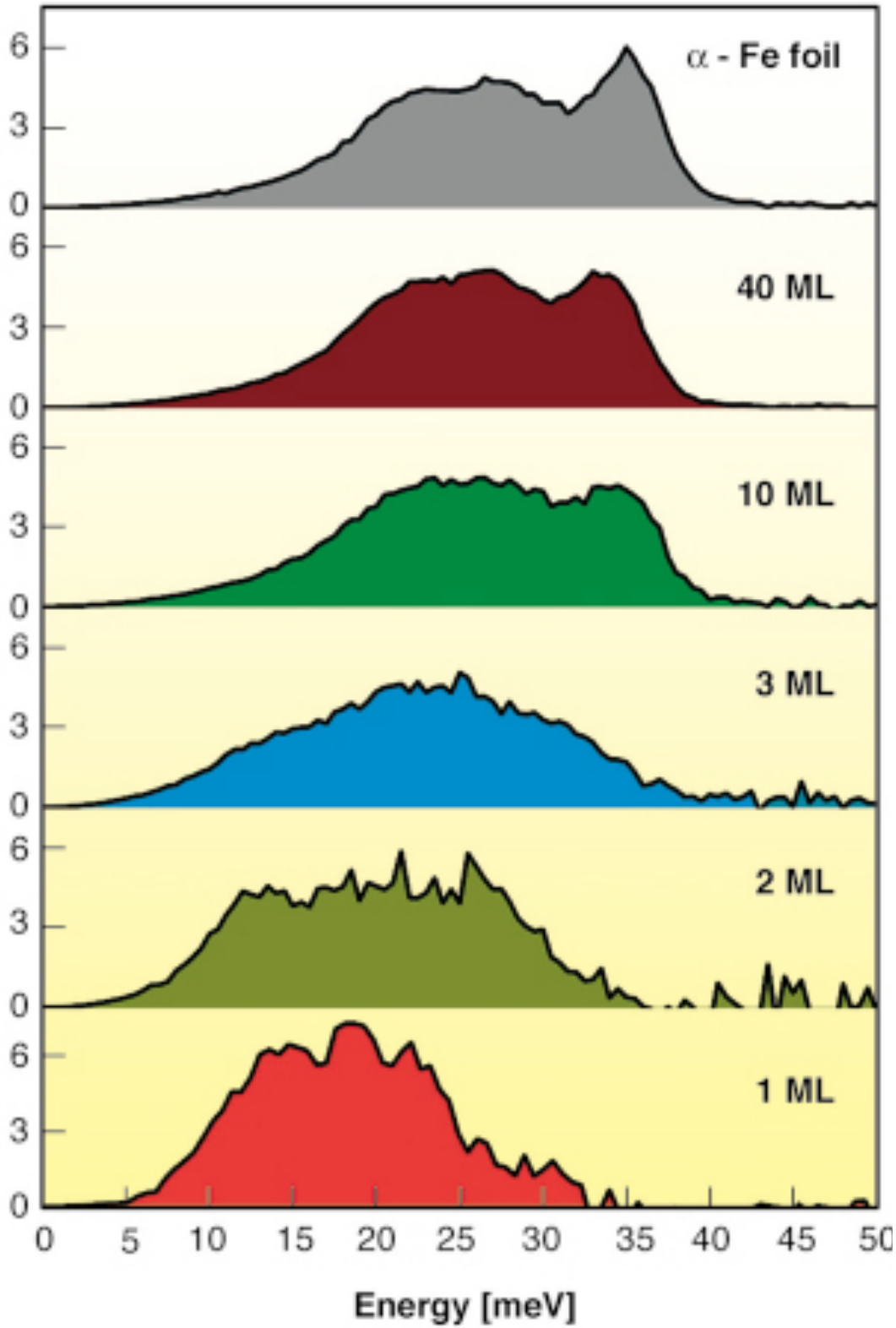
Phonon density of states of iron under high pressure



153 GPa



H.K. Mao, et al, Science, 292 (2001) 914

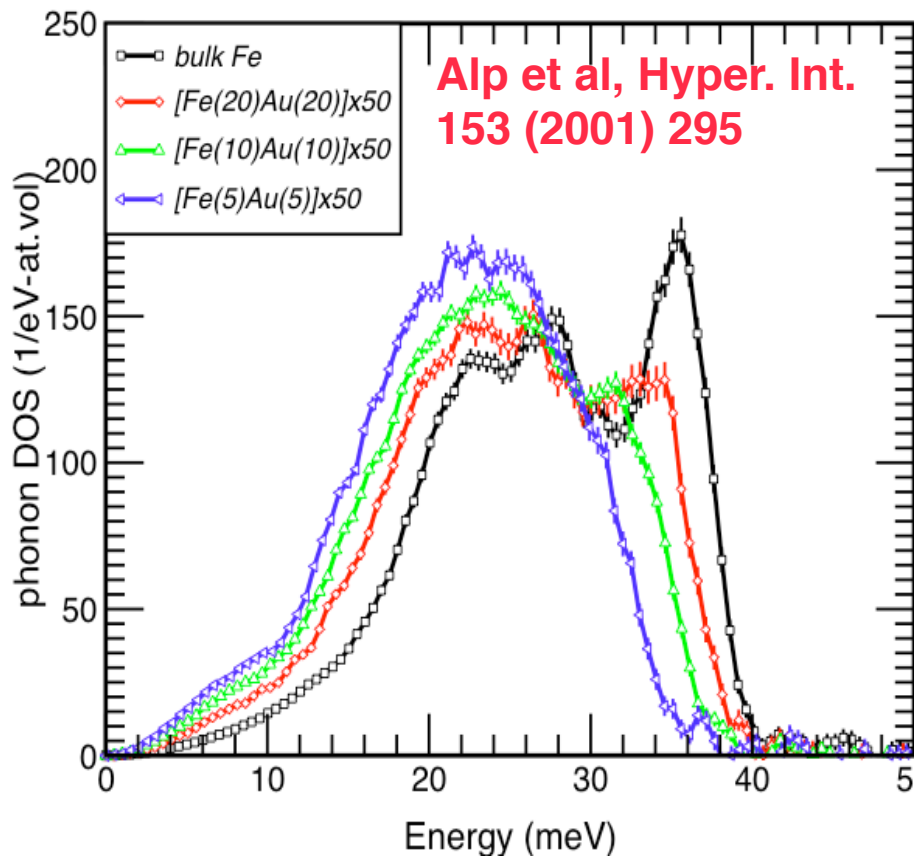


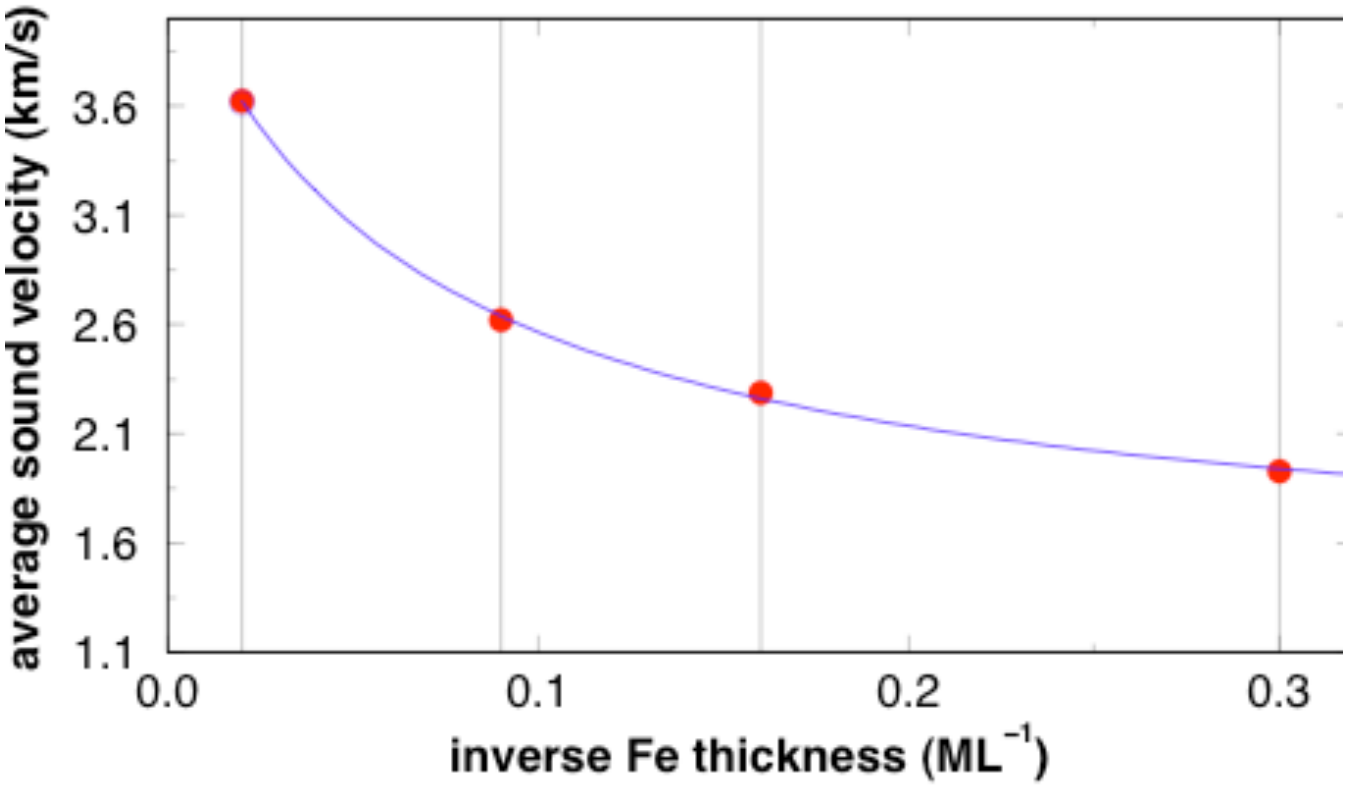
Fe films deposited on W(110)

Transition from the bulk to a single iron monolayer

S. Stankov, R. Röhlsberger, T. Slezak, M. Sladecek, B. Sepiol, G. Vogl, A. I. Chumakov, R. Rüffer, N. Spiridis, J. Lazewski, K. Parlinski, and J. Korecki,

