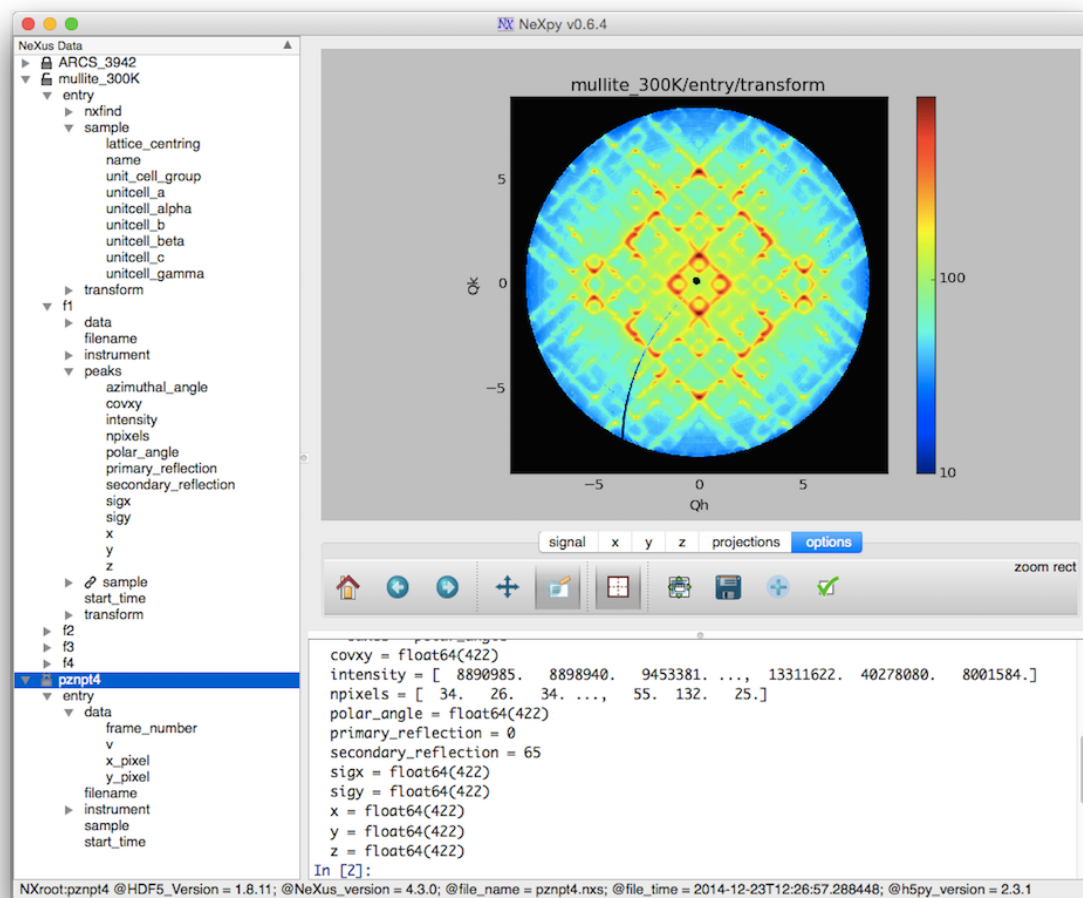


Single Crystal Diffuse Scattering

Ray Osborn

Neutron and X-ray Scattering Group
Materials Science Division
Argonne National Laboratory



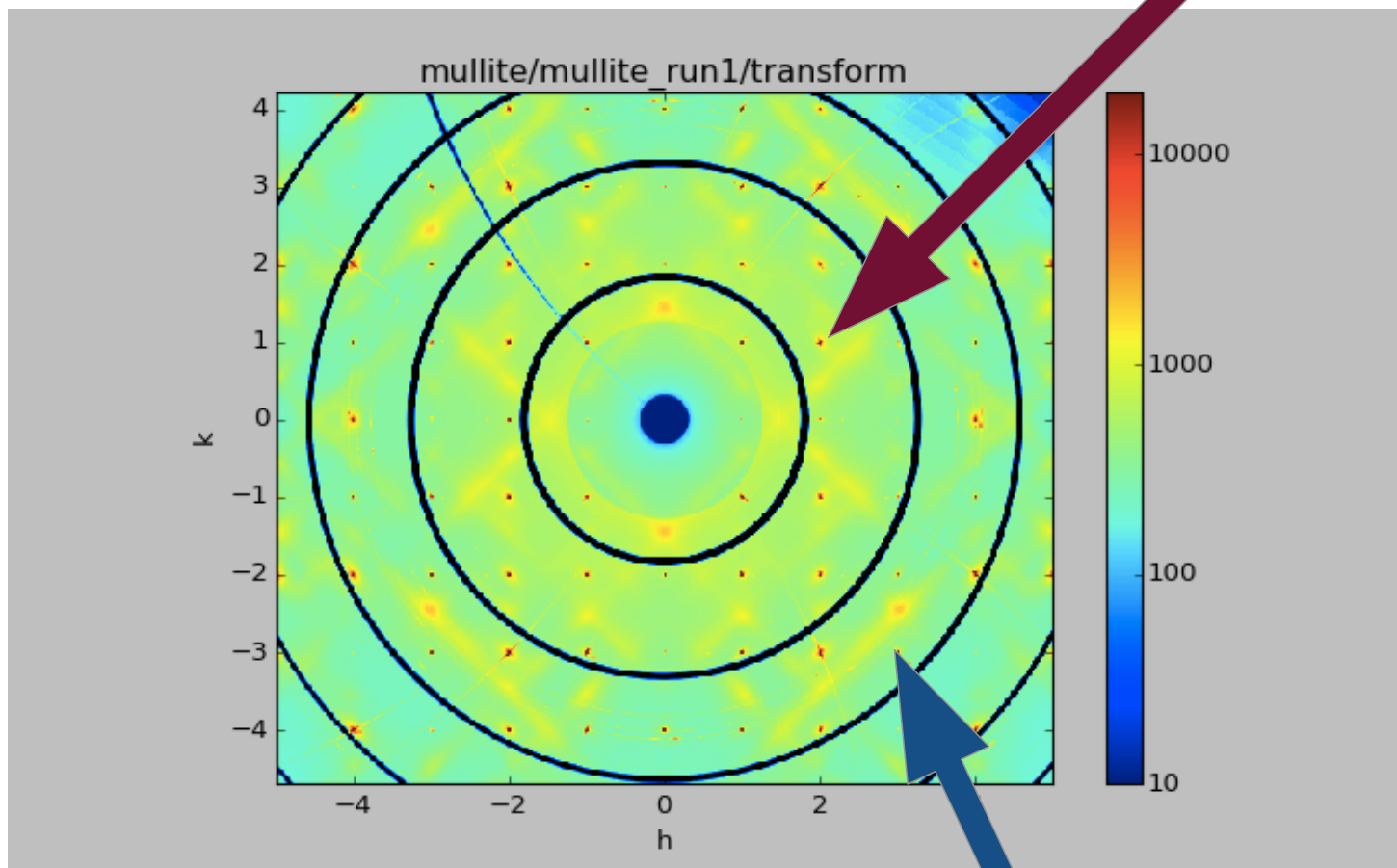
Outline

- ▶ What is diffuse scattering?
 - What does it look like?
 - What causes it?
 - Who started it?
- ▶ What is it good for?
 - A random walk through disordered materials
- ▶ How do I model it?
 - A few equations
 - Rules of thumb
- ▶ Case Study 1: Diffuse scattering from vacancies in mullite
- ▶ Case Study 2: Huang scattering in bilayer manganites
- ▶ How do I look at static disorder?
 - Neutrons vs X-rays
 - Corelli - Diffuse scattering with elastic discrimination
- ▶ Diffuse scattering - the musical



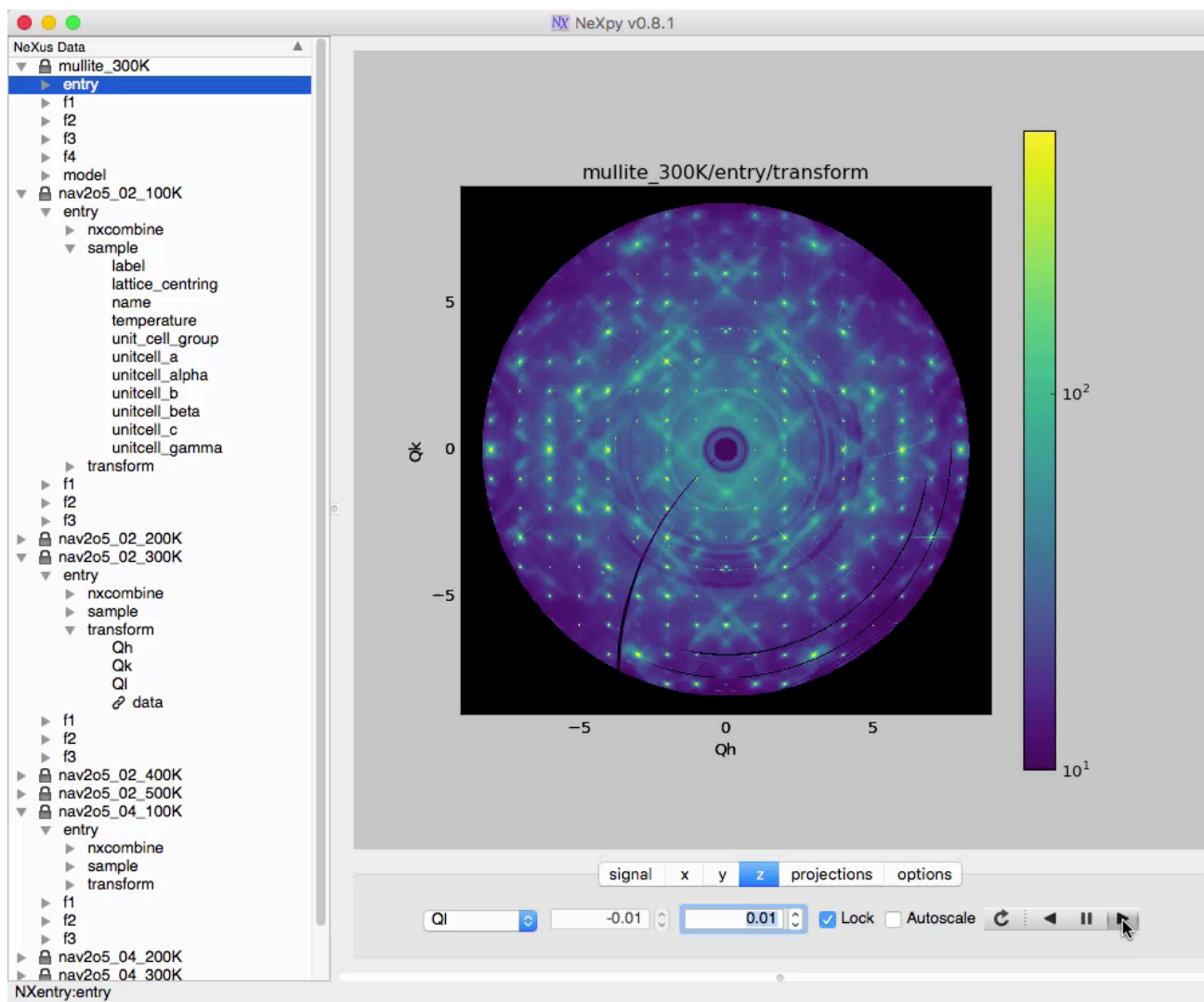
Bragg Scattering vs Diffuse Scattering

Bragg Scattering
Average Structure



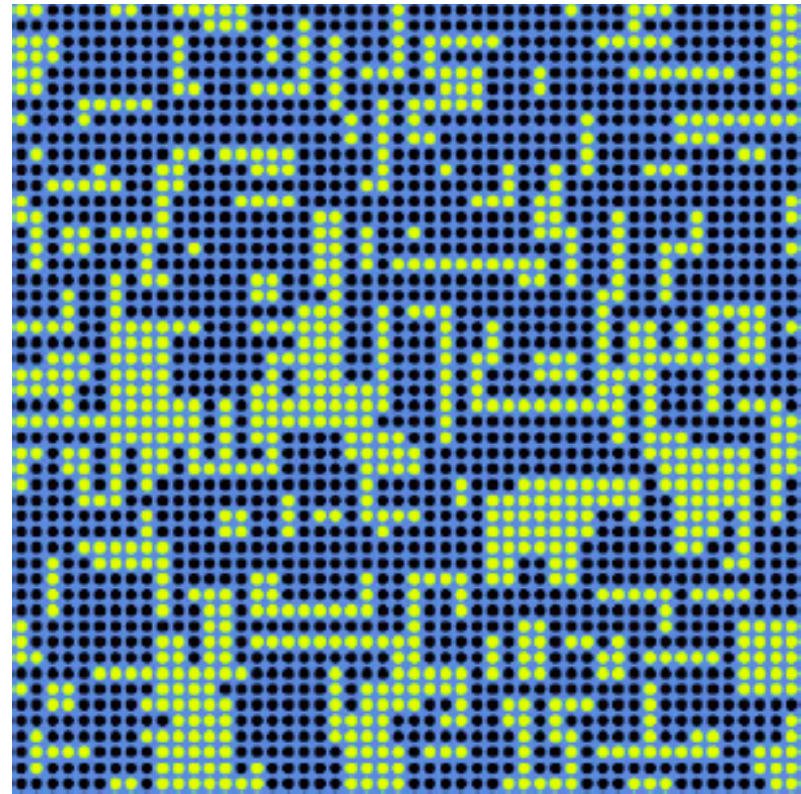
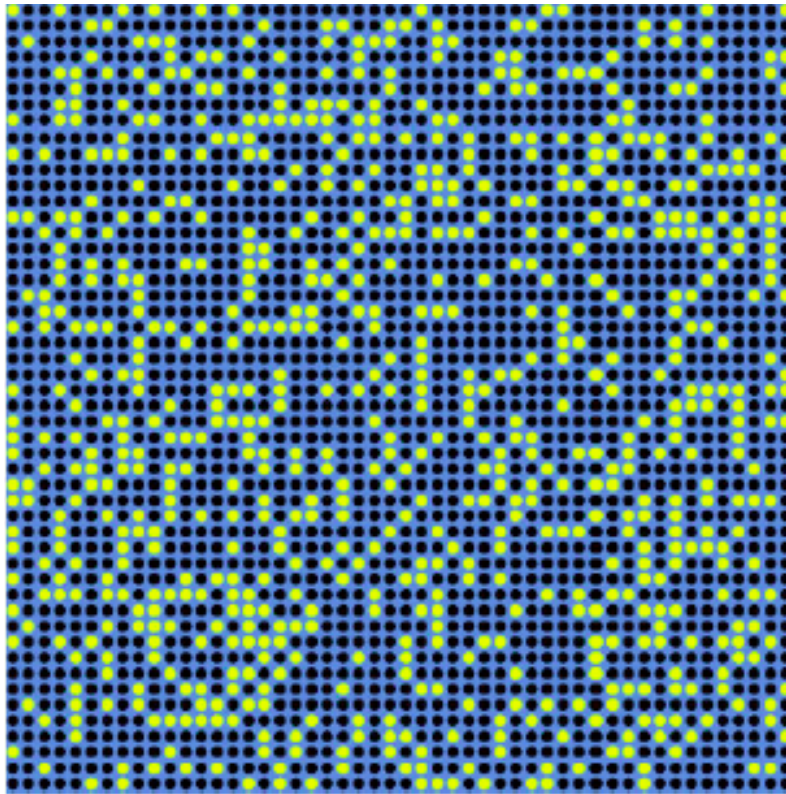
Diffuse Scattering
Deviations from the Average Structure

Single Crystal Diffuse Scattering in 3D



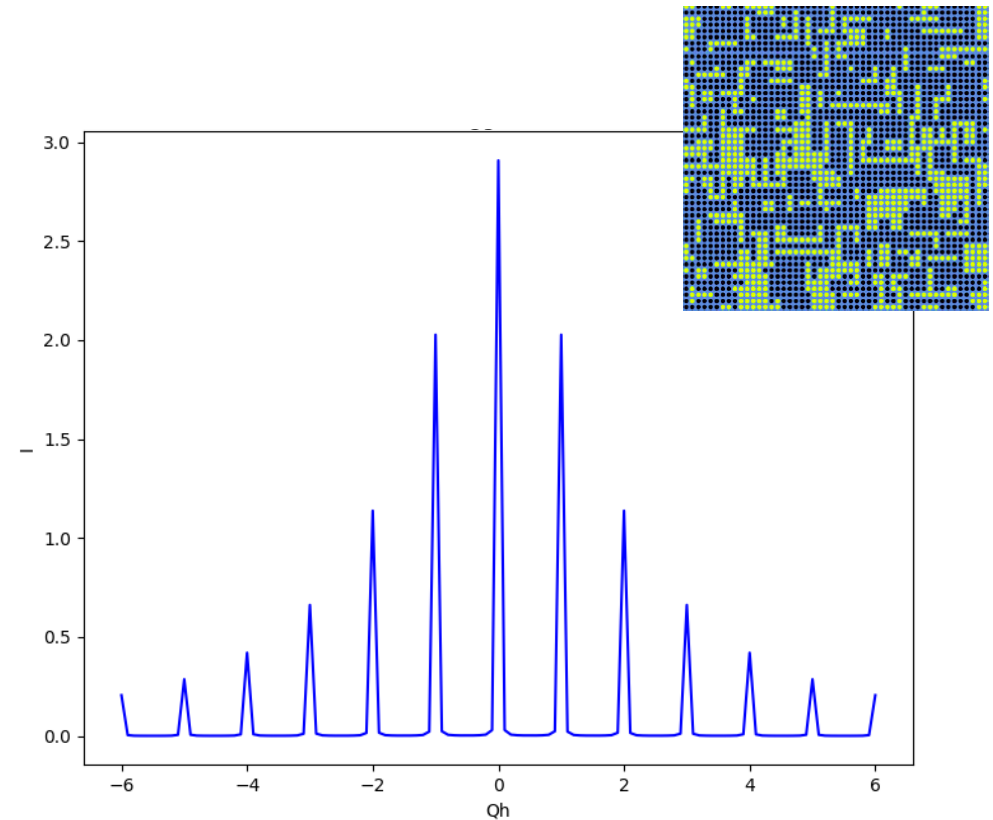
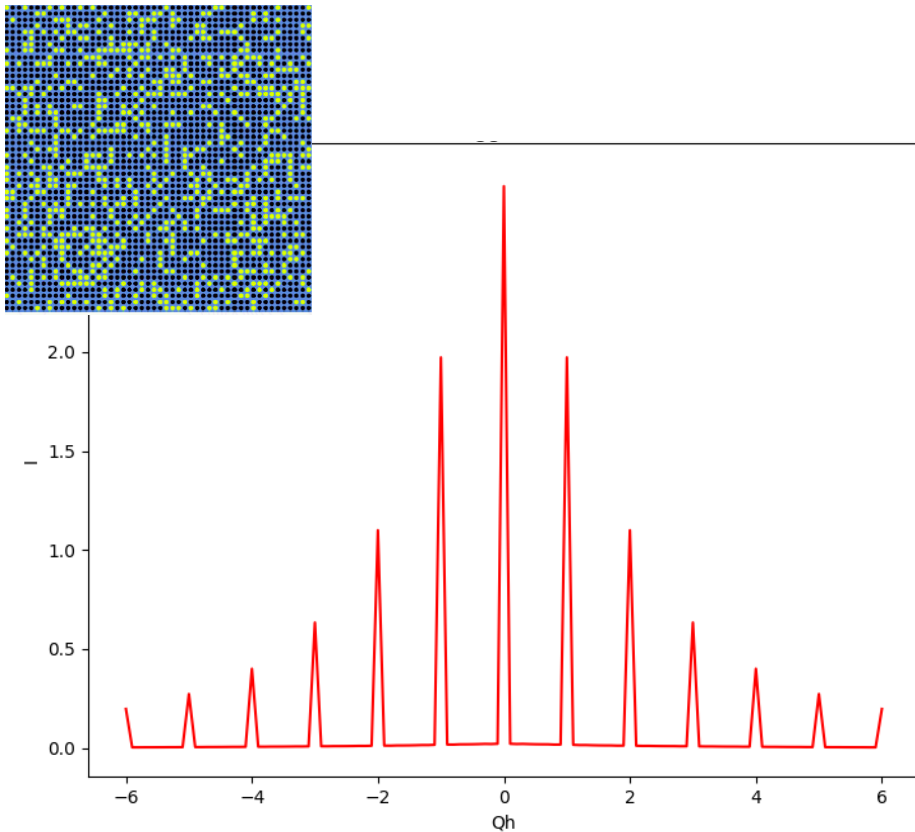
Simple Example of Disorder

- ▶ In these examples, 30% of atoms (blue dots) have been replaced by vacancies (green dots)
 - Left-Hand-Side: random substitution
 - Right-Hand-Side: high probability of vacancy clusters
 - Thanks to Thomas Proffen



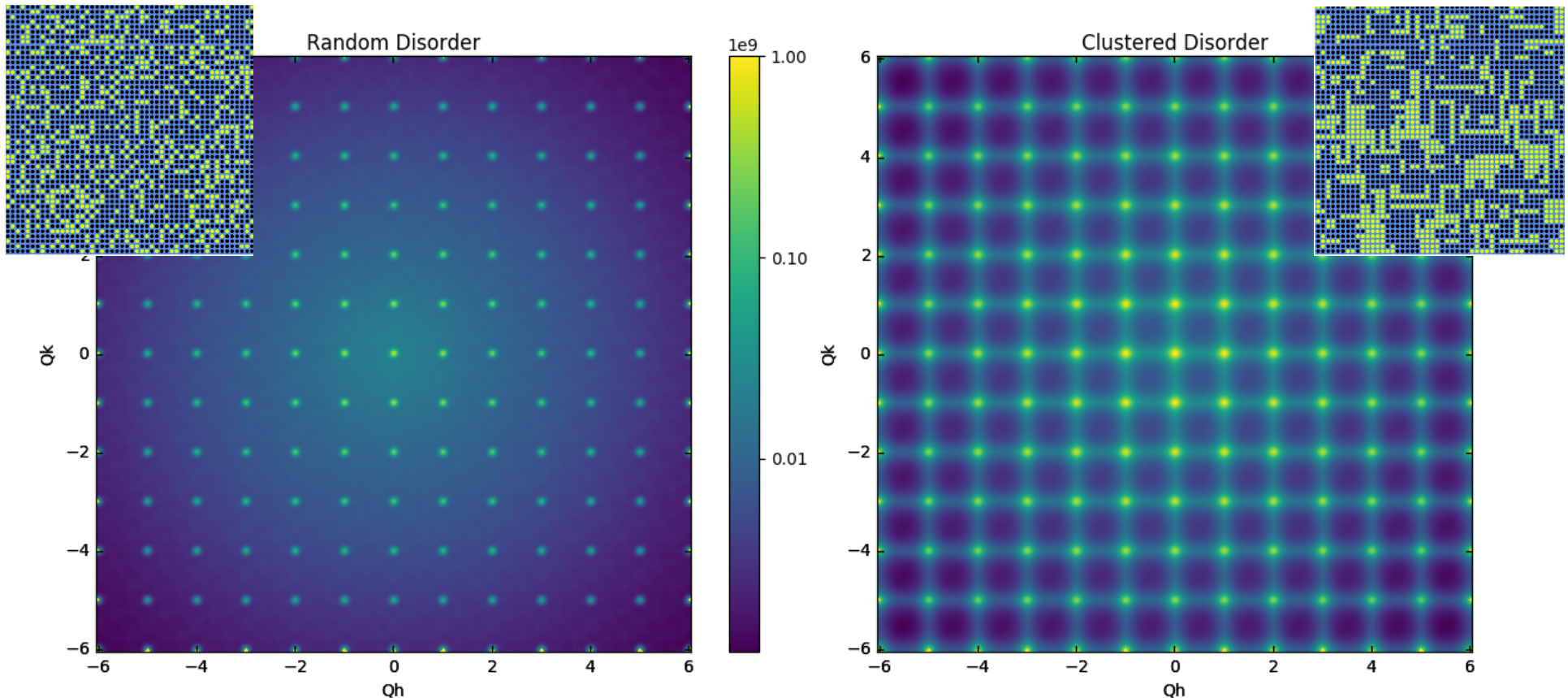
Bragg Scattering

- ▶ Bragg scattering is determined by the average structure.
 - Since the average vacancy occupation is identical, both examples have identical Bragg peaks

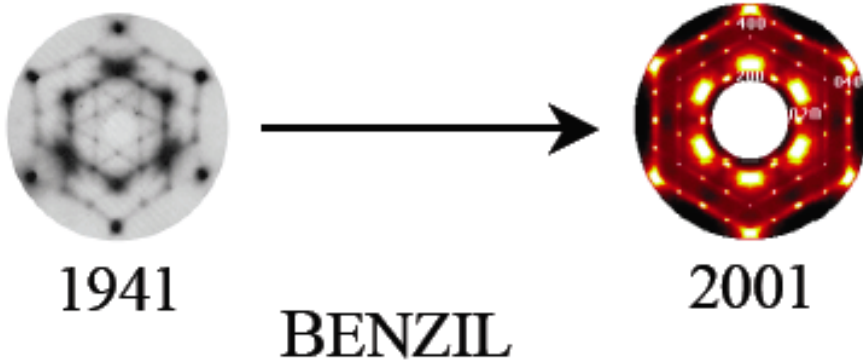


Diffuse Scattering

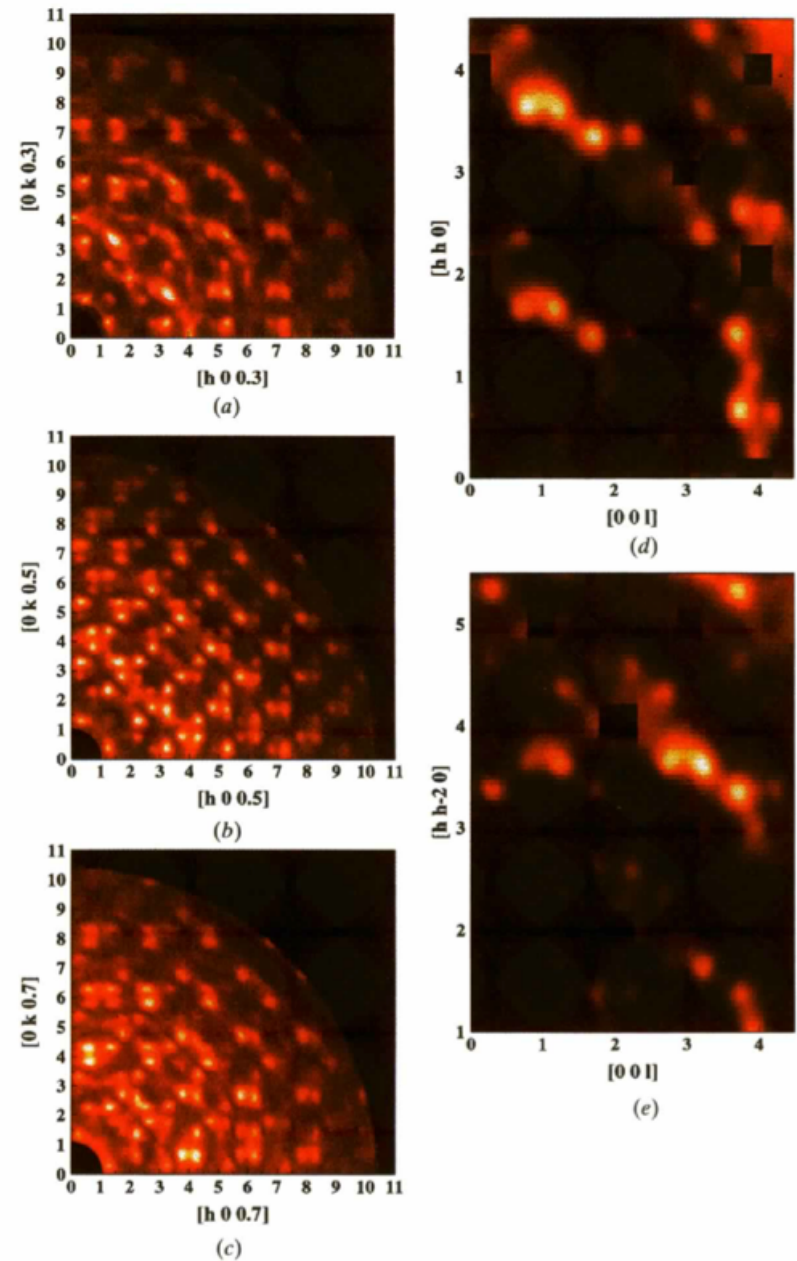
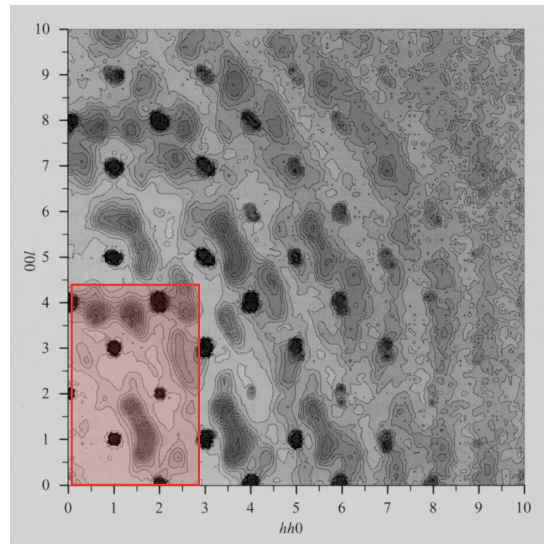
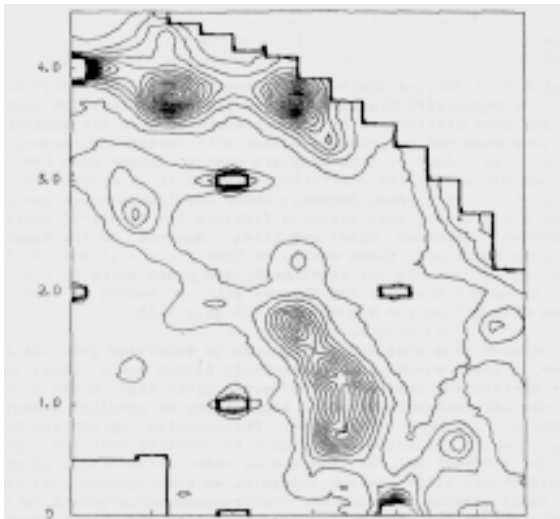
- ▶ The diffuse scattering is quite different in the two examples
 - Random vacancy distributions lead to a constant background (Laue monotonic scattering)
 - Vacancy clusters produce rods of diffuse scattering connecting the Bragg peaks