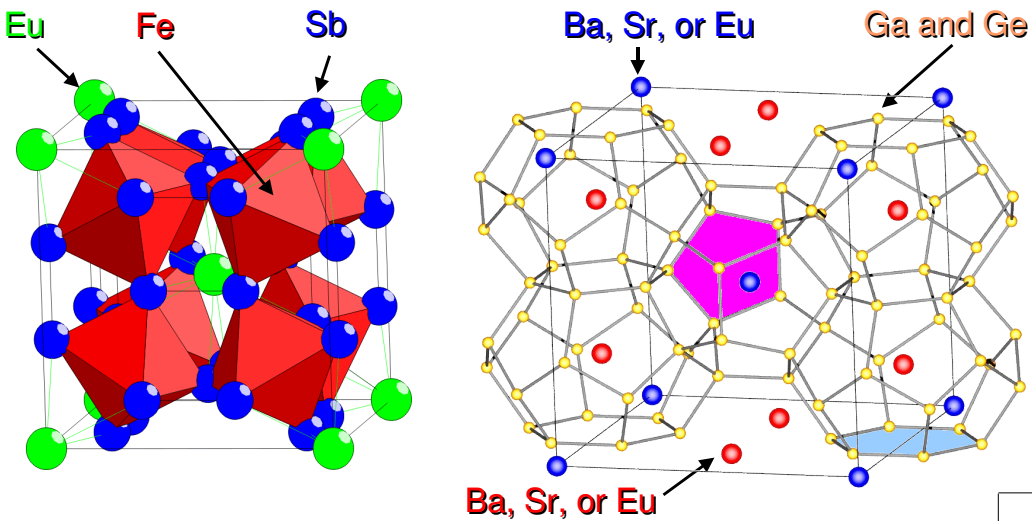
ESRF Highlights 2006




average sound velocity (km/s)
3.6
3.1
2.6
2.1
1.6
1.1
0.0
0.1
0.2
0.3
inverse Fe thickness (ML^{-1})

1. Thermoelectric materials: always something new !..

Skutterudites

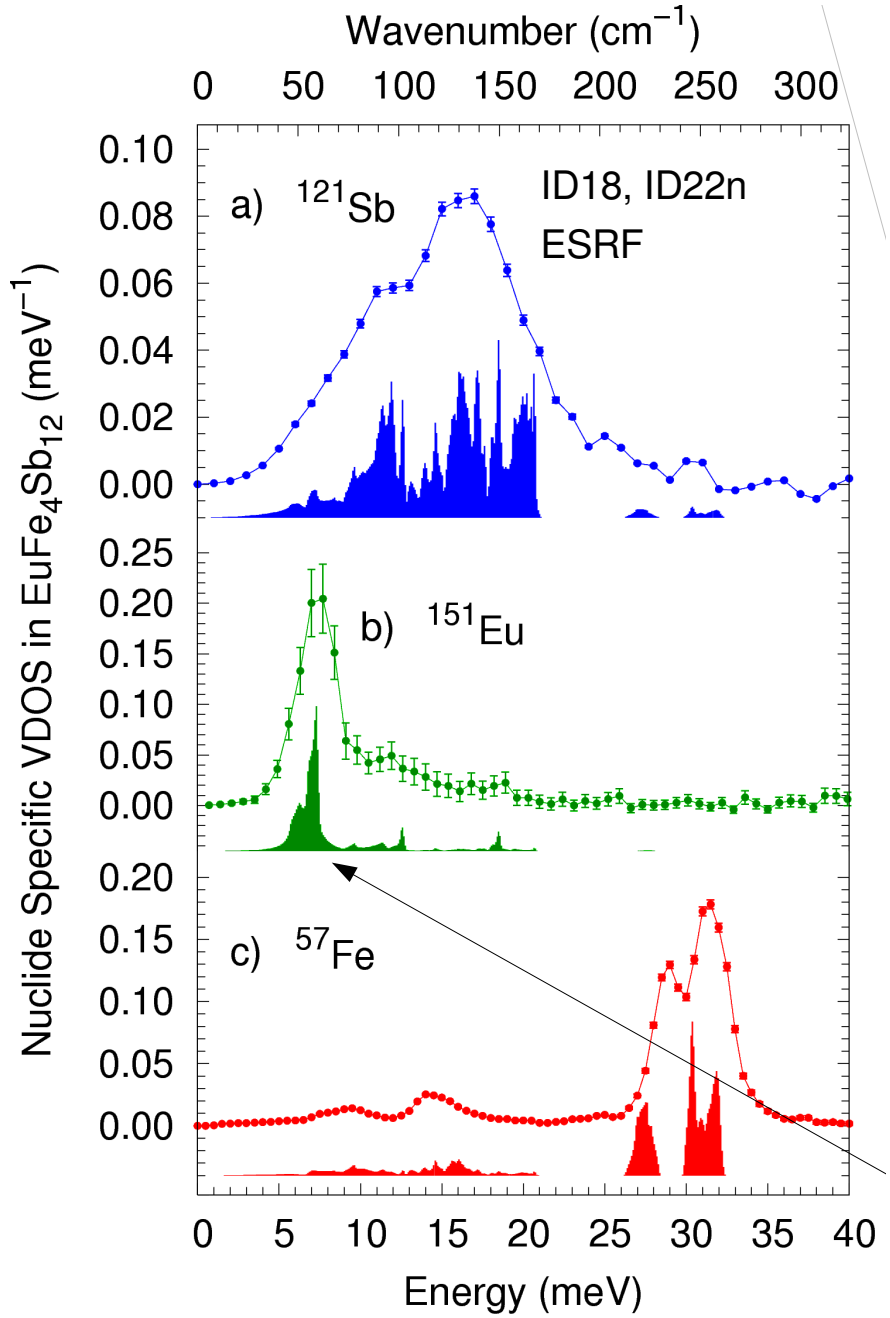


$EuFe_4Sb_{12}$

The loosely bound guests affect the characteristics of the vibrations, and change the thermal conductivity

Many elements in modern thermolelectric materials include **Fe**, rare-earth atoms like **Eu, Sm, Dy**, as well as **Sb**, and **Te**. These are all proper Mössbauer resonances we can exploit, and we do..

Courtesy: Raphael Hermann, Jülich



PHYSICAL REVIEW B **90**, 104304 (2014)

Vibrational dynamics of the host framework in Sn clathrates

Bogdan M. Leu,[1,*] Mihai Sturza,[2] Michael Y. Hu,[1] David Gosztola,[3] Volodymyr Baran,[4] Thomas F. Fässler,[4] and E. Ercan Alp[1]

$K_8Zn_4Sn_{42}$

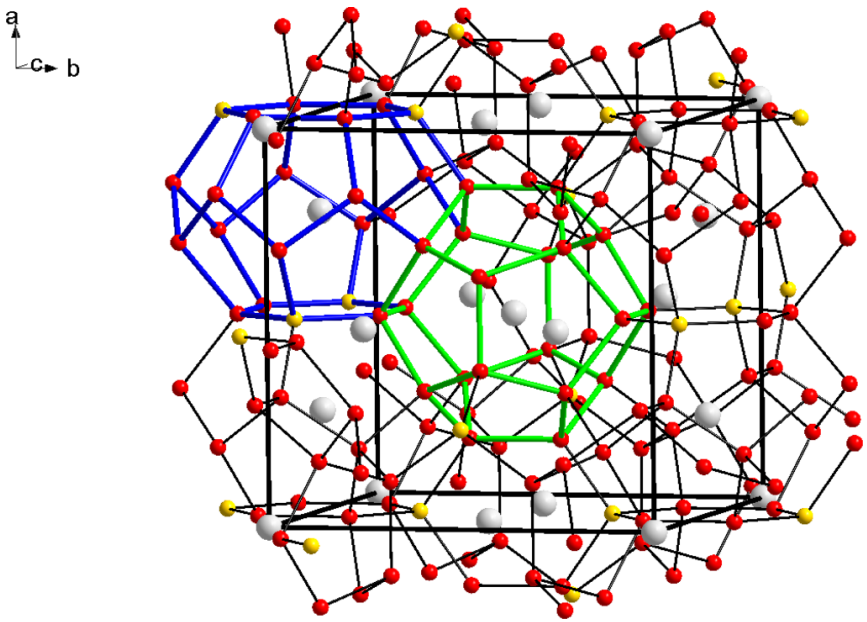


FIG. 1. (Color online) Structure of type-I clathrate $K_8Zn_4Sn_{42}$. Color scheme: gray = K, yellow = Zn/Sn, red = Sn. One small (pentagonal dodecahedron) and large (tetrakaidecahedron) host framework cage are highlighted in green and blue, respectively.