An Ultra-Short History of Advances in Diffuse Scattering



Yttria-Stabilized Zirconia



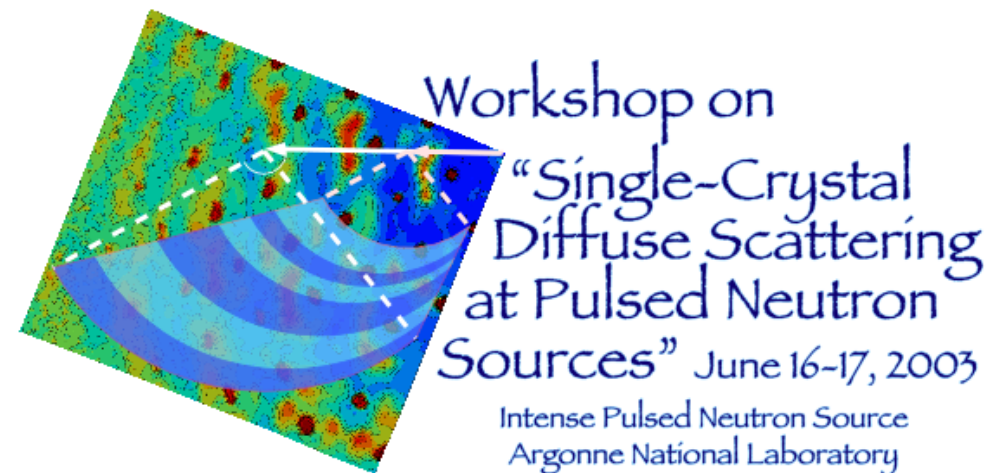
T. Proffen and T. R. Welberry J. Appl. Cryst. **31**, 318 (1998)

What is it good for?



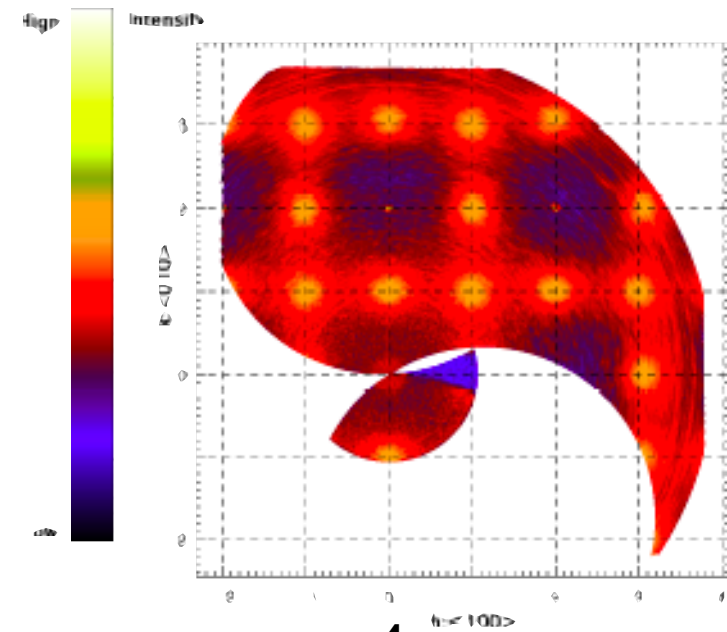
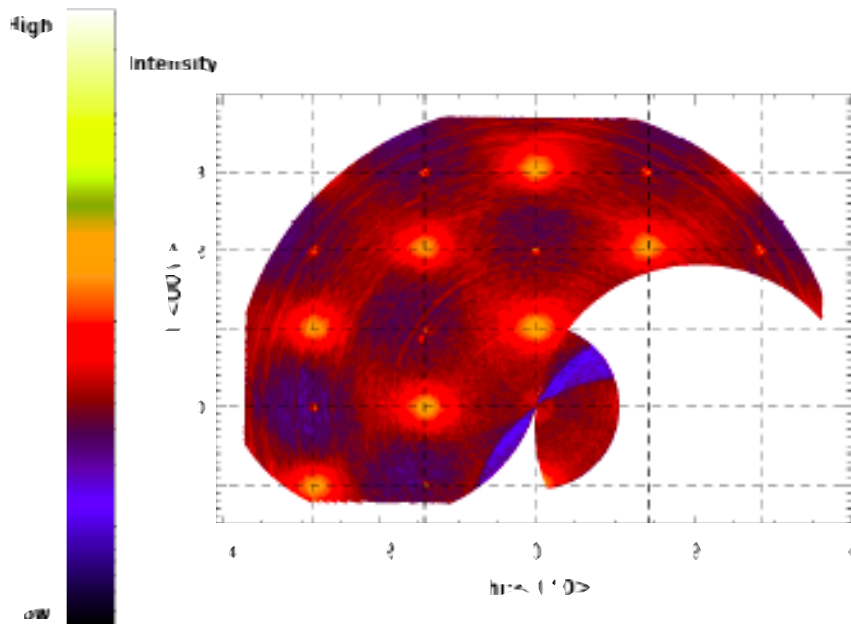
Science Impacted by Diffuse Scattering

- ▶ Subjects identified at the *Workshop on Single Crystal Diffuse Scattering at Pulsed Neutron Sources*
 - Stripes in cuprate superconductors
 - Orbital correlations in transition metal oxides (including CMR)
 - Nanodomains in relaxor ferroelectrics
 - Defect correlations in fast-ion conductors
 - Geometrically frustrated systems
 - Critical fluctuations at quantum phase transitions
 - Orientational disorder in molecular crystals
 - Rigid unit modes in framework structures
 - Quasicrystals
 - Atomic and magnetic defects in metallic alloys
 - Molecular magnets
 - Defect correlations in doped semiconductors
 - Microporous and mesoporous compounds
 - Host-guest systems
 - Hydrogen-bearing materials
 - Soft matter - protein configurational disorder using polarization analysis of spin-incoherence
 - Low-dimensional systems
 - Intercalates
 - Structural phase transitions in geological materials



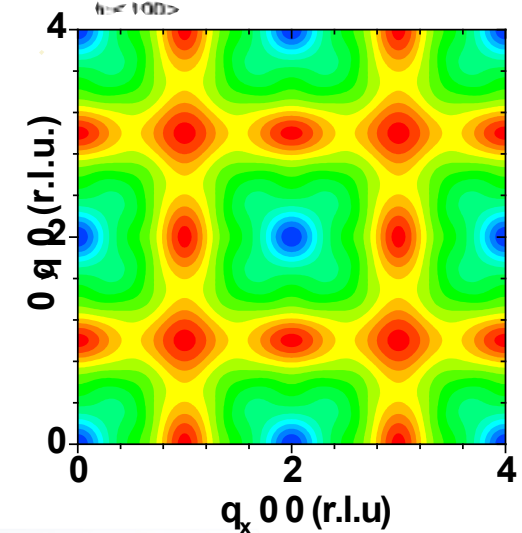
<http://www.neutron.anl.gov/diffuse/>

Diffuse Scattering from Metallic Alloys

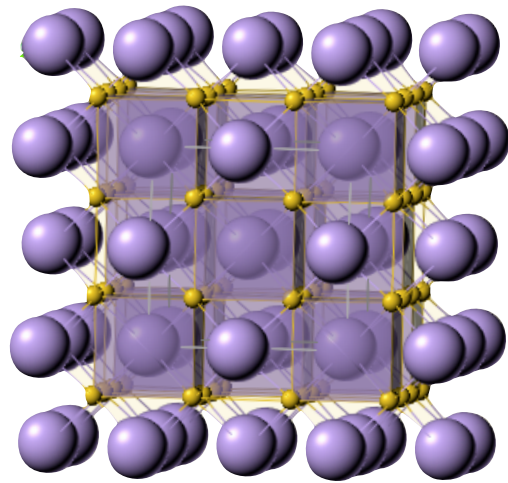


Short-range Order in Null Matrix $^{62}\text{Ni}_{0.52}\text{Pt}_{0.52}$

J. A. Rodriguez, S. C. Moss, J. L. Robertson, J. R. D. Copley, D. A. Neumann, and J. Major
Phys. Rev. B **74**, 104115

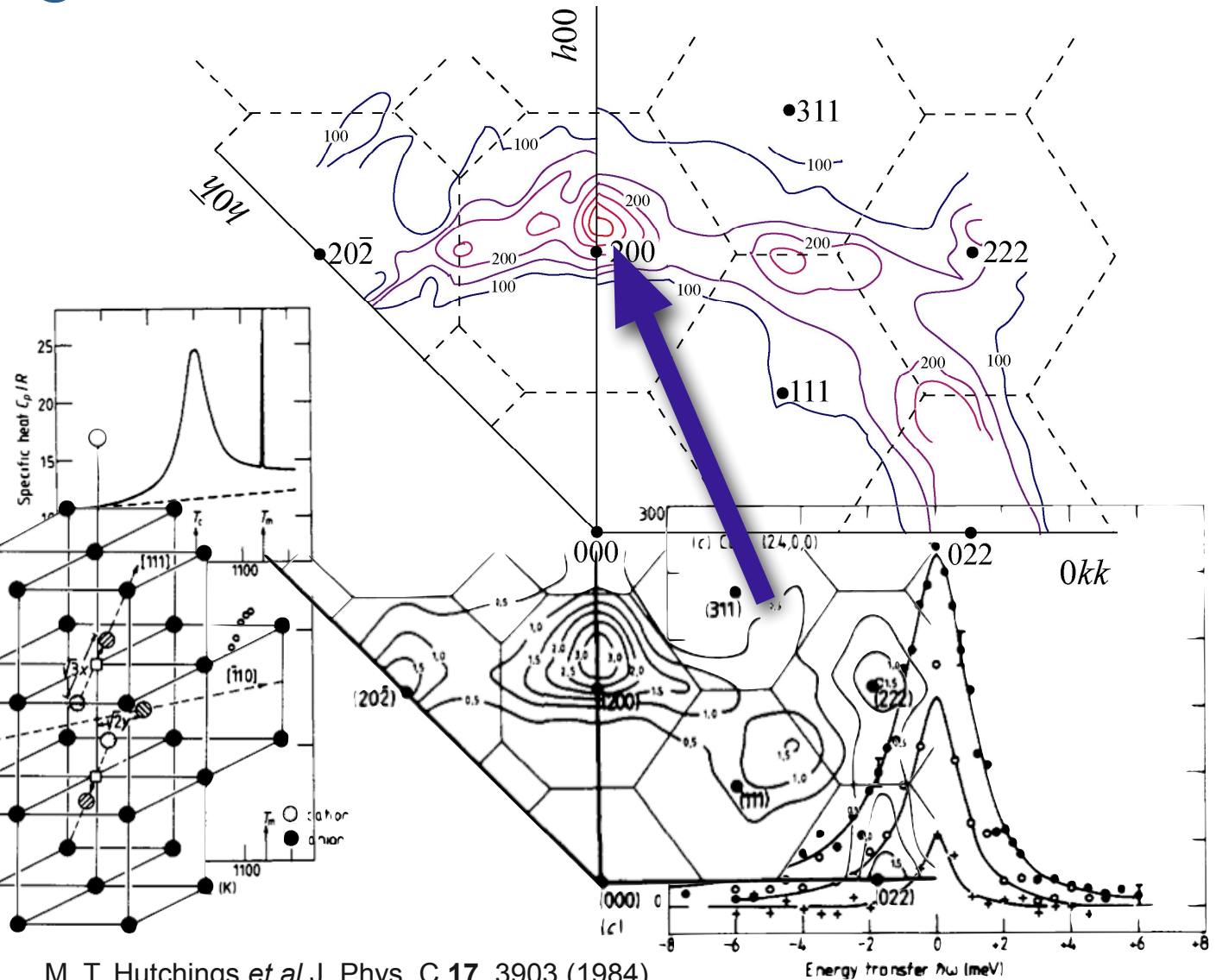
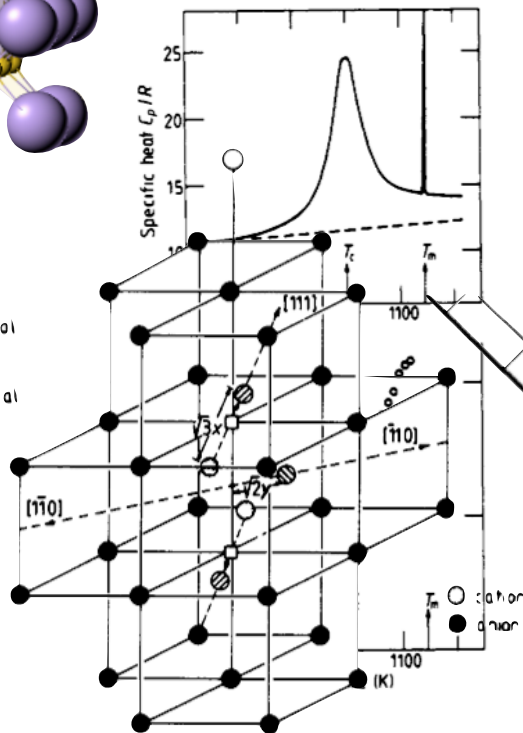