$Ba_{8Ga4}Sn_{30}$

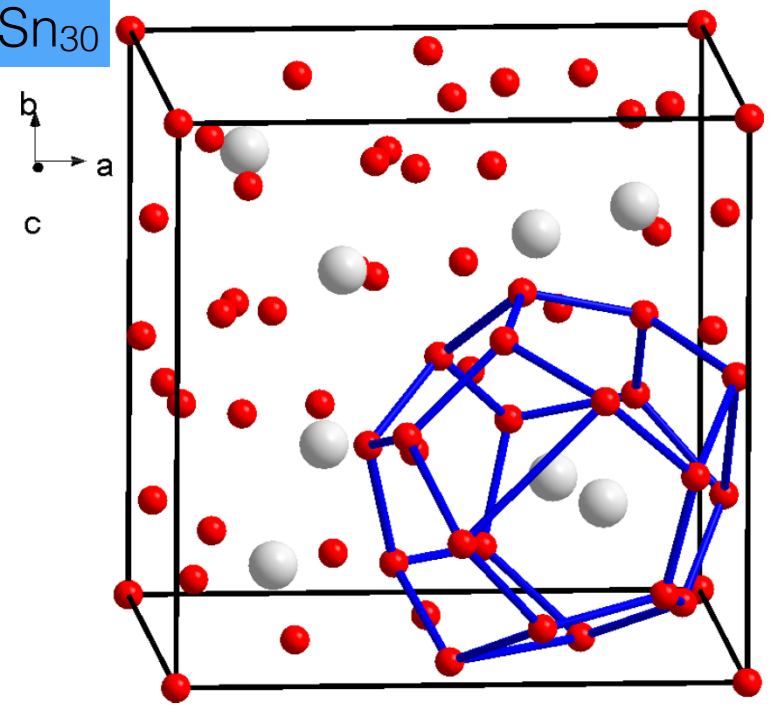
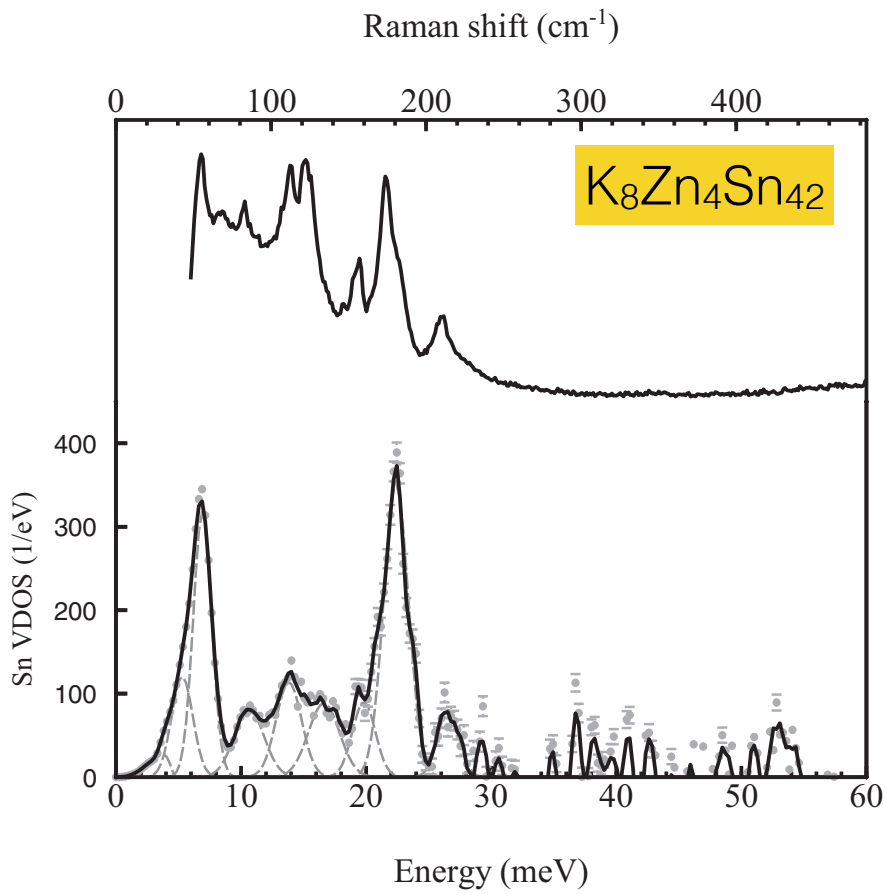
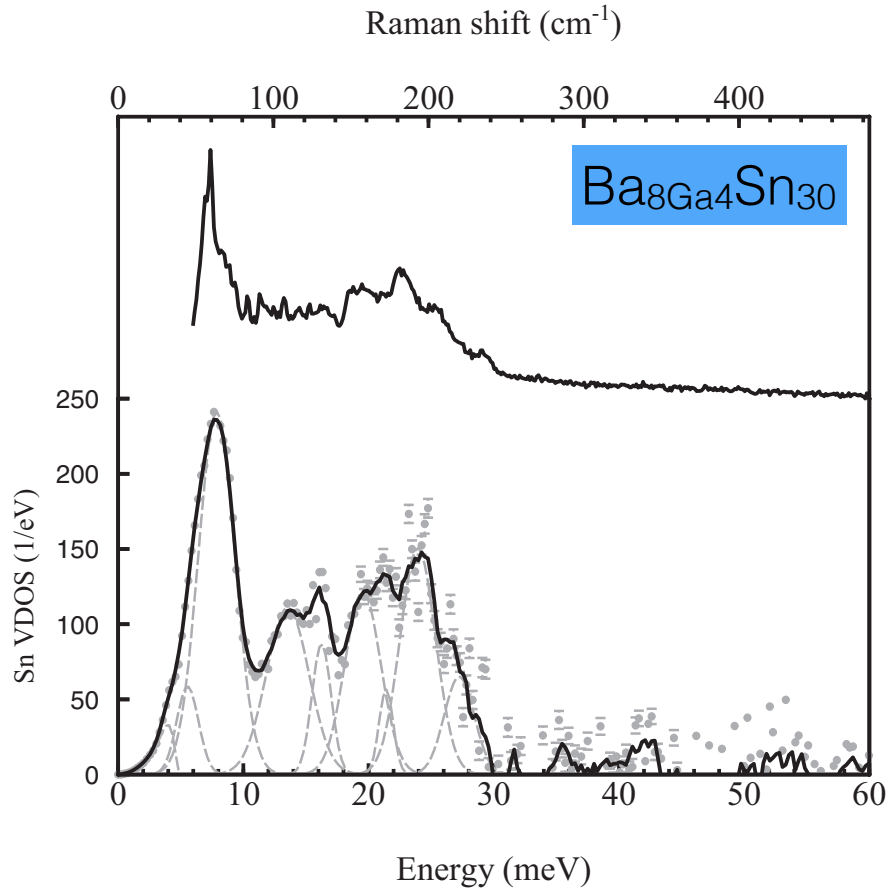


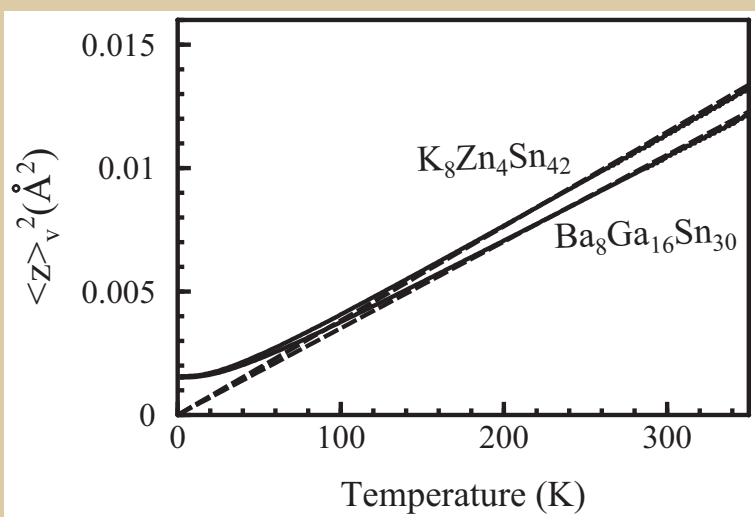
FIG. 2. (Color online) Structure of type-VIII clathrate $Ba_8Ga_{16}Sn_{30}$. Color scheme: gray = Ba, red = Sn/Ga. One host framework cage (pentagonal dodecahedron) is highlighted in blue.

type-I clathrate: pentagonal dodecahedra and tetrakaidecahedra alternating in a 1:3 ratio

type VIII : pentagonal dodecahedra; however, BGS adopts the type-I clathrate structure at high-temperature