Diffuse Scattering from a Fast-Ion Conductor



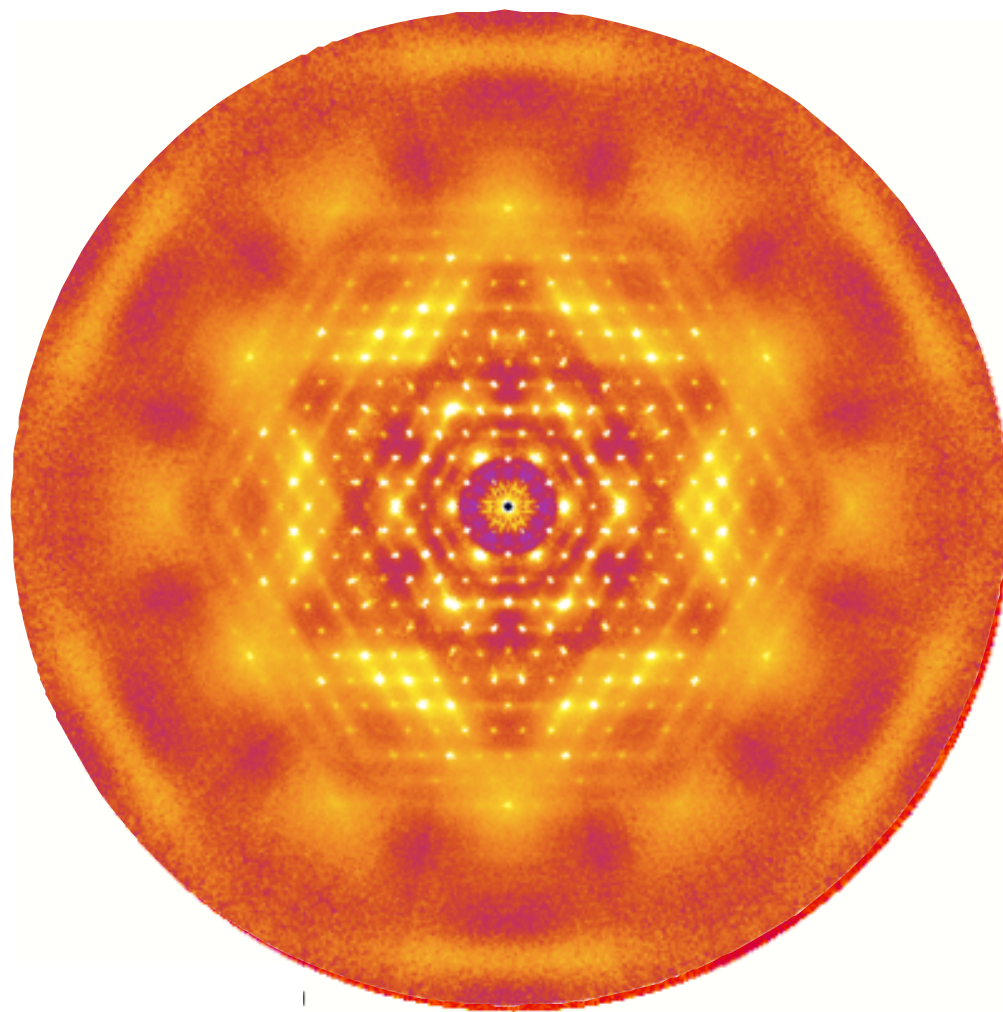
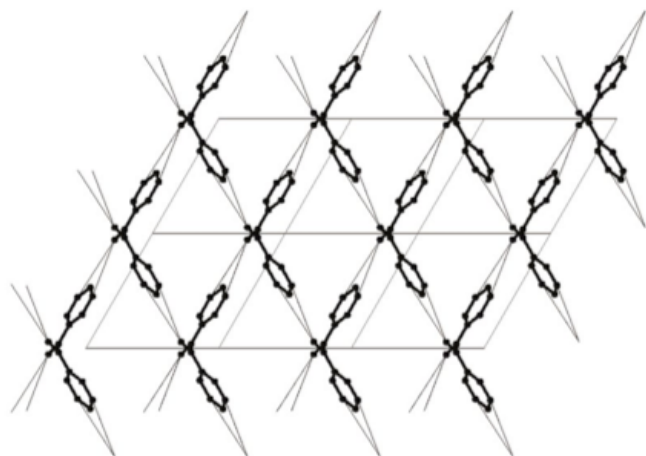
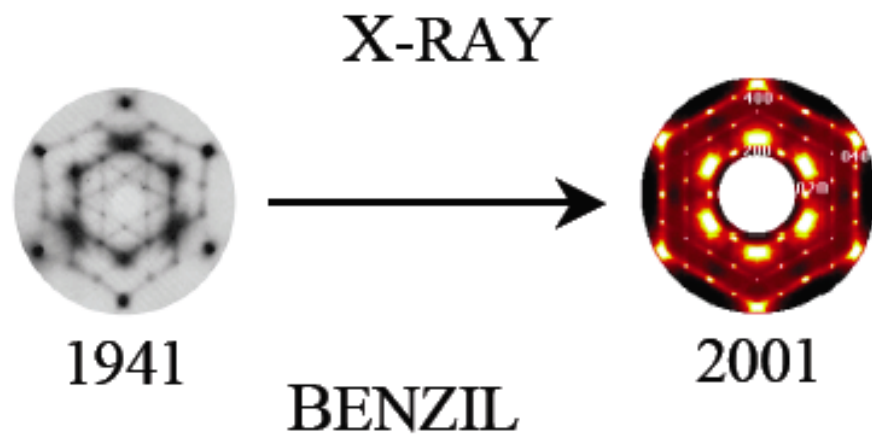
CaF₂

- Frenkel pair
 - vacancy
 - ⊗ interstitial
- Relaxed anion
 - vacancy
 - ⊗ interstitial



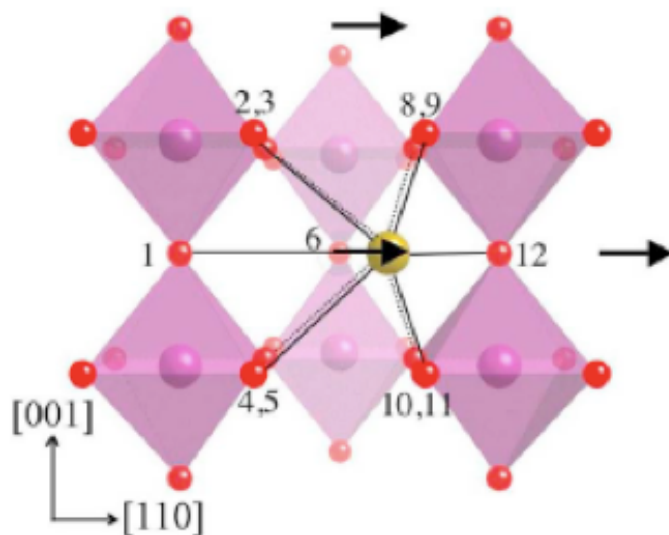
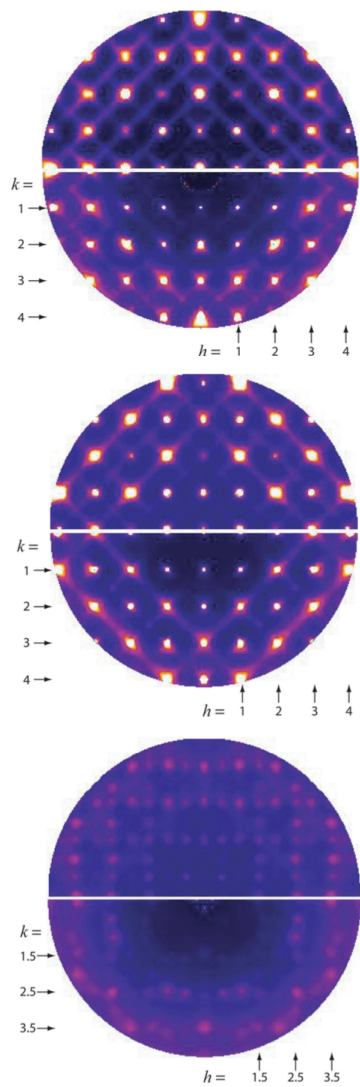
M. T. Hutchings *et al* J. Phys. C 17, 3903 (1984)

Diffuse Scattering from Molecular Solids

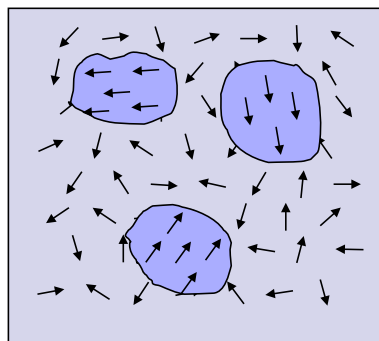


T. R. Welberry *et al* J. Appl. Cryst. **36**, 1400 (2003)

Diffuse Scattering from Relaxor Ferroelectrics

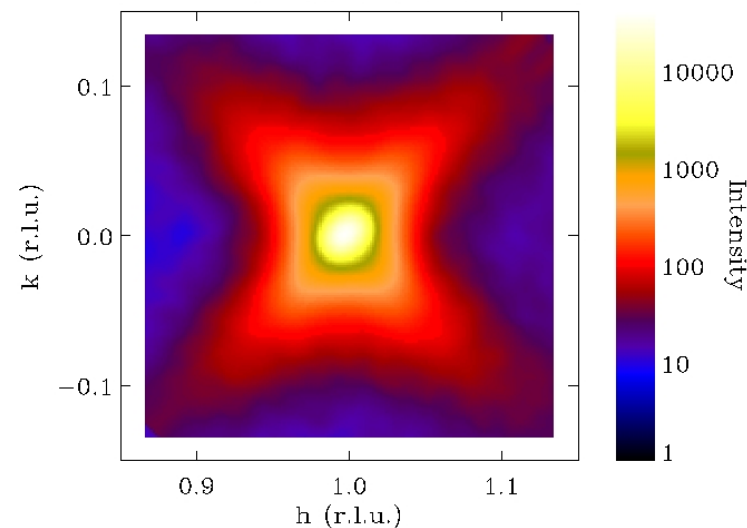


Lead Zinc-Niobate



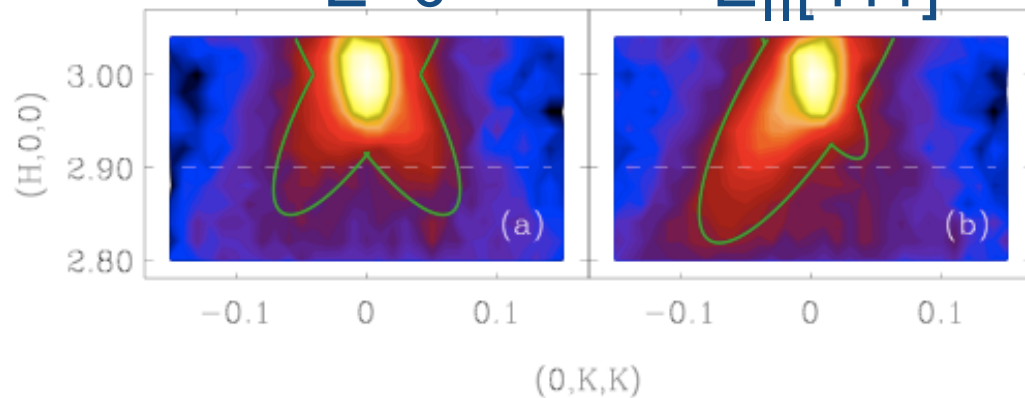
T. R. Welberry *et al* J. Appl. Cryst. **38**, 639 (2005)

Lead Magnesium-Niobate



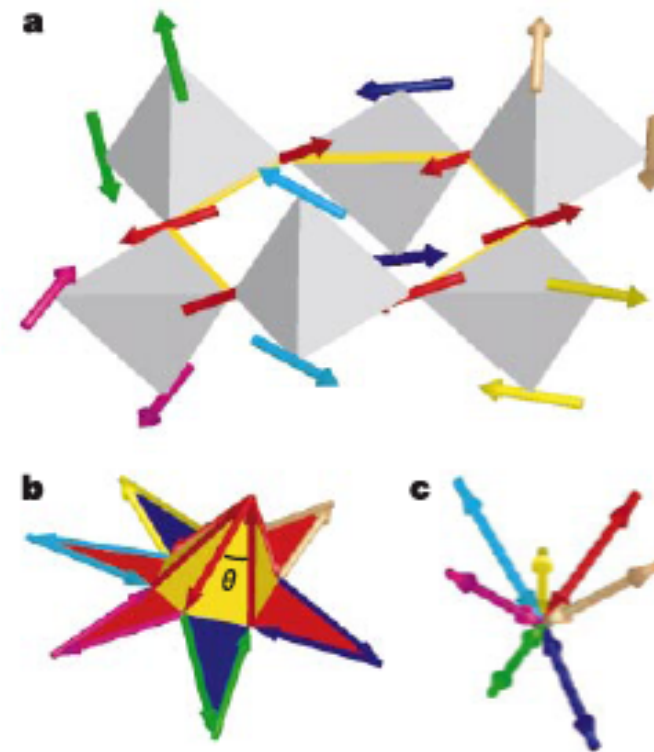
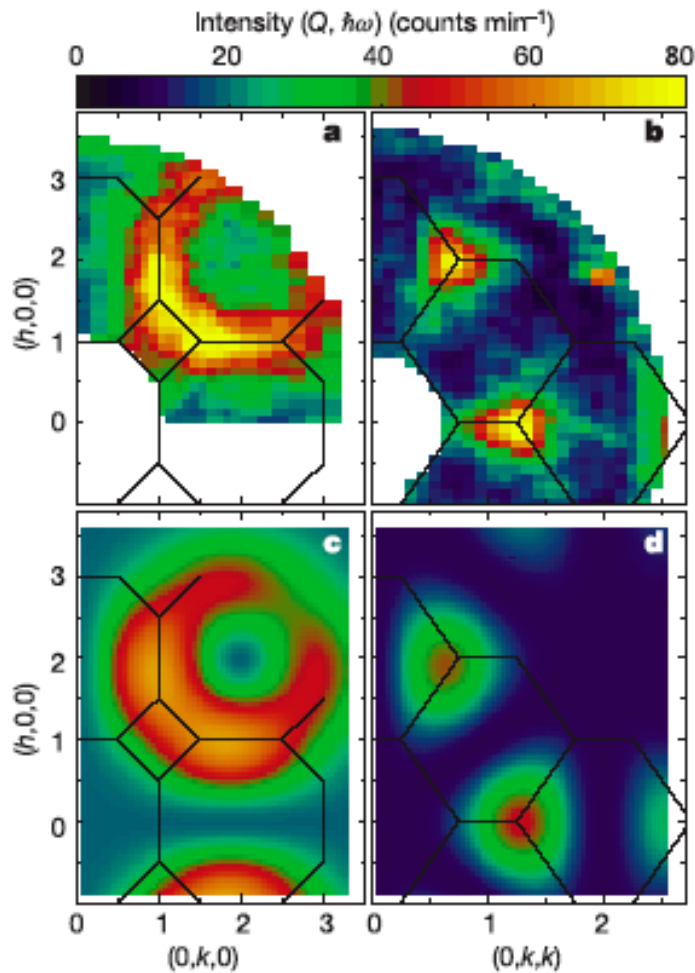
E=0

E||[111]



G. Xu, P. M. Gehring, G. Shirane, Phys. Rev. B **72**, 214106 (2005).

Magnetic Diffuse Scattering from Geometric Frustration



S.-H. Lee *et al* Nature **418**, 856 (2002)

How do I model it?