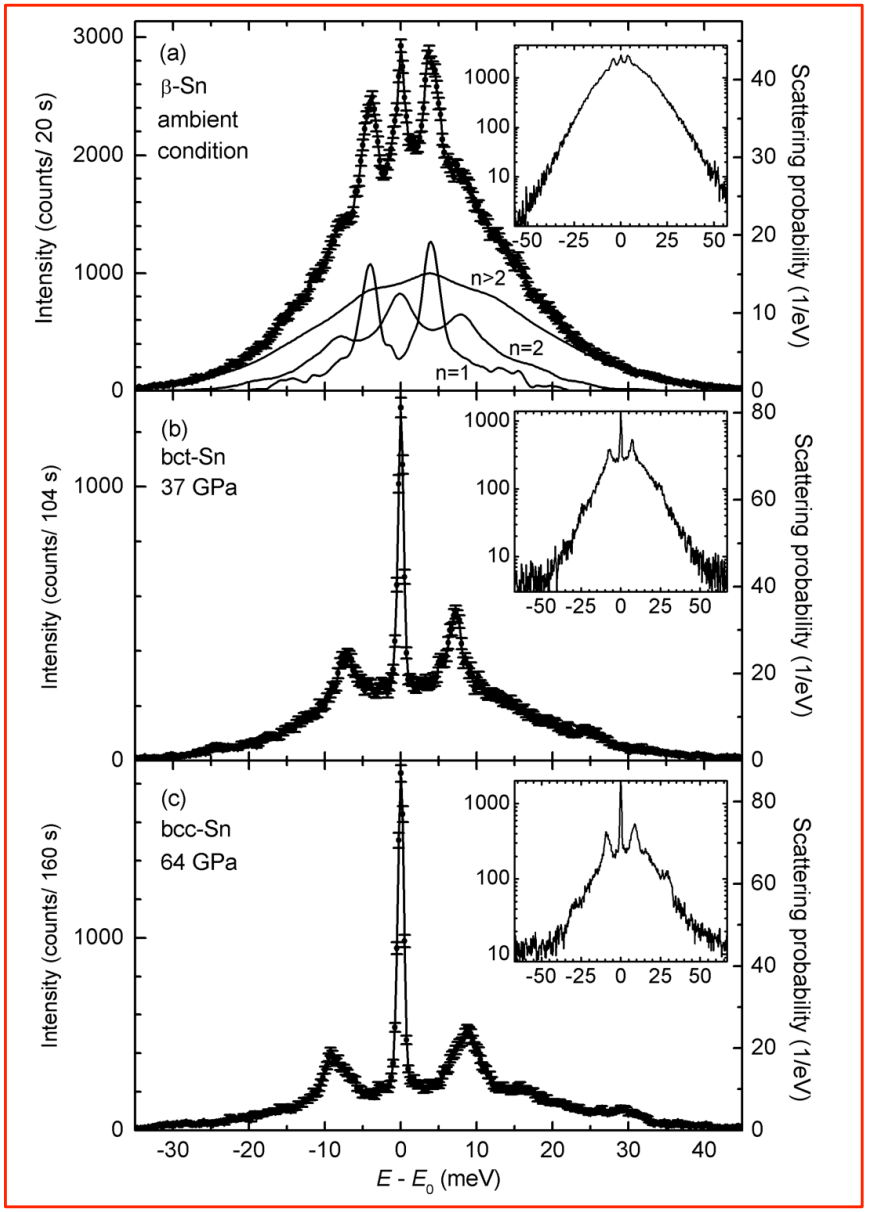


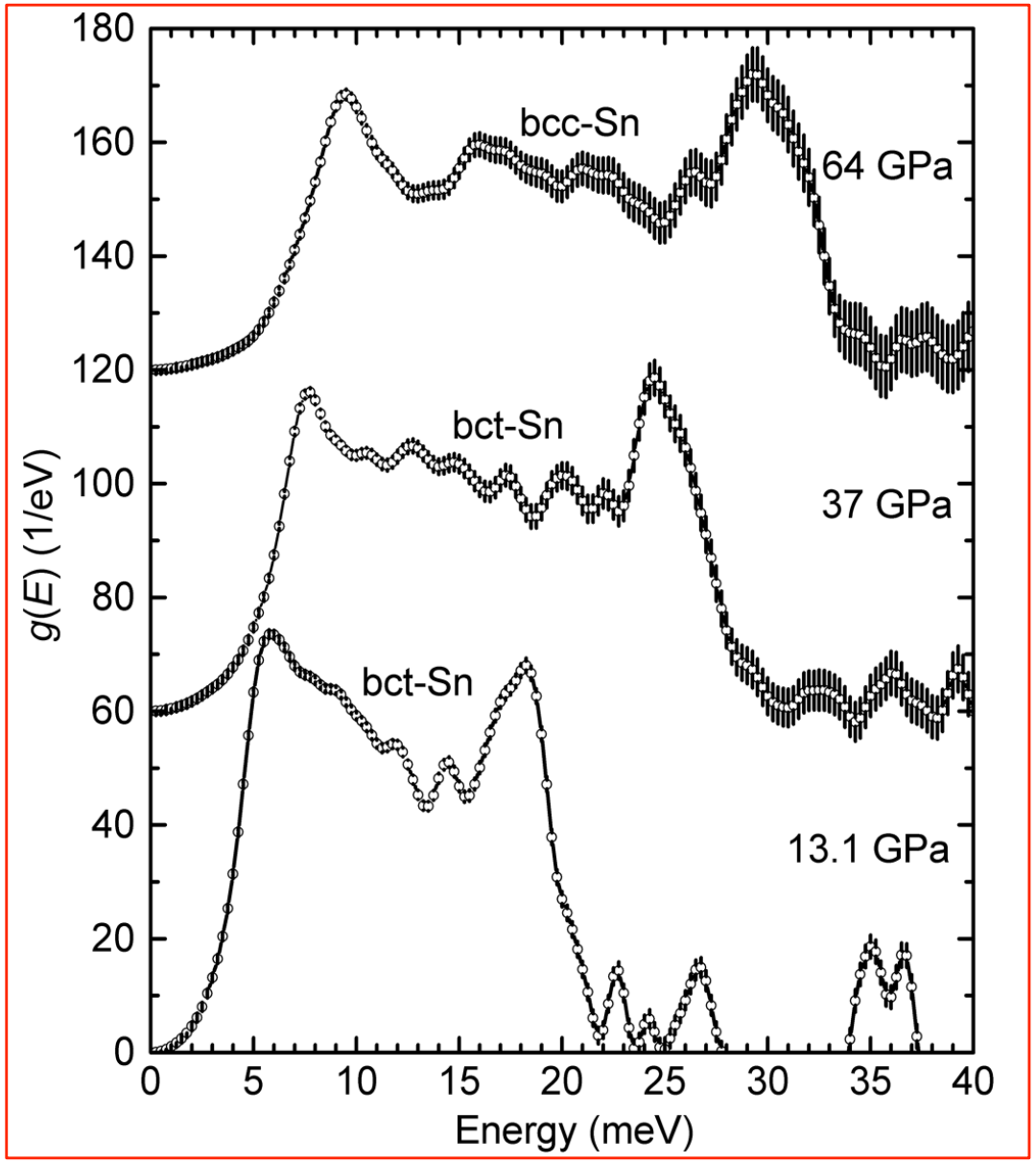
mean square displacement via phonon dos

$$\langle z^2 \rangle_v = \frac{1}{3k^2} \int [2\bar{n}(\bar{\nu}) + 1] \frac{\bar{\nu}_R}{\bar{\nu}} D(\bar{\nu}) d\bar{\nu},$$

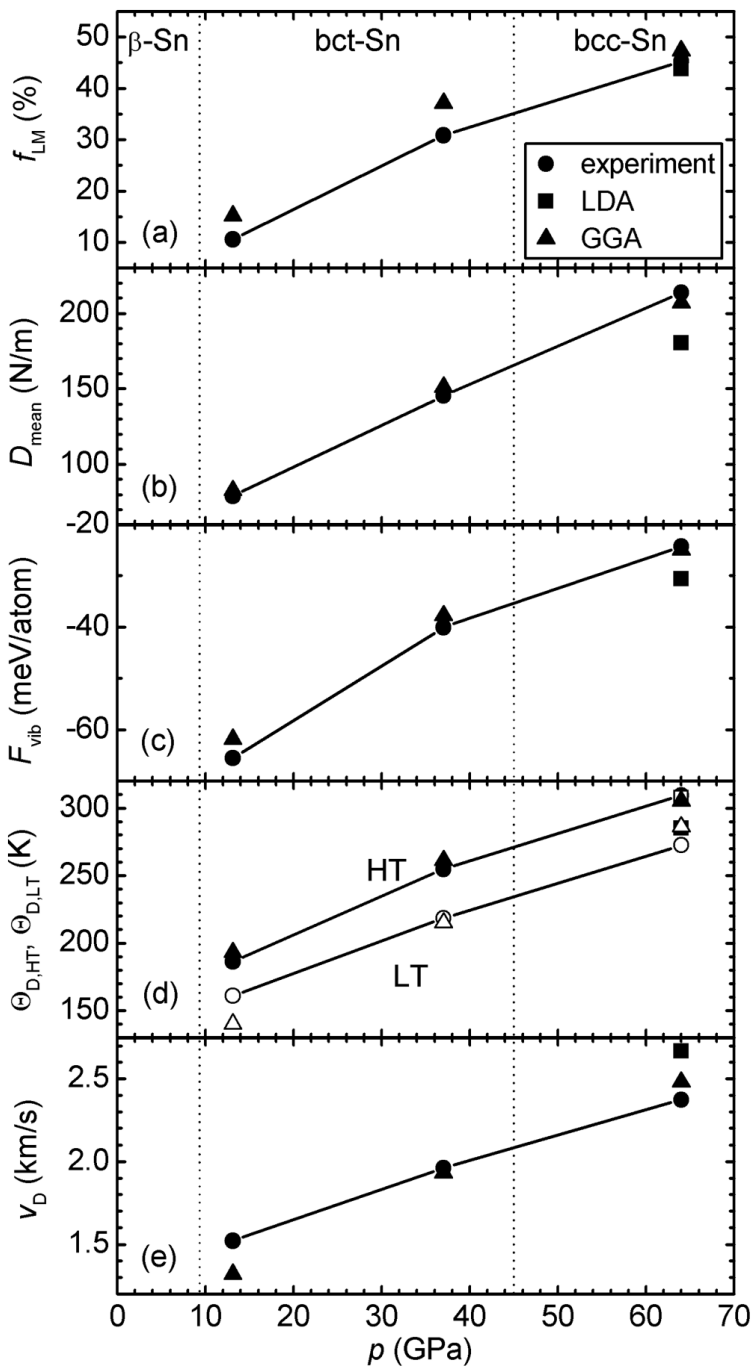
3.8×10^{-5} Å^2/K for KZS and 3.5×10^{-5} Å^2/K for BGS.

H. Giefers, et al, Phys. Rev. Lett. 98 (2007) 245502





H. Giefers, et al, Phys. Rev. Lett. 98 (2007) 245502



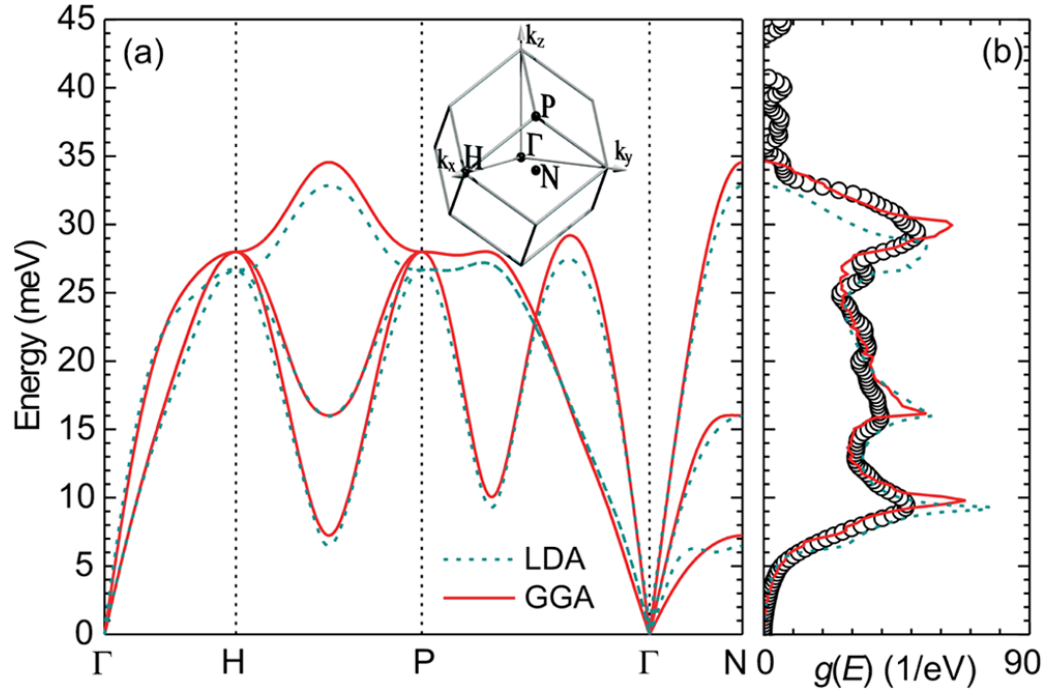
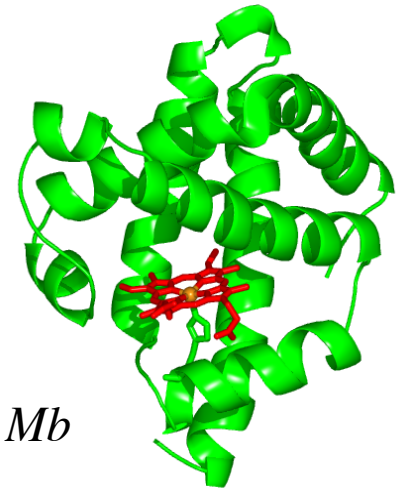


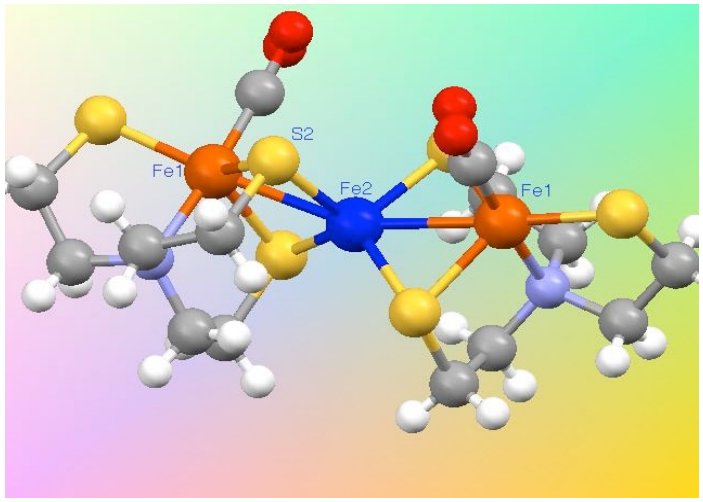
FIG. 3 (color online). (a) Theoretical phonon dispersion relation of bcc-Sn at 64 GPa. The inset shows the Brillouin zone of the bcc-Sn lattice. (b) Comparison between the theoretically calculated phonon DOS (lines) and the experimentally derived phonon DOS at 64 GPa (circles).