A Few Equations

V. M. Nield and D. A. Keen *Diffuse Neutron Scattering From Crystalline Materials* (2001)
 T. R. Welberry *Diffuse X-ray Scattering and Models of Disorder* (2004)

$$I = \sum_i \sum_j b_i b_j \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij})$$

▶ Laue Monotonic Diffuse Scattering

$$I = \bar{b}^2 \sum_{ij} \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij}) + N(\bar{b}^2 - \bar{b}^2); \quad \bar{b}^2 = (c_A b_A + c_B b_B)^2; \quad \bar{b}^2 = c_A c_B (b_B - b_A)^2$$

▶ Cowley Short-Range Order

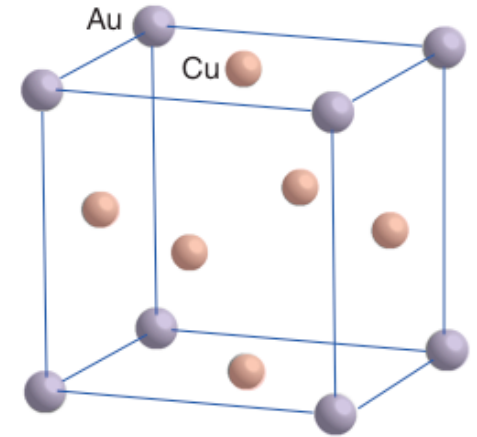
$$I_{diffuse} = N c_A c_B (b_B - b_A)^2 + \sum_{ij} \alpha_i c_B c_A (b_B - b_A)^2 \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij}); \quad \alpha_i = \left(1 - \frac{p_i}{c_A}\right)$$

▶ Warren Size Effect

$$I_{diffuse} = N c_A c_B (b_B - b_A)^2 \left(1 + \sum_{ij} \alpha_i \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij}) + \sum_{ij} \beta_i \exp(i\mathbf{Q} \cdot \mathbf{r}_{ij})\right); \quad \beta_i = f(\epsilon_{AA}^i, \epsilon_{BB}^i)$$

▶ Borie and Sparks Correlations

$$I = \sum_i \sum_j b_i b_j \exp(i\mathbf{Q} \cdot (\mathbf{R}_i - \mathbf{R}_j)) \left[1 + i\mathbf{Q} \cdot (\mathbf{u}_i - \mathbf{u}_j) - \frac{1}{2} (\mathbf{Q} \cdot (\mathbf{u}_i - \mathbf{u}_j))^2 + \dots\right]$$



J. M. Cowley, *J. Appl. Phys.* **21**, 24 (1950)

Three-Dimensional Pair Distribution Functions

- ▶ The ability to measure three-dimensional $S(\mathbf{Q})$ over a wide range of reciprocal space provides the 3D analog of PDF measurements.
 - Total PDFs if Bragg peaks and diffuse scattering can be measured simultaneously
 - Δ -PDFs if the Bragg peaks are eliminated
 - using the punch and fill method
- ▶ This would allow a model-independent view of the measurements in real space.

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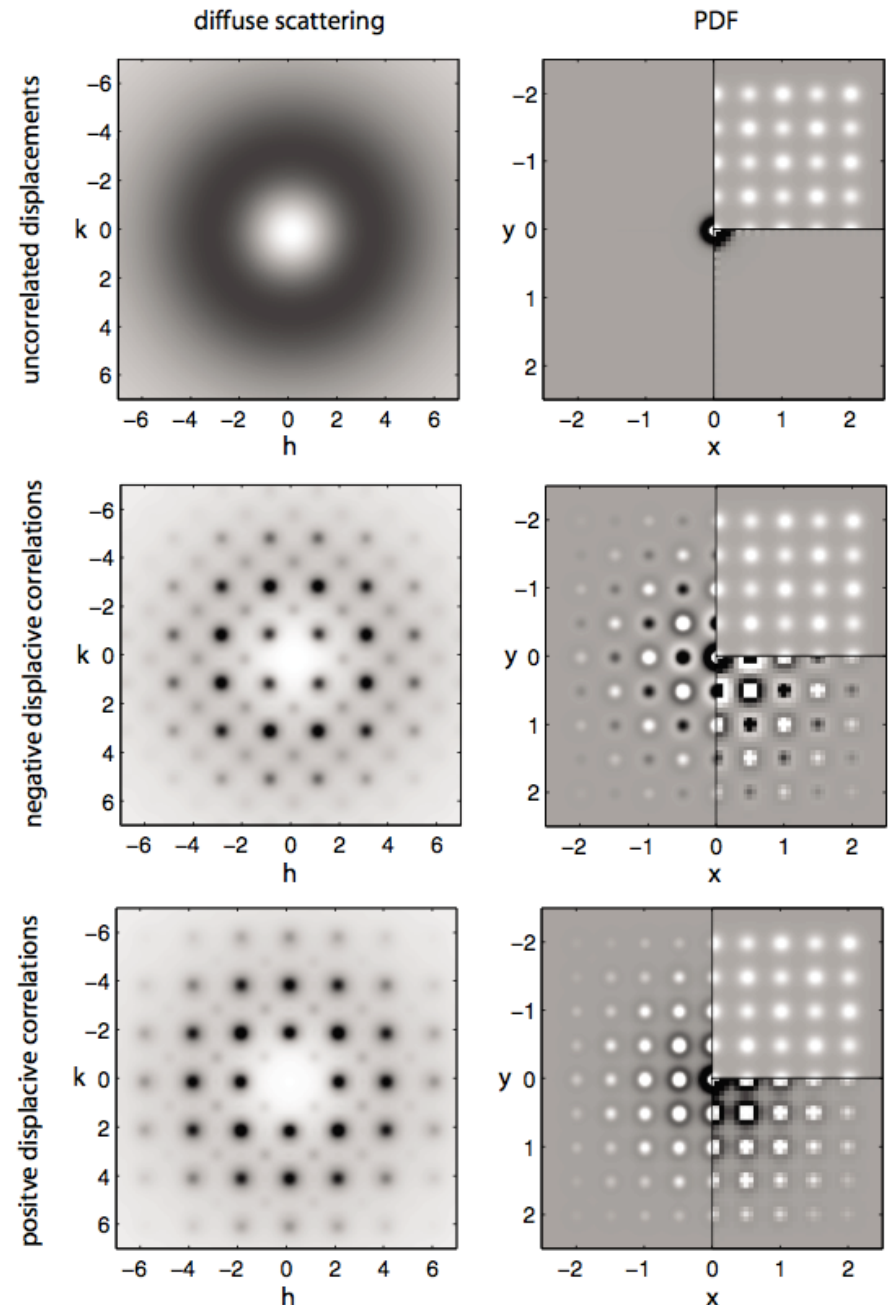
Z. Kristallogr. 2012, 227, 238–247 / DOI 10.1524/zkri.2012.1504

© by Oldenbourg Wissenschaftsverlag, München

The three-dimensional pair distribution function analysis of disordered single crystals: basic concepts

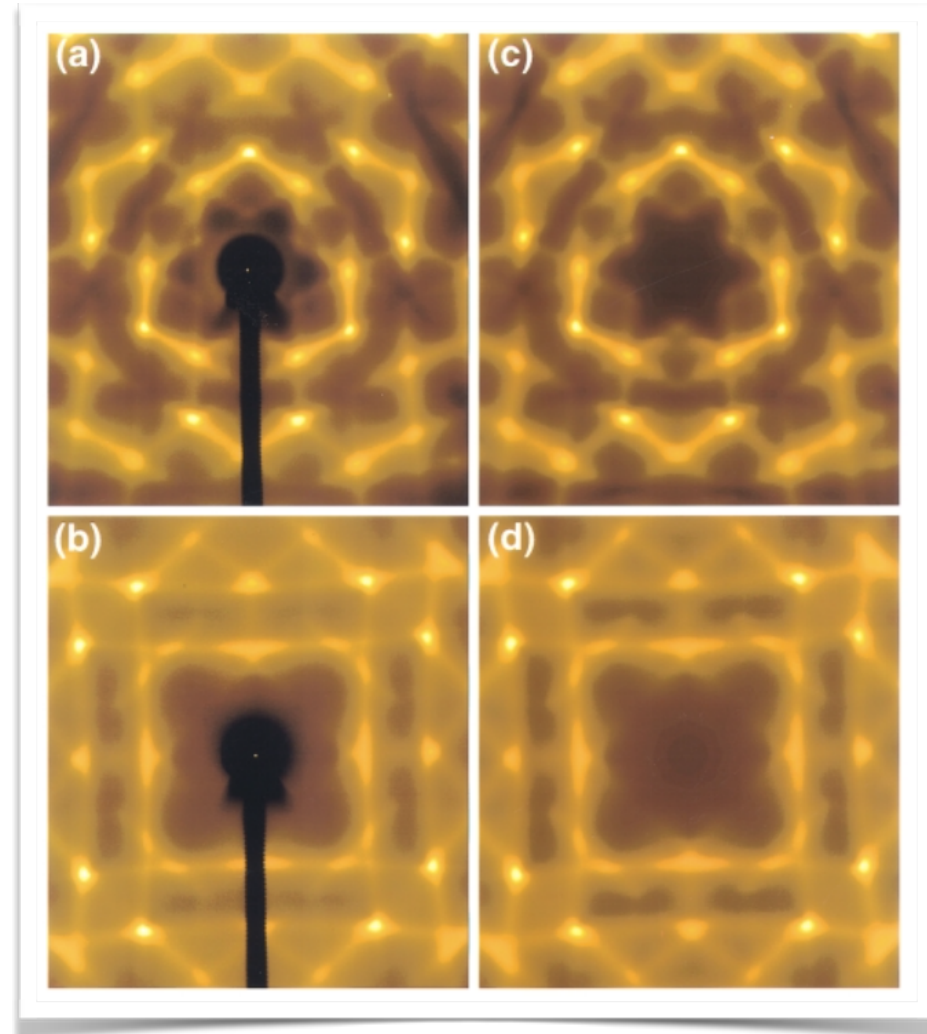
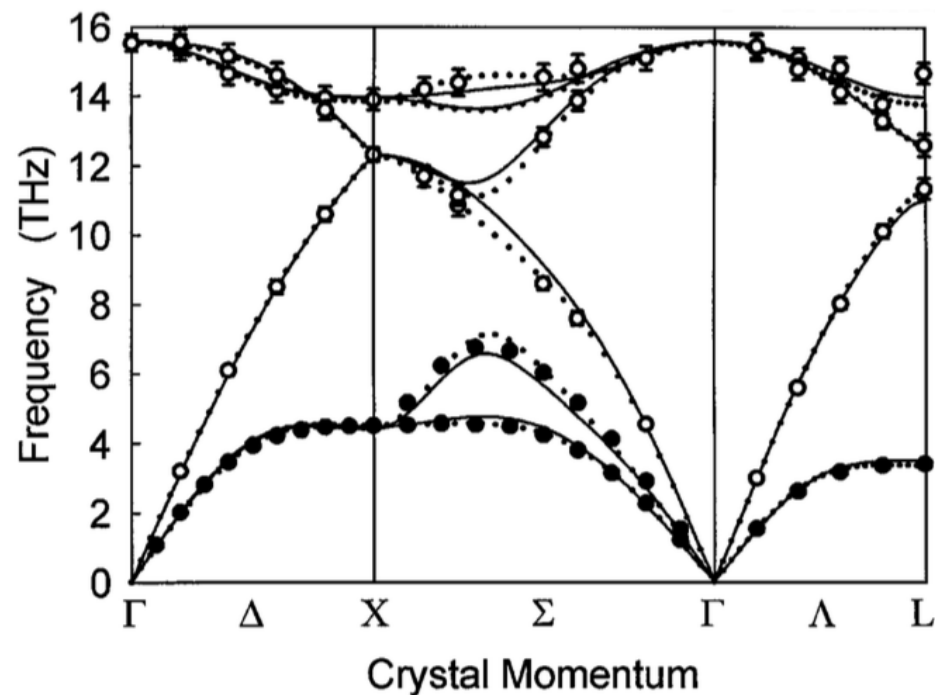
Thomas Weber* and Arkadiy Simonov

Laboratory of Crystallography, ETH Zurich Wolfgang-Pauli-Str. 10, 8093 Zurich, Switzerland



Thermal Diffuse Scattering

- ▶ Lattice vibrations produce deviations from the average structure even in perfect crystals
- ▶ X-ray scattering intensity is given by the integral over all the phonon branches at each \mathbf{Q}



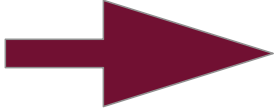



$$I_0 \propto f^2 e^{-2M} \sum_{j=1}^6 \frac{|\mathbf{q} \cdot \hat{\mathbf{e}}_j|^2}{\omega_j} \coth\left(\frac{\hbar\omega_j}{2k_B T}\right).$$

M. Holt, *et al*, Phys Rev Lett **83**, 3317 (1999).

Some Rules of Thumb (*thanks to Hans Beat Bürgi*)

Reciprocal space

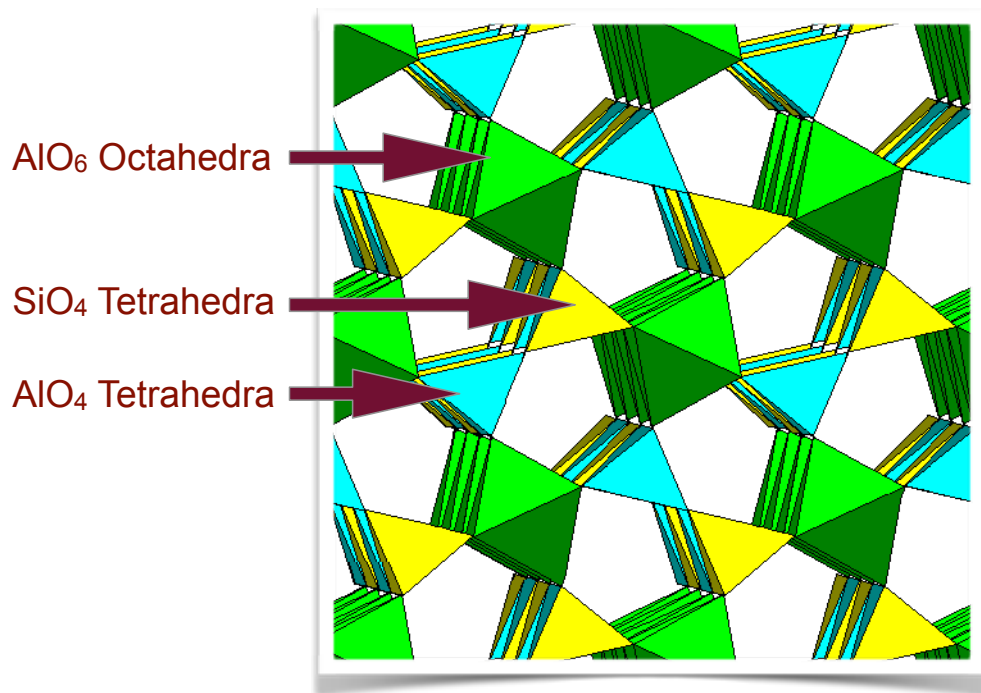
Direct space

- ▶ Only sharp Bragg reflections  ▶ 3D-periodic structure
 - ▶ no defects
- ▶ Sharp diffuse rods  ▶ 2D-periodic structure
 - ▶ perpendicular to the streaks
 - ▶ disordered in streak directions
- ▶ Sharp diffuse planes  ▶ 1D-periodic structure
 - ▶ perpendicular to the planes
 - ▶ disordered within the plane
- ▶ Diffuse clouds  ▶ 0D-periodic structure
 - ▶ no fully ordered direction