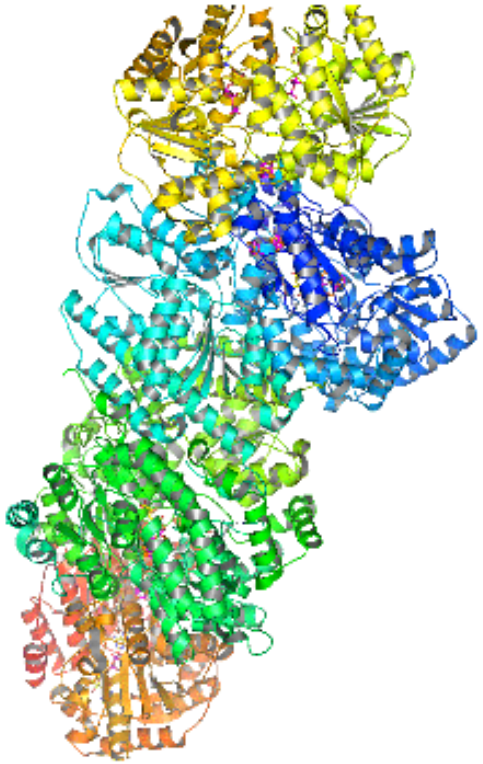
Biology & bio-inorganic chemistry

S. Cramer	University of California-Davis
E. Solomon	Stanford University
T. Sage	Northeastern University
E. Munck	University of Pittsburg
DeBeer George	Cornell University
Nicolai Lehnert	University of Michigan
R. Scheidt	University of Notre Dame

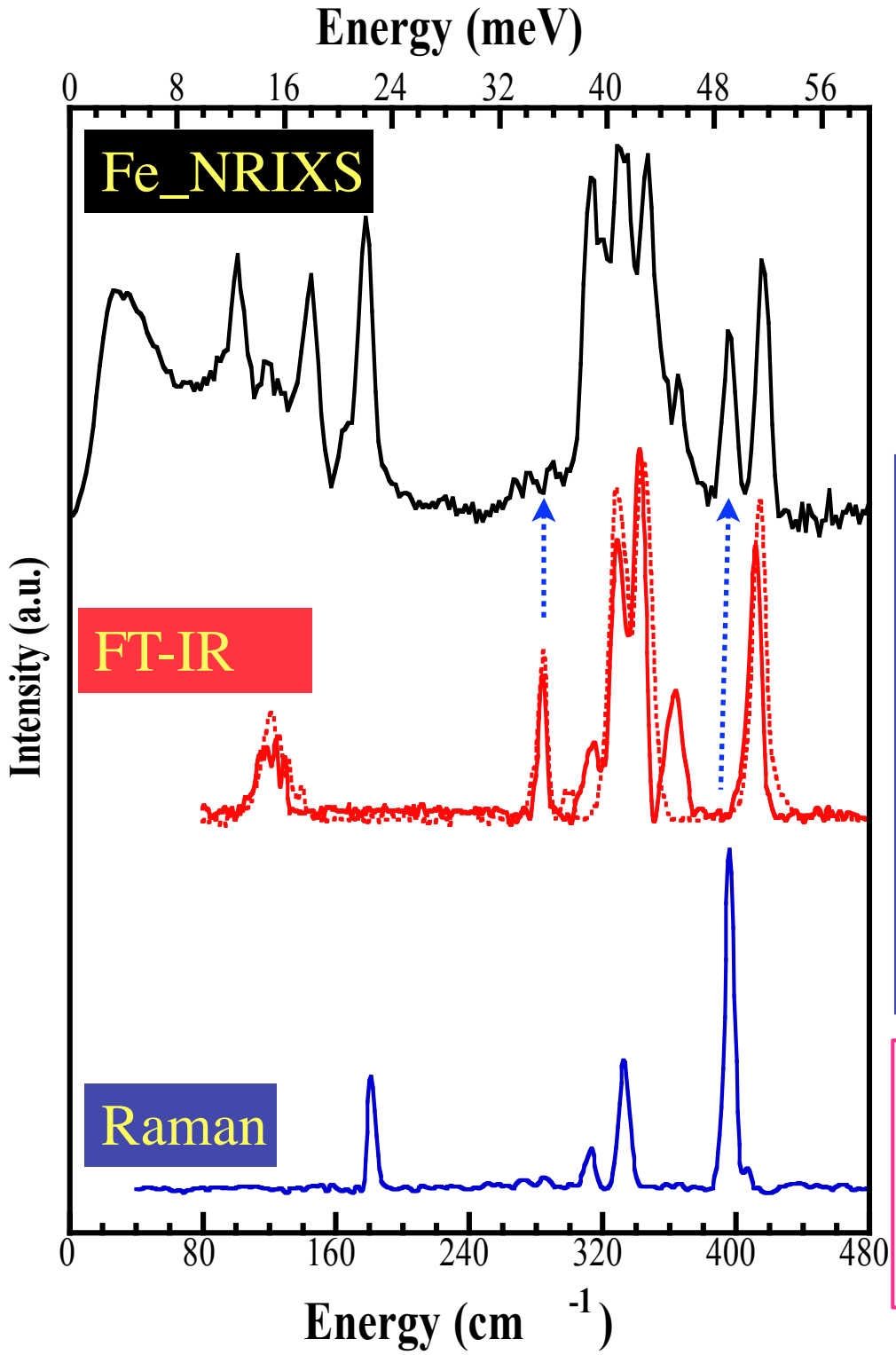


Mb

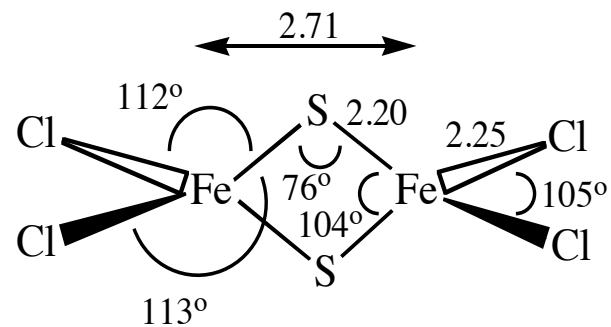




Vibrational spectroscopy of proteins, enzymes and biomimic model porphyrins and cubanes



$[NEt_4]_2[Fe_2S_2Cl_4]$.



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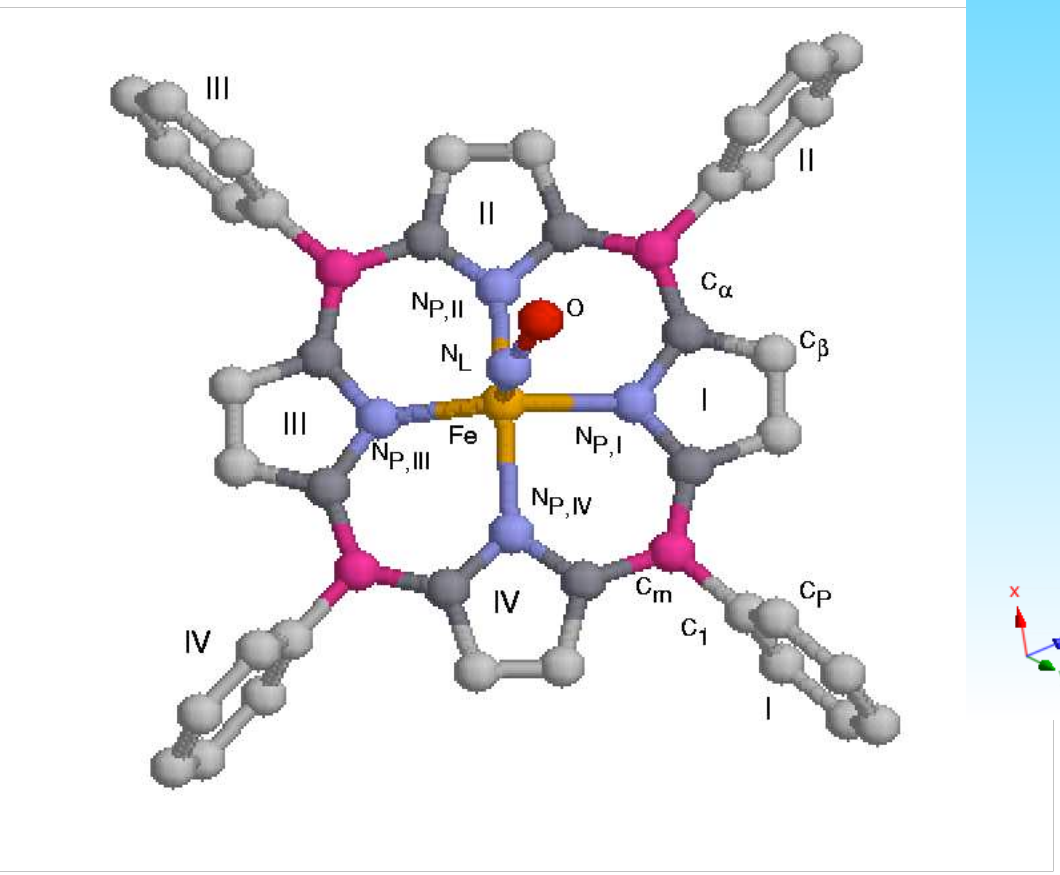
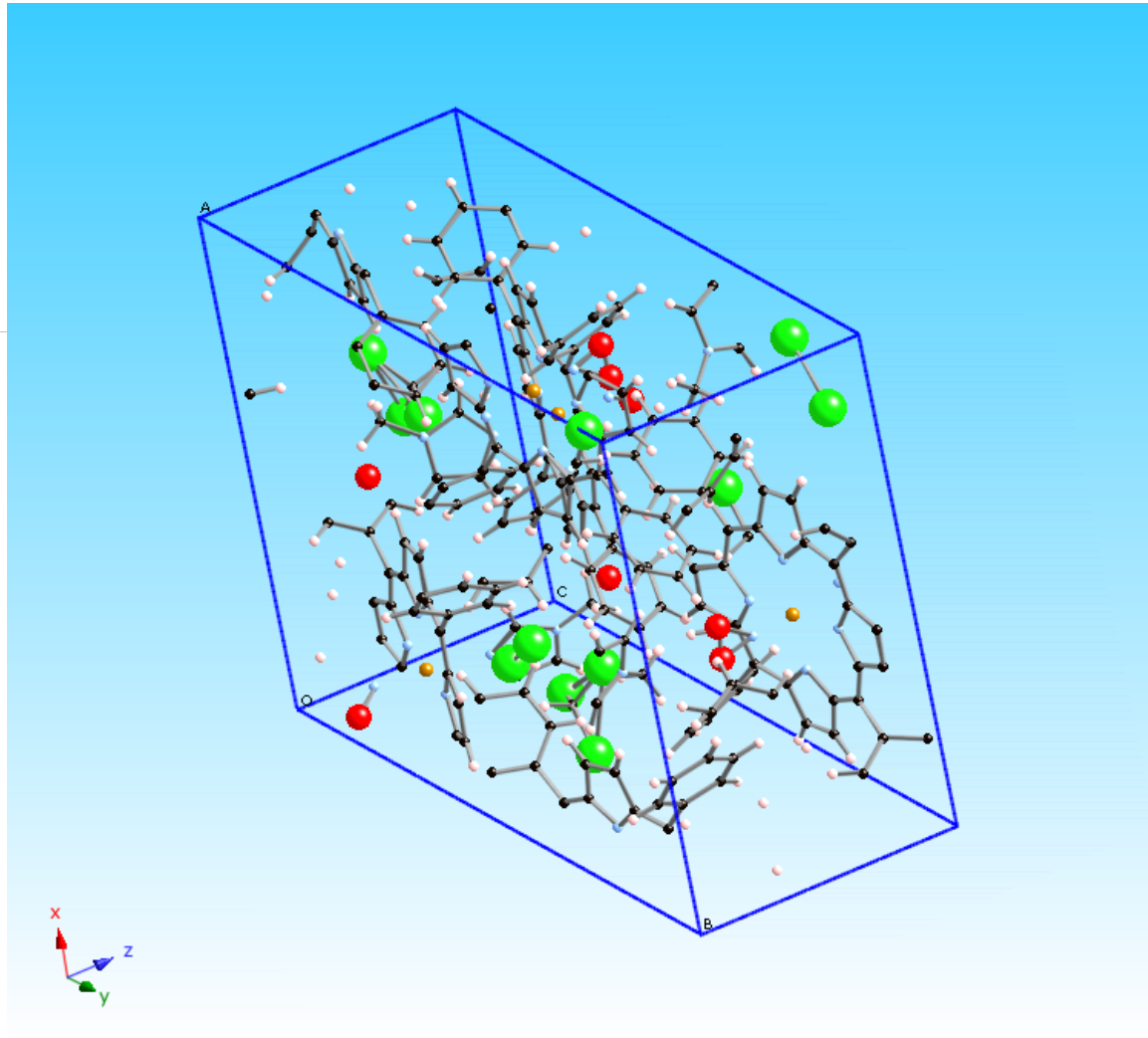
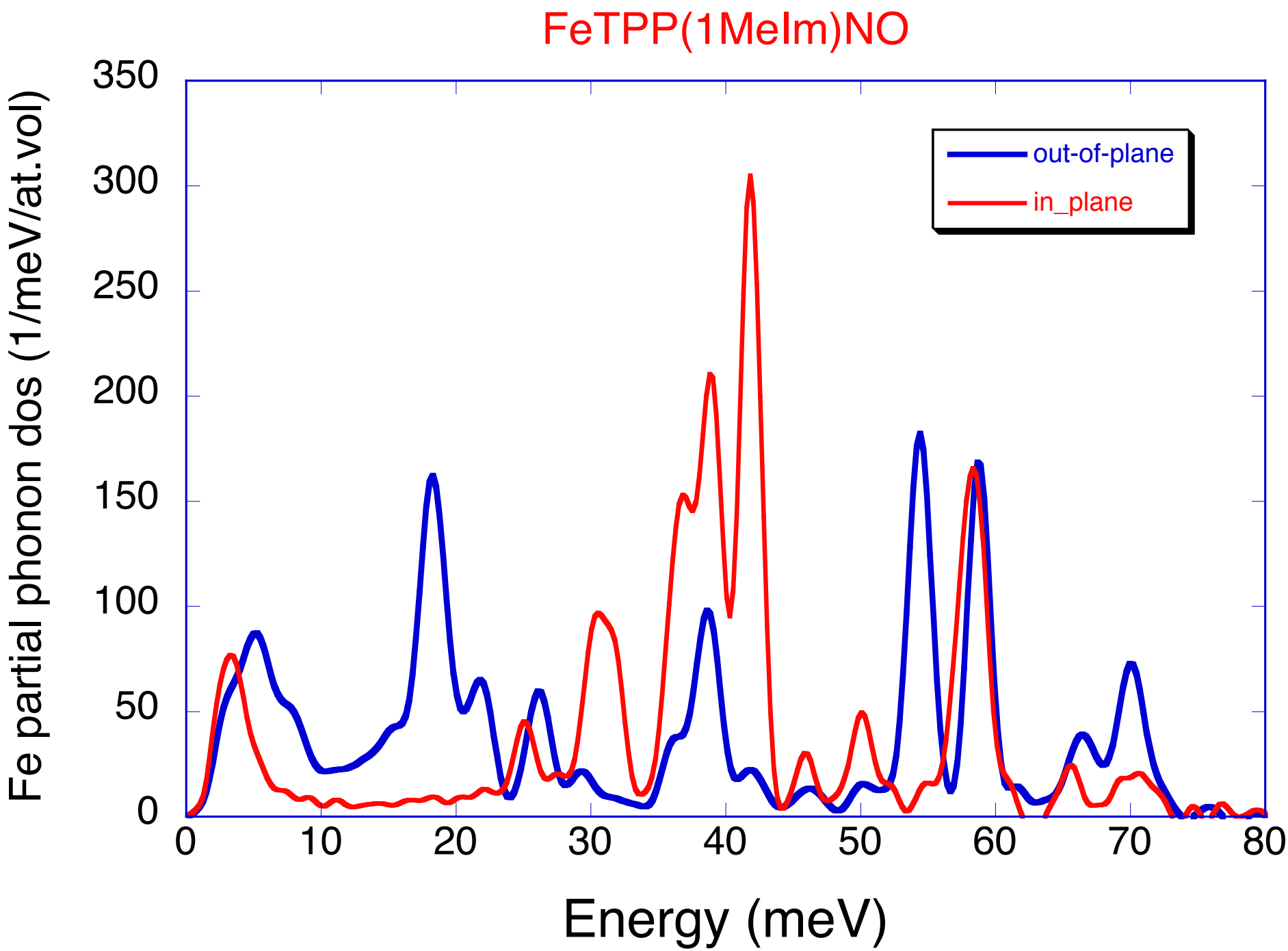
$$\phi_\alpha = \frac{1}{3}\frac{\bar{\nu}_R}{\bar{\nu}_\alpha} e_{j\alpha}^2 (\bar{n}_\alpha + 1) f$$