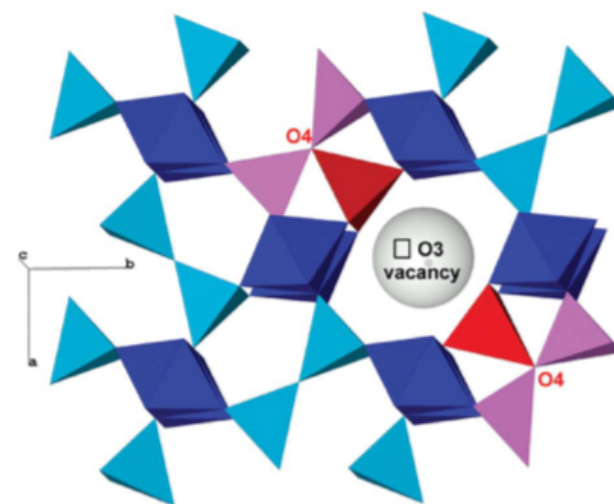
Case Study 1: Mullite

Mullite - A Case Study

- ▶ Mullite is a ceramic that is formed by adding O^{2+} vacancies to Sillimanite
 - Sillimanite has alternating AlO_4 and SiO_4 tetrahedra
 - Mullite has excess Al^{3+} occupying Si^{2+} sites for charge balance
- ▶ This results in strong vacancy-vacancy correlations



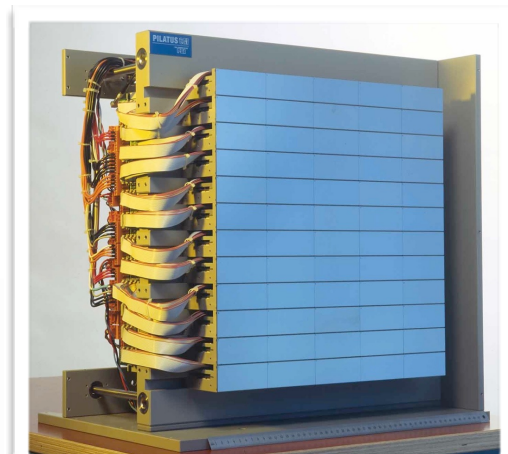
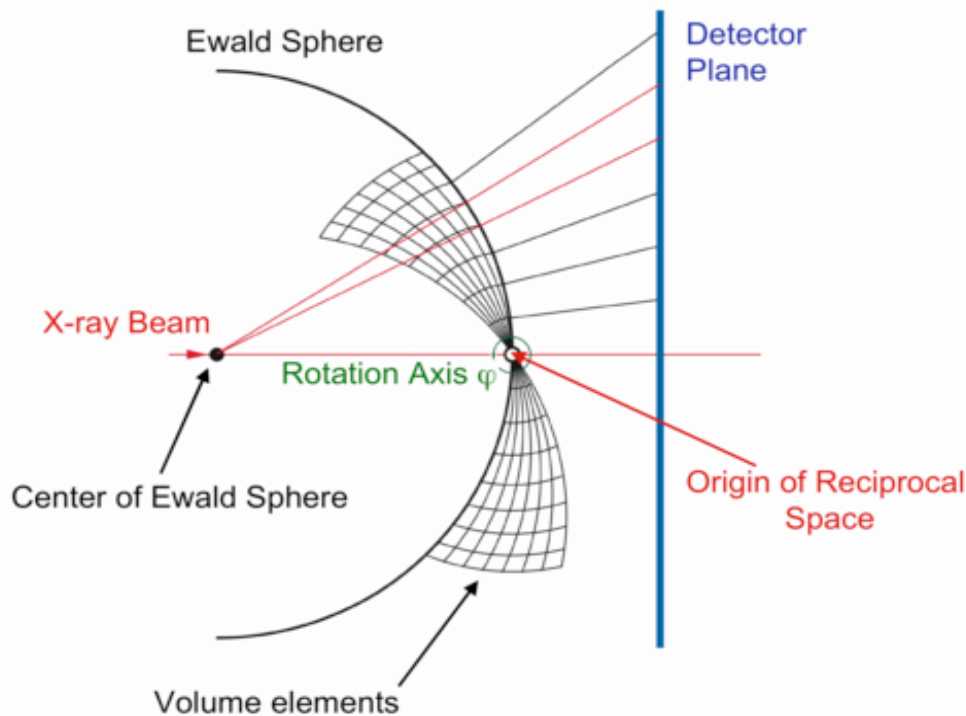
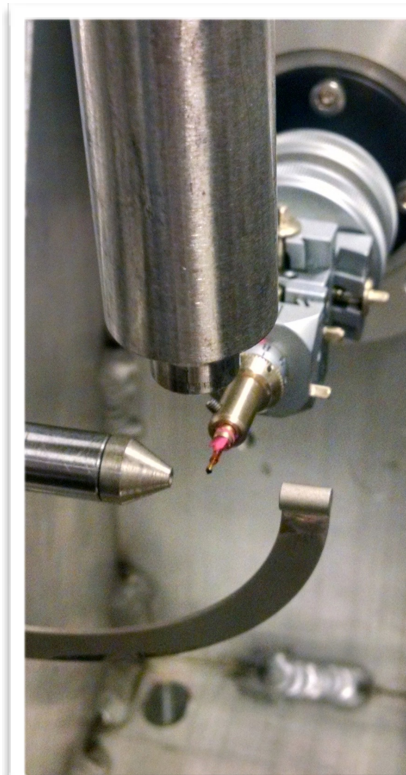
Sillimanite: Al_2SiO_5



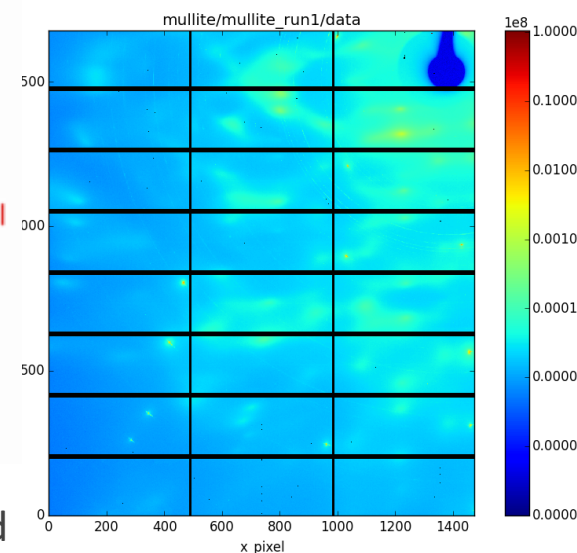
Mullite: $Al_2(Al_{2+2x}Si_{2-2x})O_{10+x}$

B. D. Butler, T. R. Welberry, & R. L. Withers, Phys Chem Minerals **20**, 323 (1993)

Measuring X-ray Diffuse Scattering with Continuous Rotation Method

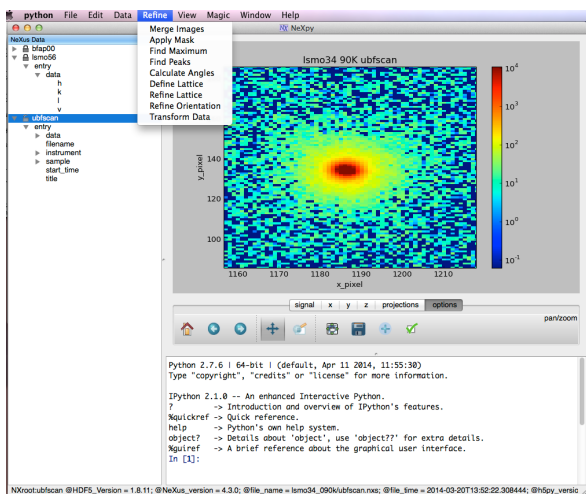


Pilatus 2M Detector

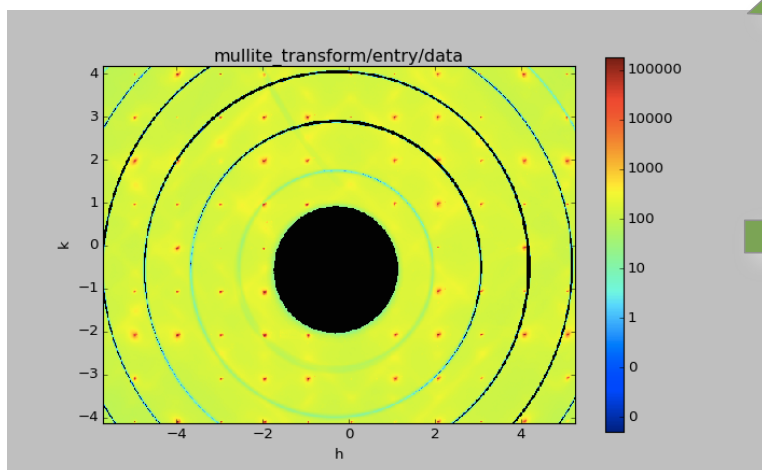


- ▶ The sample is continuously rotated in shutterless mode at 1° per second
- ▶ A fast area detector (e.g., a Pilatus 2M) acquires images at 10 frames per second
 - i.e., 3600 x 8MB frames \sim 30GB every 6 minutes
- ▶ The detector needs low background, high dynamic range, and energy discrimination
 - Ideally, this is performed with high-energy x-rays, e.g., 80 to 100 keV

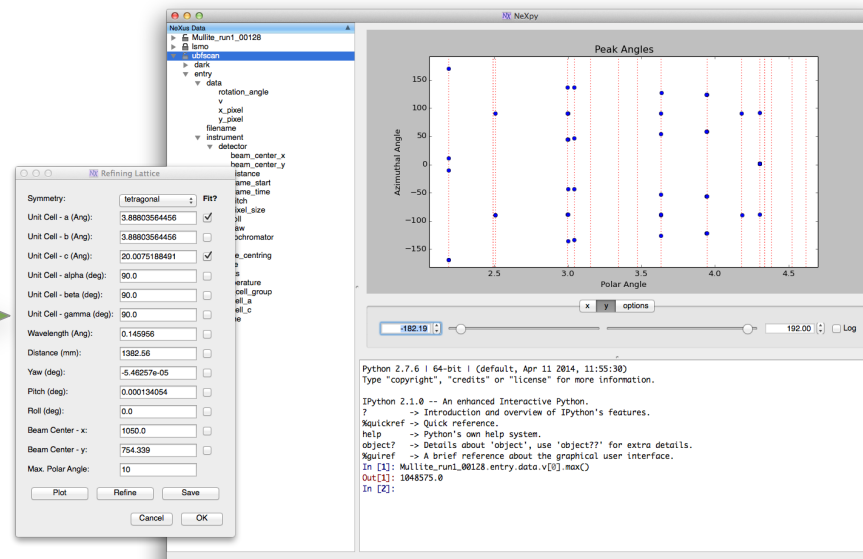
Data Reduction Workflows



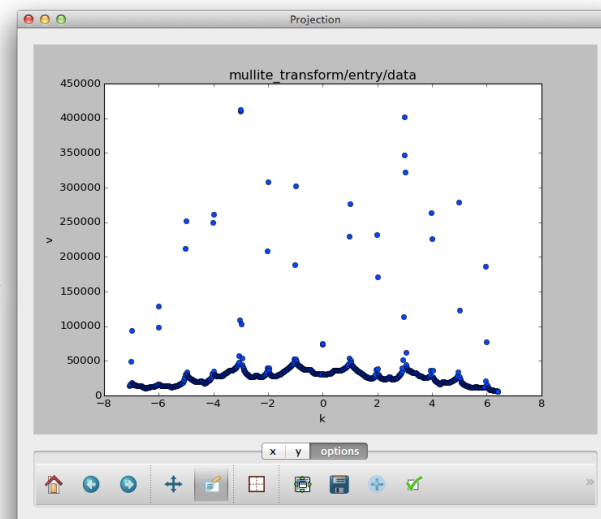
Peak Search



Coordinate Transformation



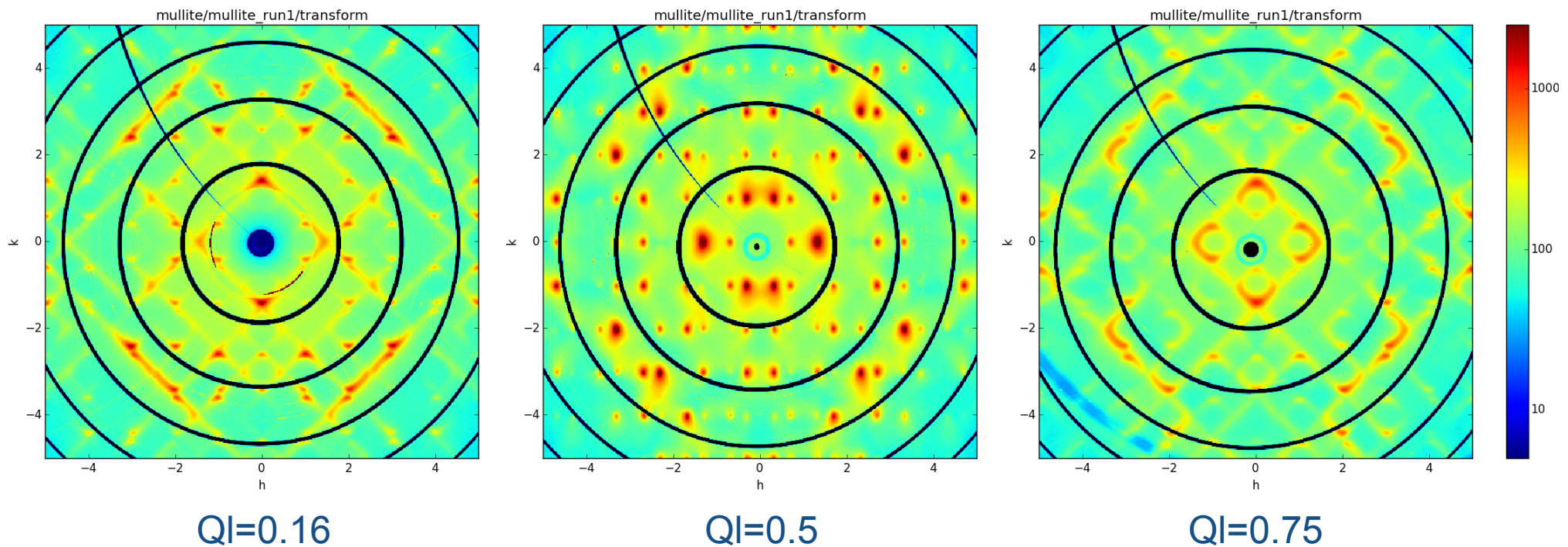
Refinement and Orientation



Data Projections

3D Diffuse Scattering in Mullite

- ▶ There is strong diffuse scattering throughout reciprocal space
- ▶ The shape of the diffuse scattering is strongly dependent on the value of Ql
- ▶ There are incipient superlattice peaks at $\mathbf{Q} = 0.5 c^* + 0.31 a^*$