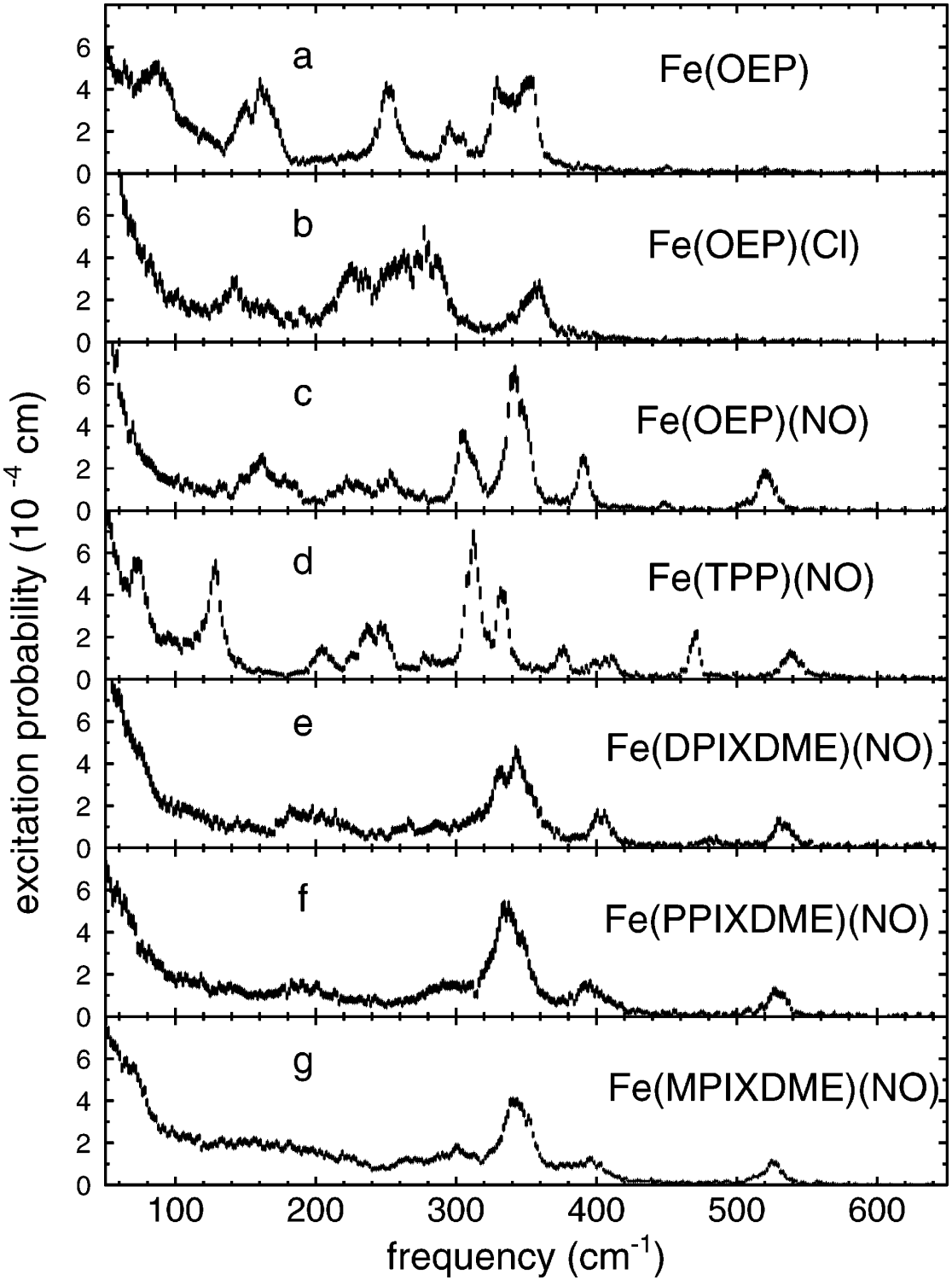
Matt Smith, et al, Inorganic Chemistry, 2005, 44,5562





Porphyrins:	A	B
Tetraphenylporphyrin (TPP)	Phenyl	H
Octaethylporphyrin (OEP)	H	Ethyl


FeTPP(1MeIm)NO
Fe partial phonon dos (1/meV/at.vol)
Energy (meV)
out-of-plane
in_plane
0
50
100
150
200
250
300
350
0
10
20
30
40
50
60
70
80



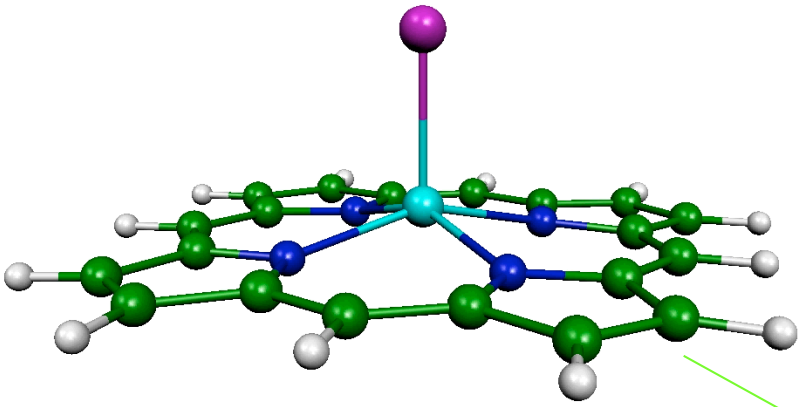
OEP: octaethylporphyrin;