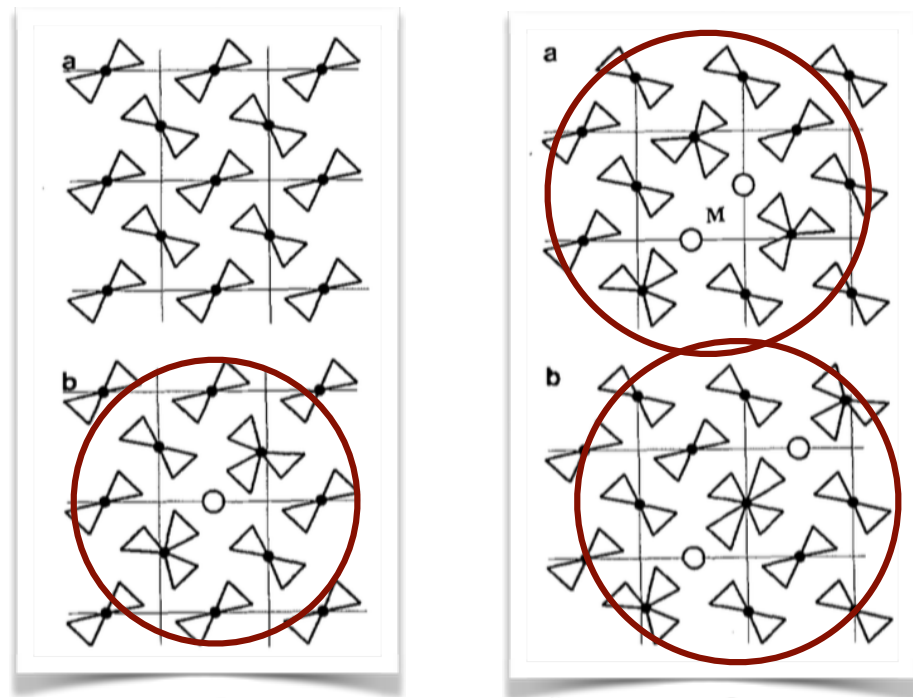
Monte Carlo Analysis

- ▶ In a classic analysis, Richard Welberry and colleagues developed a set of interaction energies to model mullite disorder
- ▶ Interaction energies were initialized:
 - ▶ insights from chemical intuition
 - ▶ insights from the measured diffuse scattering
- ▶ The diffuse scattering was calculated using a Monte Carlo algorithm to generate vacancy distributions first in 2D slices and then in 3D

$$P_i = \frac{e^{-V_i}}{1 + e^{-V_i}},$$

where,

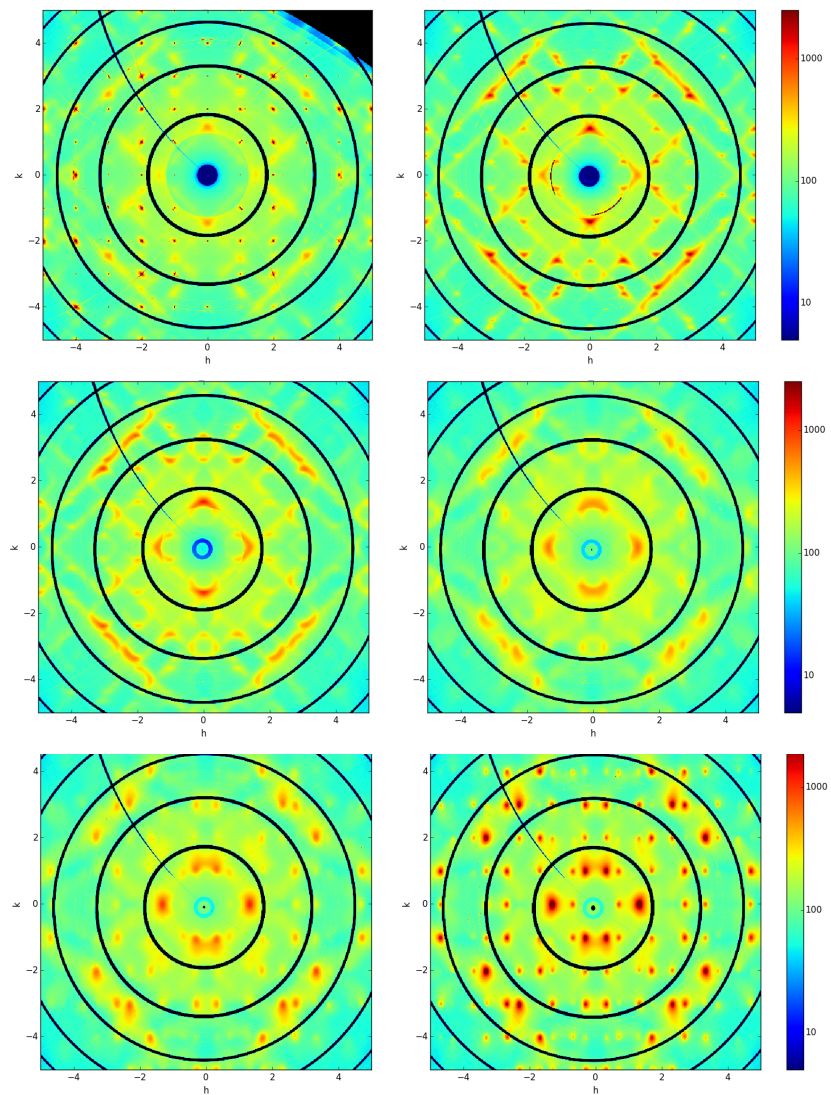
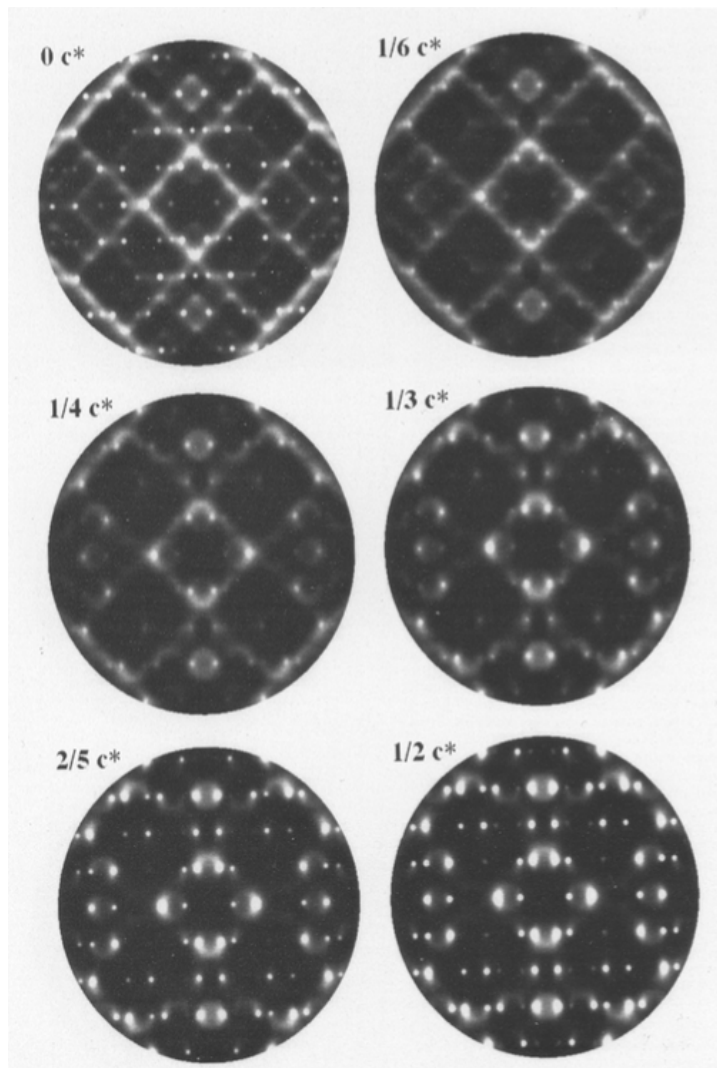
$$V_i = \frac{\sum_j E_{ij}}{kT} + \frac{(N_v - N_v^o)^2}{N_v^o} \text{sgn}(N_v - N_v^o).$$



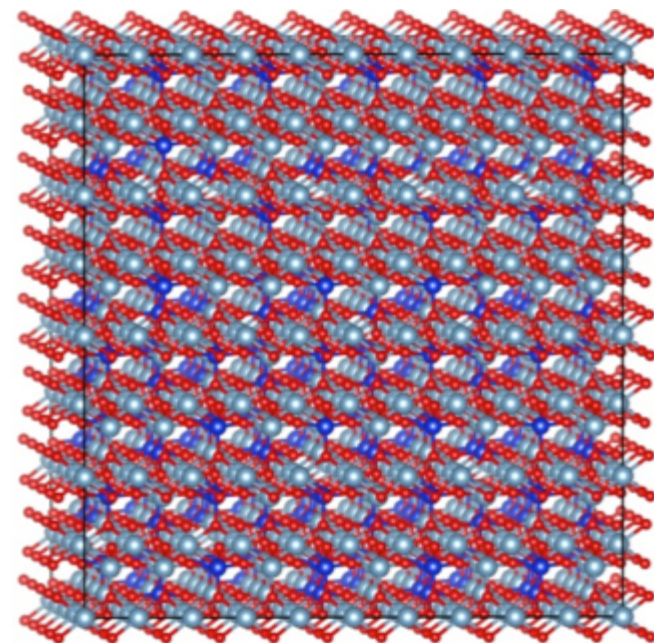
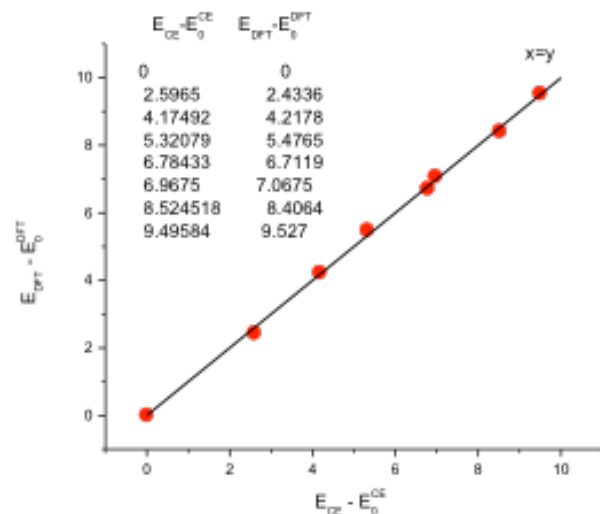
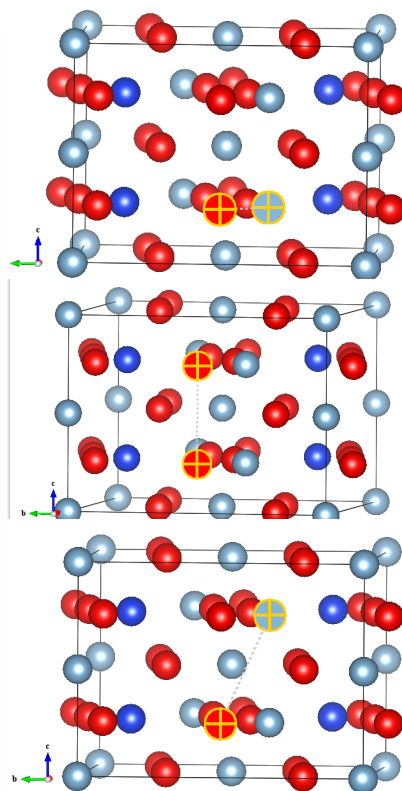
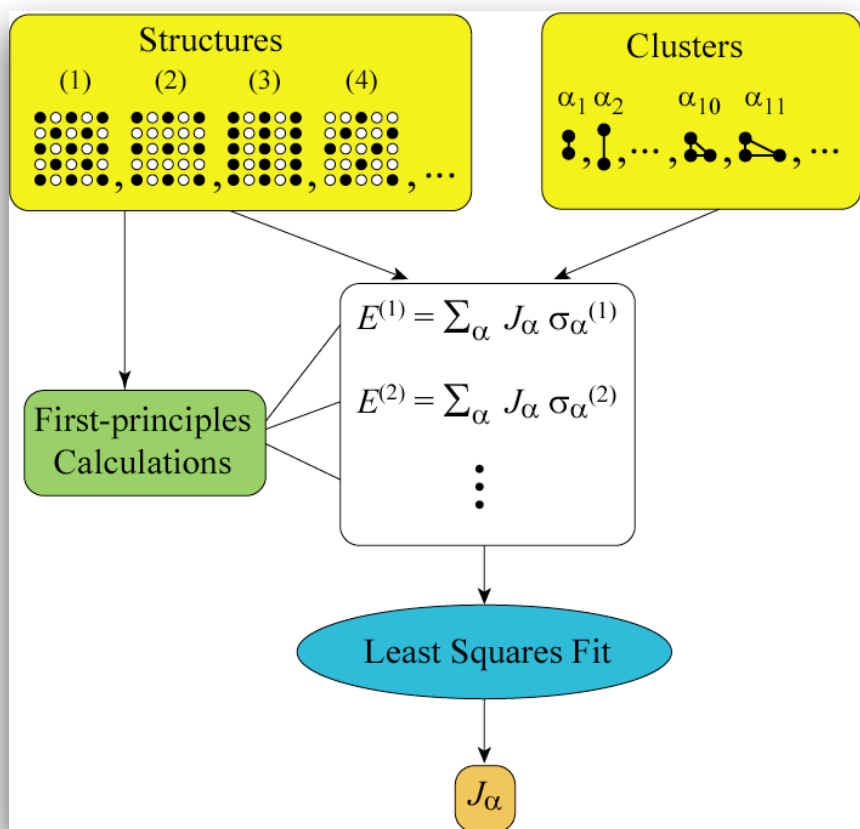
Interatomic vector	α_{lmn}	Interatomic vector	α_{lmn}
$\frac{1}{2}\langle 1\ 1\ 0 \rangle$	-0.24	$\langle 0\ 2\ 0 \rangle$	+0.13
$[1\ 1\ 0]$	-0.23	$\frac{1}{2}\langle 3\ 1\ 0 \rangle$	+0.22
$[1\ -1\ 0]$	-0.05	$\frac{1}{2}\langle 1\ 3\ 0 \rangle$	-0.01
$\langle 1\ 0\ 0 \rangle$	-0.06	$\langle 1\ 0\ 1 \rangle$	+0.07
$\langle 0\ 1\ 0 \rangle$	+0.22	$\langle 0\ 1\ 1 \rangle$	-0.12
$\langle 0\ 0\ 1 \rangle$	-0.03	$\frac{1}{2}\langle 3\ 3\ 0 \rangle$	+0.17
$\frac{1}{2}[1\ -1\ 2]$	+0.12	$\langle 1\ 1\ 1 \rangle$	-0.01
$\frac{1}{2}[1\ 1\ 2]$	+0.12	$\frac{1}{2}\langle 3\ 1\ 2 \rangle$	-0.11
$\langle 2\ 0\ 0 \rangle$	-0.12	$\frac{1}{2}\langle 3\ 3\ 2 \rangle$	-0.07