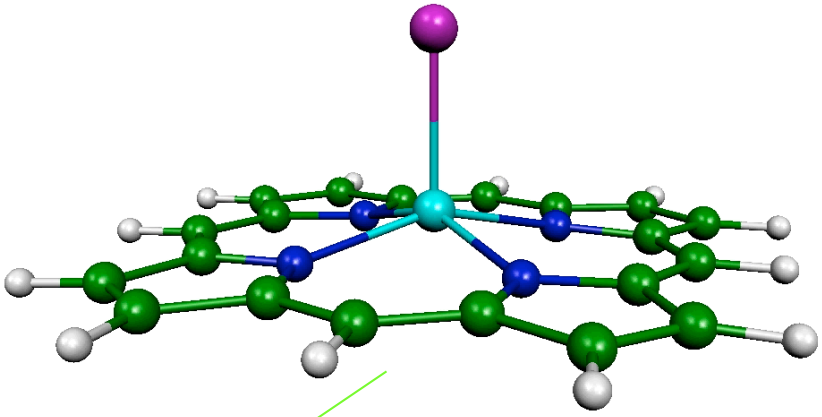
TPP: Tetraphenylporphyrin

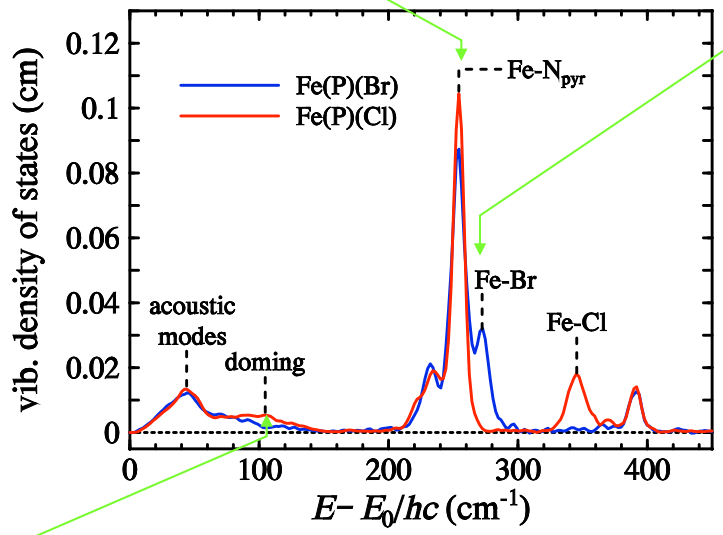
DPIXDME, deuteroporphyrin IX dimethyl ester

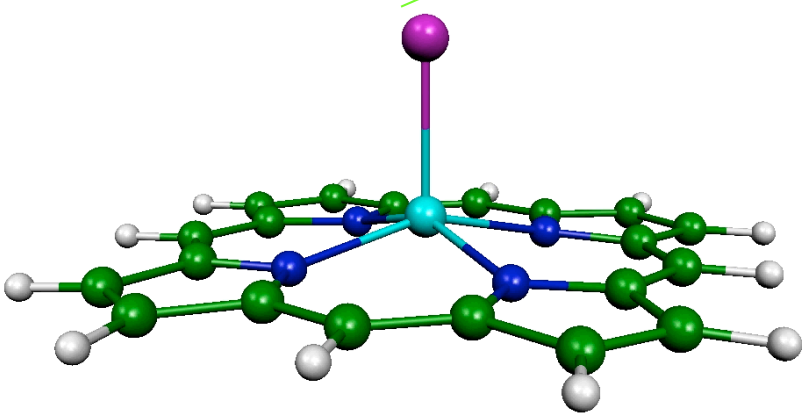
PPIXDME, protoporphyrin IX dimethyl ester

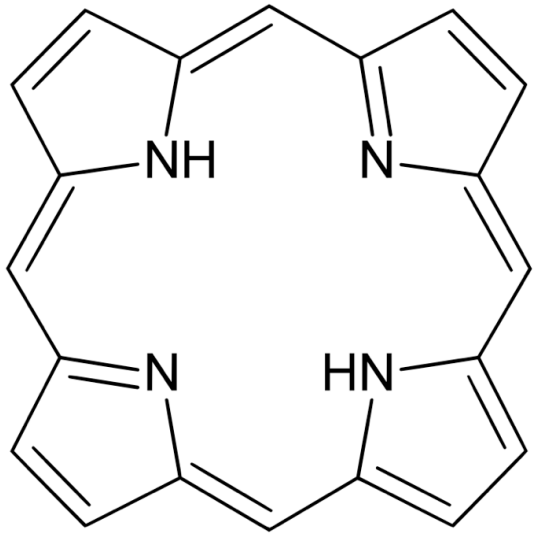
MPIXDME, mesoporphyrin IX dimethyl ester

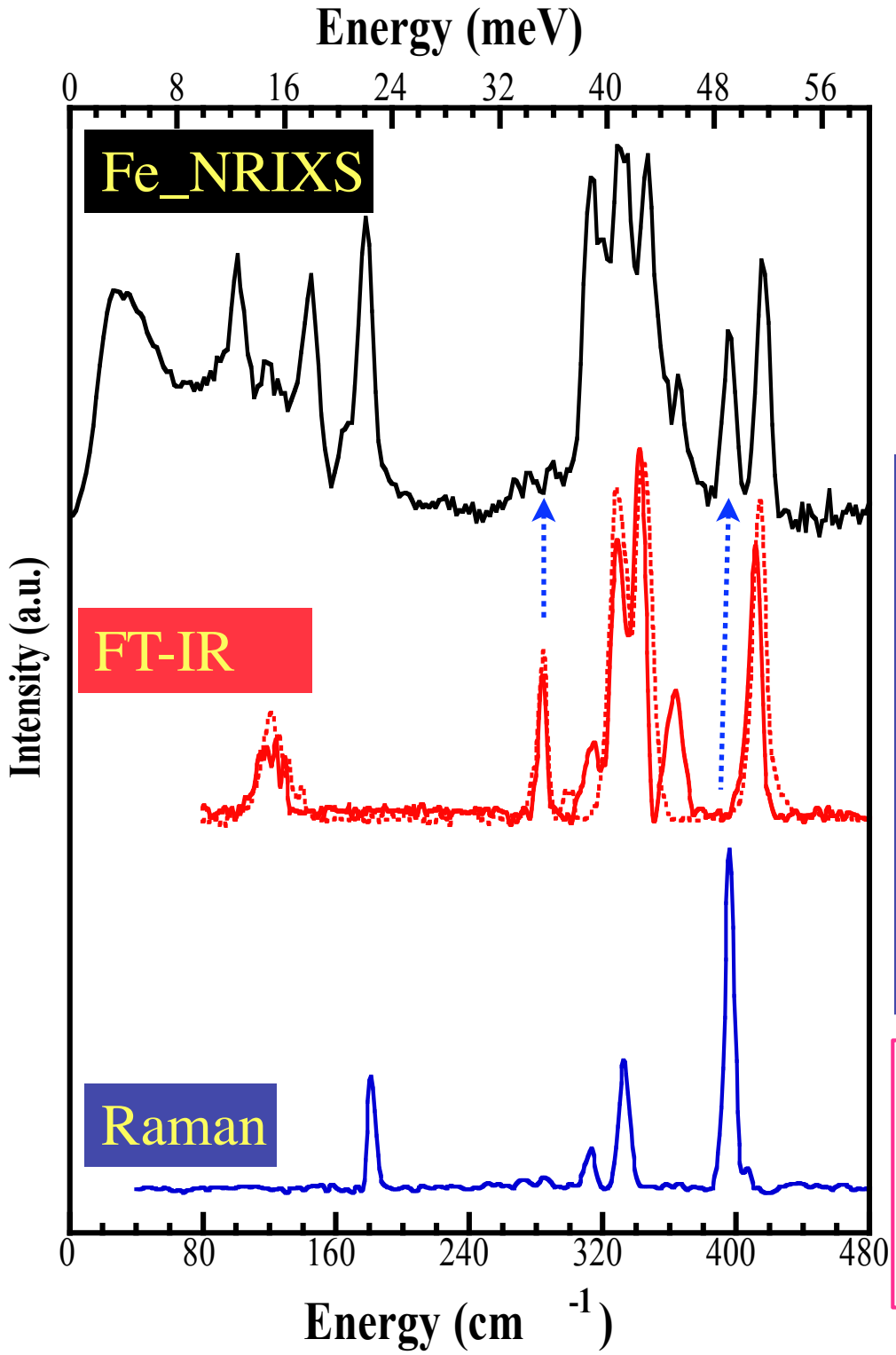





vib. density of states (cm)
0.12
0.1
0.08
0.06
0.04
0.02
0
Fe(P)(Br)
Fe(P)(Cl)
Fe-N$_{pyr}$
Fe-Br
Fe-Cl
acoustic
modes
doming
0
100
200
300
400
$E - E_0/hc$ (cm^{-1})




NH
N
N
HN



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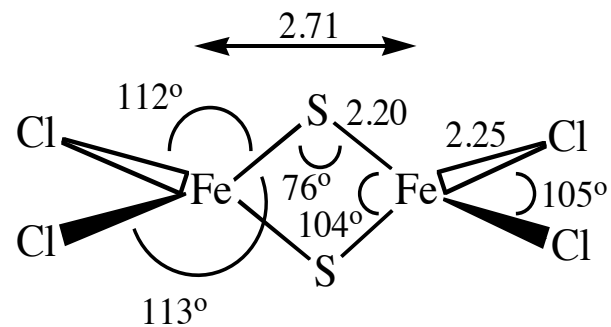

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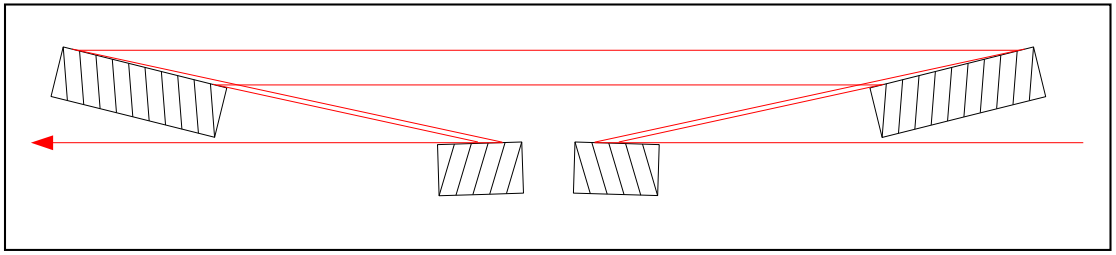
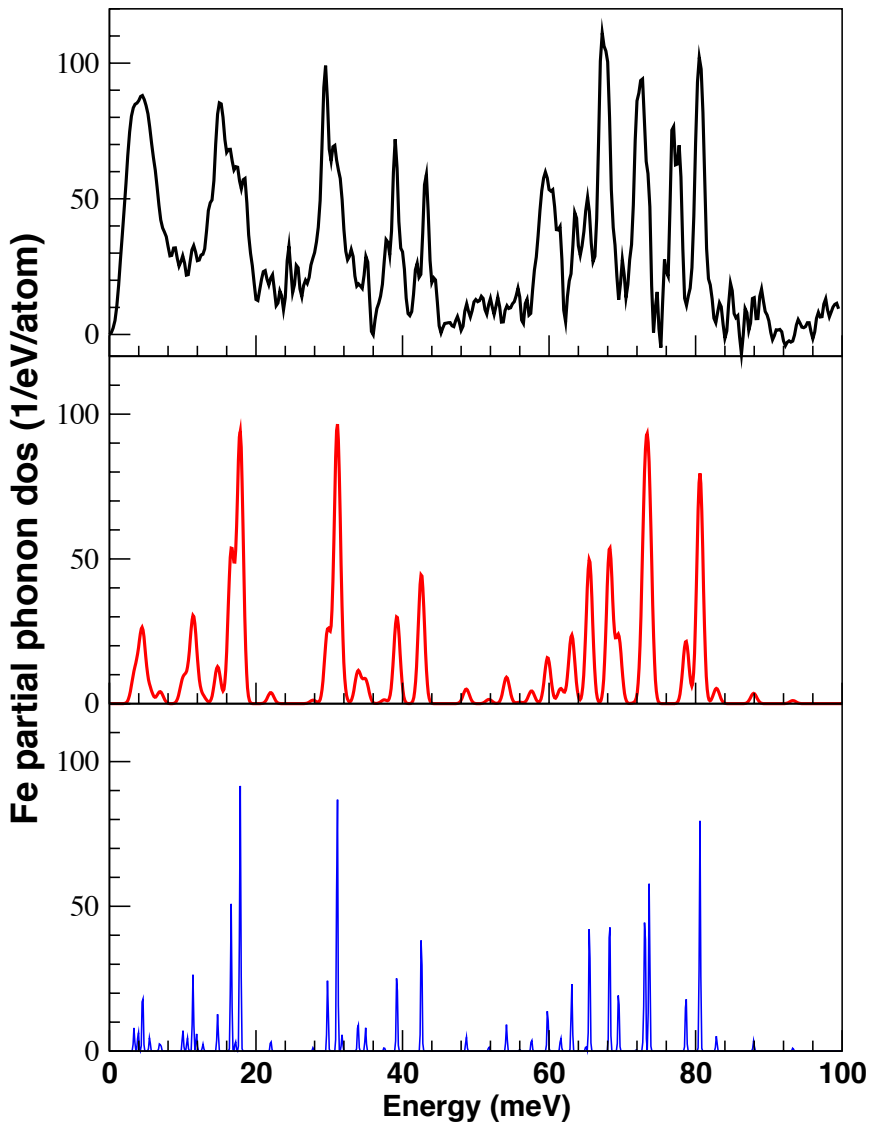
Matt Smith, et al, Inorganic Chemistry, 2005, 44,5562

~ 0.1meV, all-vacuum high resolution monochromator





Protonation state of oxo-ligand in heme protein intermediates:

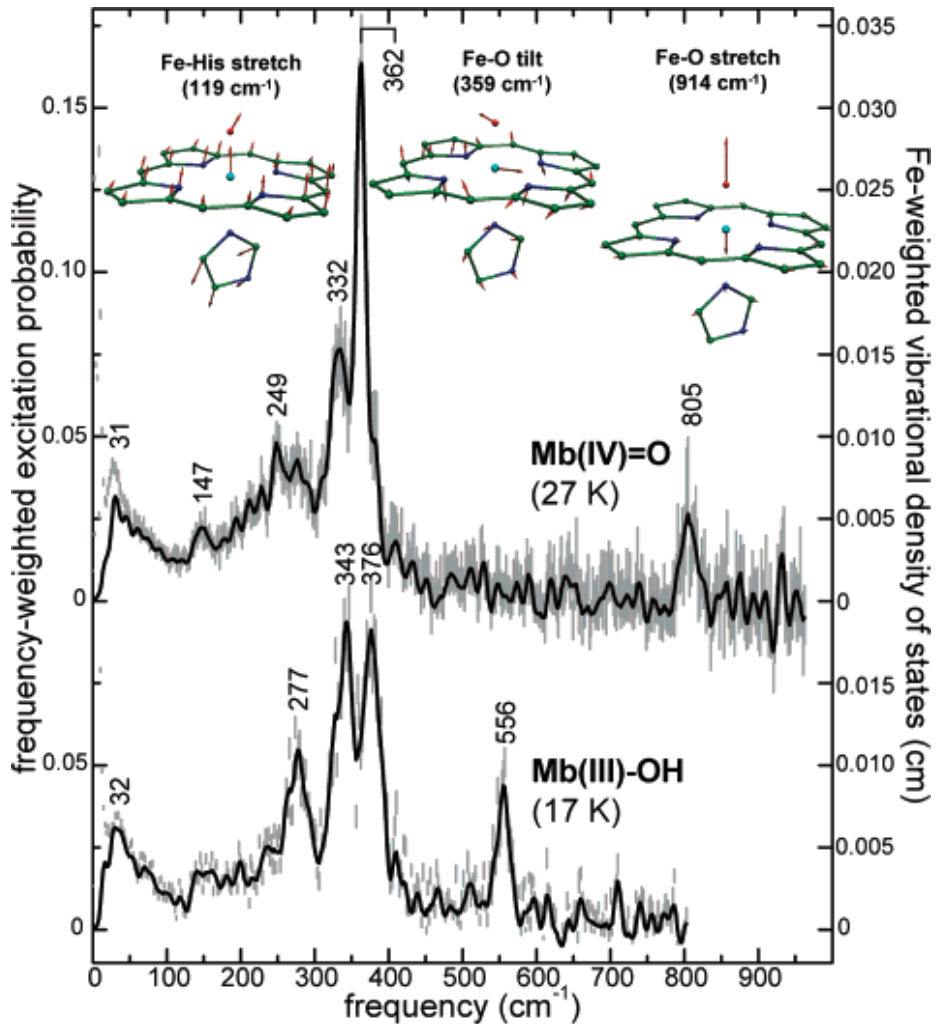
J|A|C|S
COMMUNICATIONS

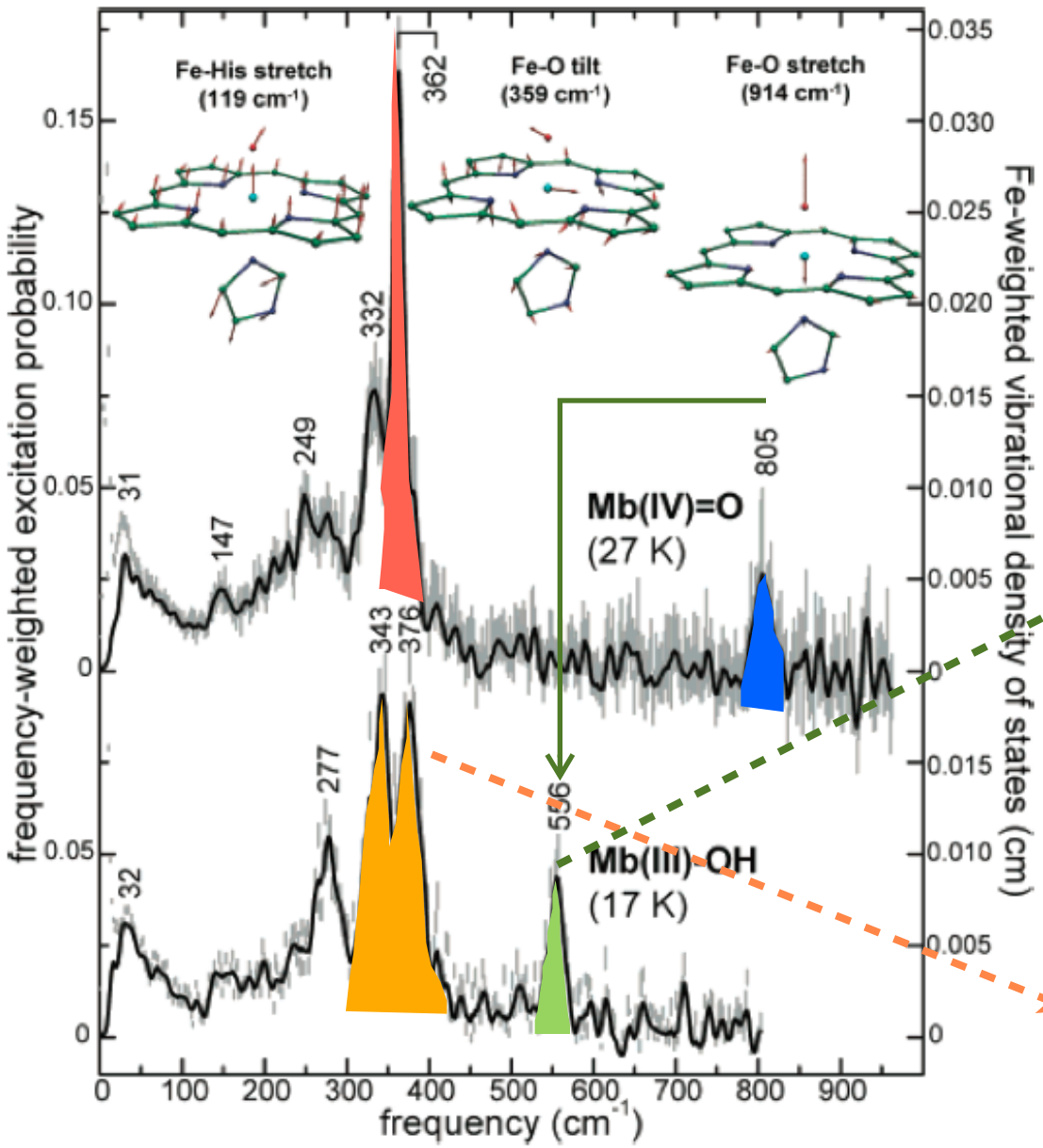
Published on Web 01/18/2008

Synchrotron-Derived Vibrational Data Confirm Unprotonated Oxo Ligand in Myoglobin Compound II

Weiqiao Zeng,† Alexander Barabanschikov,† Yunbin Zhang,† Jiyong Zhao,‡ Wolfgang Sturhahn,‡ E. Ercan Alp,‡ and J. Timothy Sage*,†

J. AM. CHEM. SOC. 2008, *130*, 1816−1817





Fe-O stretching

Fe-O tilting

Fe-O tilting

Downshift of Fe-O stretching frequency by protonation of oxo-ligand

Lifting of degeneracy of tilting frequency by protonation of the oxo-ligand

Figure 1. Vibrational dynamics of the heme Fe reveal an unprotonated oxo ligand in Mb(IV)=O, in contrast with the bound hydroxyl group in Mb(III)−OH. Protonation of the oxo ligand results in a downshift of the Fe−O stretching frequency from 805 cm^{-1} to 556 cm^{-1}, and splits the Fe−O tilting vibrations, which are degenerate near 362 cm^{-1} in Mb(IV)=O, but are separated by 33 cm^{-1} in the asymmetrically protonated heme Mb(III)−OH complex. Error bars represent the normalized experimental signal,