B. D. Butler, T. R. Welberry, & R. L. Withers, Phys Chem Minerals **20**, 323 (1993)

Monte Carlo Analysis Results



Vacancy Short-Range Order in Mullite A First-Principles Approach

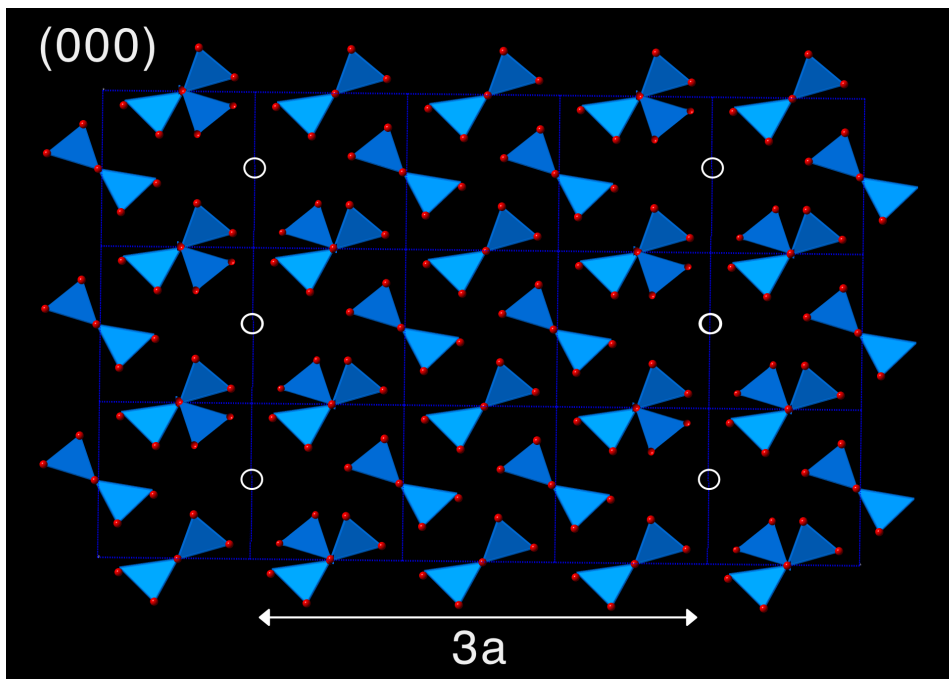


$$E(\sigma) = J_0 + \sum_i \sigma_i J_i + \sum_{i,j} J_{ij} \sigma_i \sigma_j + \sum_{ijk} J_{ijk} \sigma_i \sigma_j \sigma_k + \dots$$

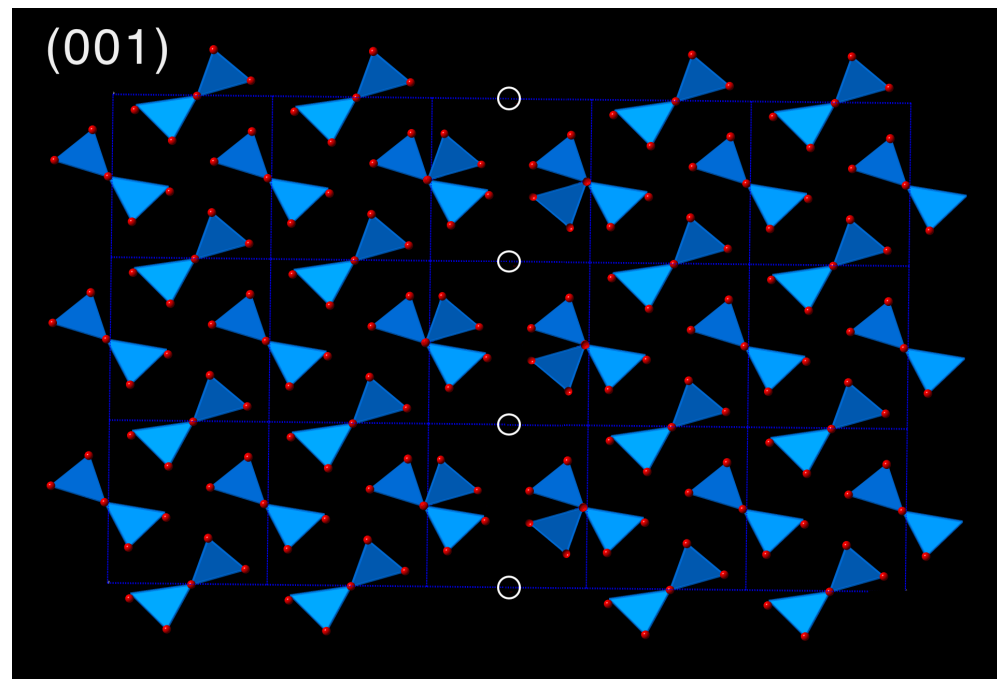
Peter Zapol & Anh Ngo

Lowest Energy 3:2 Mullite Structure
from Kinetic Monte Carlo Calculation

Nearly-Commensurate Vacancy Stripes in Mullite



$$c = 0$$

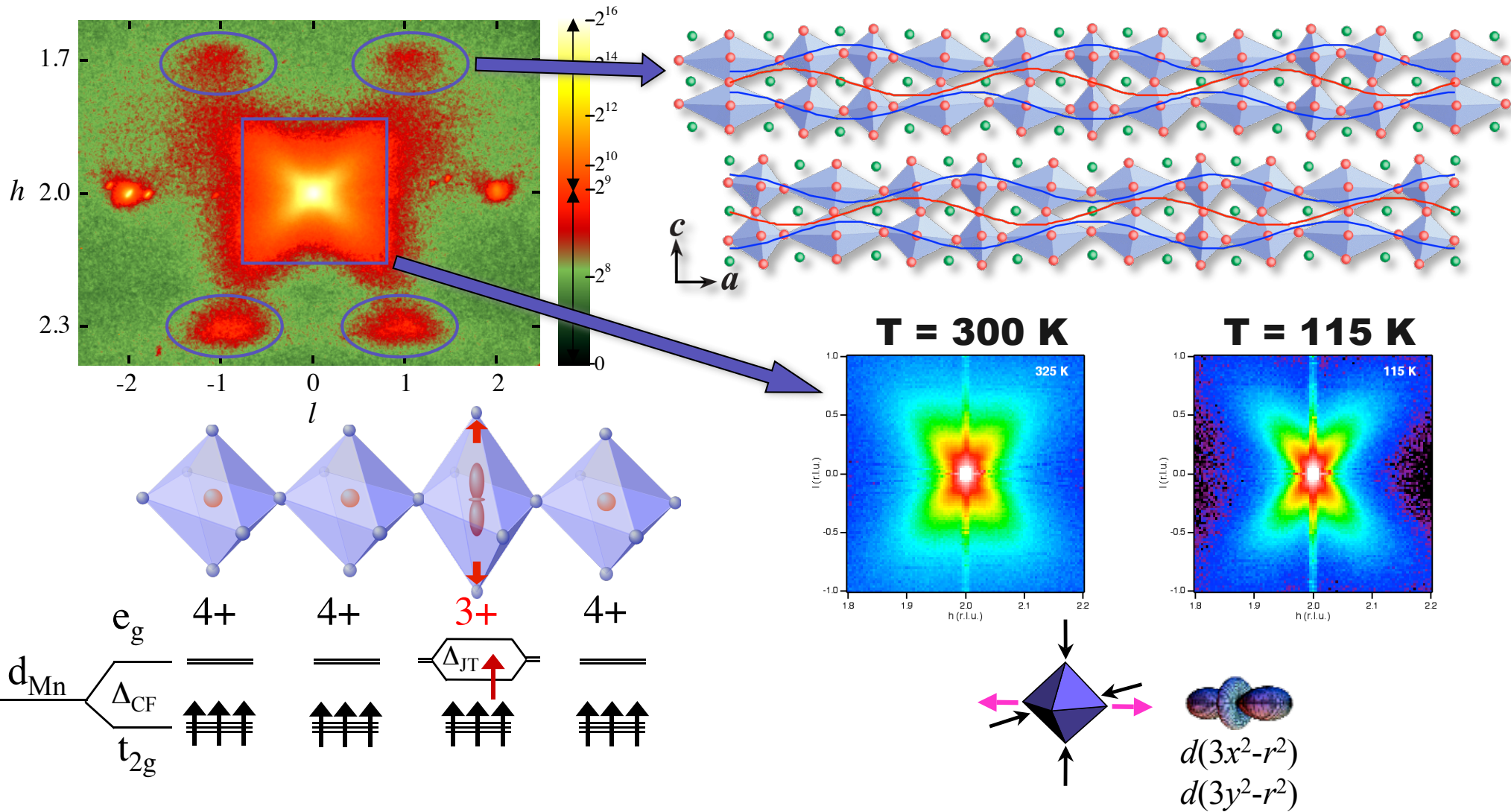


$$c = 1.0$$

$$\mathbf{q} = \pm \frac{1}{2} \mathbf{c}^* \pm \frac{1}{3} \mathbf{a}^*$$

Case Study 1: Bilayer Manganites

Diffuse Scattering from Jahn-Teller Polarons



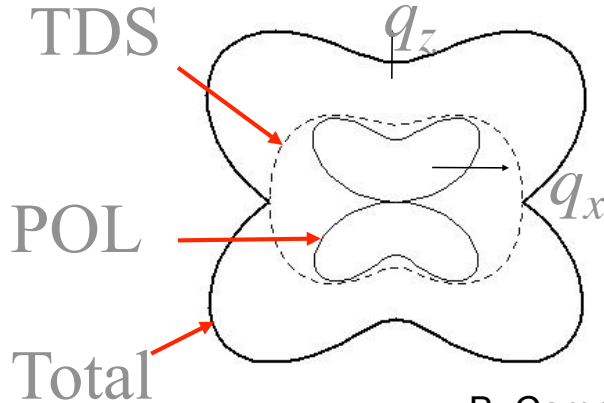
Huang Scattering

$$I(\mathbf{Q}) = \sum_{m,n} e^{i\mathbf{Q}\cdot(\mathbf{R}_m - \mathbf{R}_n)} f_m f_n e^{-W_m} e^{-W_n} \langle (\mathbf{Q} \cdot \mathbf{u}_m)(\mathbf{Q} \cdot \mathbf{u}_n) \rangle$$

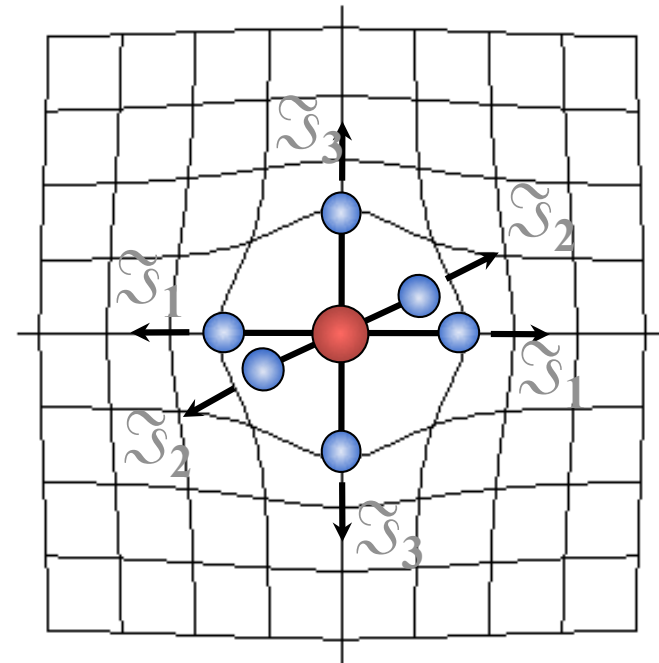
$$I_{POL}(\mathbf{Q}) = N|F_G|^2 \sum_{\alpha,\beta,\gamma,\delta} Q_\beta Q_\delta \left(\sum_{j,j'} \frac{\epsilon_{\alpha,\mathbf{q},j} \epsilon_{\beta,\mathbf{q},j}^* \epsilon_{\gamma,\mathbf{q},j'} \epsilon_{\delta,\mathbf{q},j'}^*}{\omega_{\mathbf{q},j}^2 \omega_{\mathbf{q},j'}^2} \right) \frac{1}{j} \sum_{m,n} \mathfrak{S}_{m,\alpha} \mathfrak{S}_{n,\gamma} e^{i\mathbf{q}\cdot(\mathbf{R}_m - \mathbf{R}_n)}$$

$$I_{TDS}(\mathbf{Q}) = N|F_G|^2 \left(\frac{kT}{2M} \right) \sum_{\beta,\delta} Q_\beta Q_\delta \left(\sum_j \frac{\epsilon_{\beta,\mathbf{q},j}^* \epsilon_{\delta,\mathbf{q},j}}{\omega_{\mathbf{q},j}^2} \right) \frac{1}{j}$$

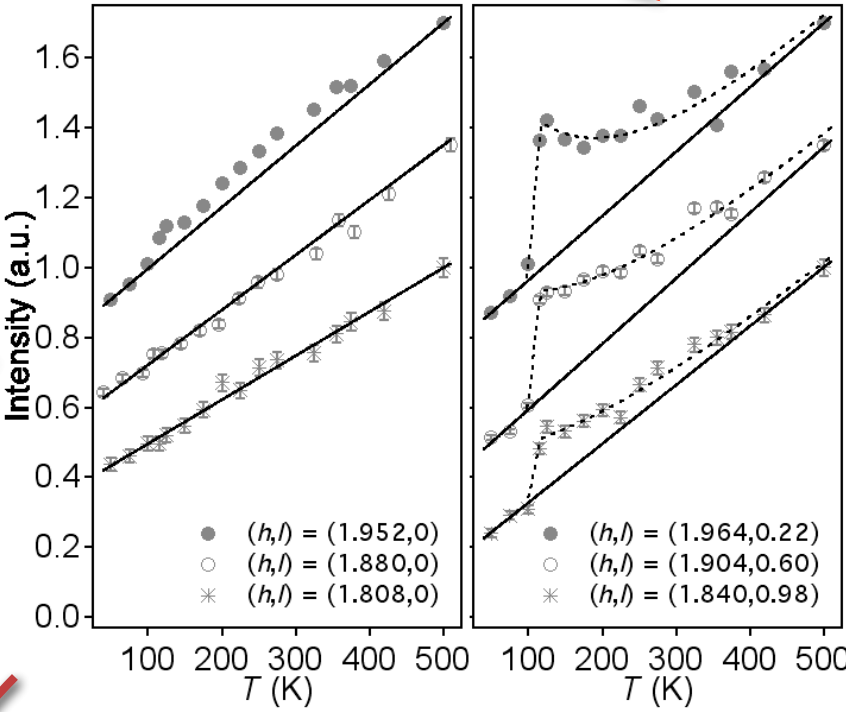
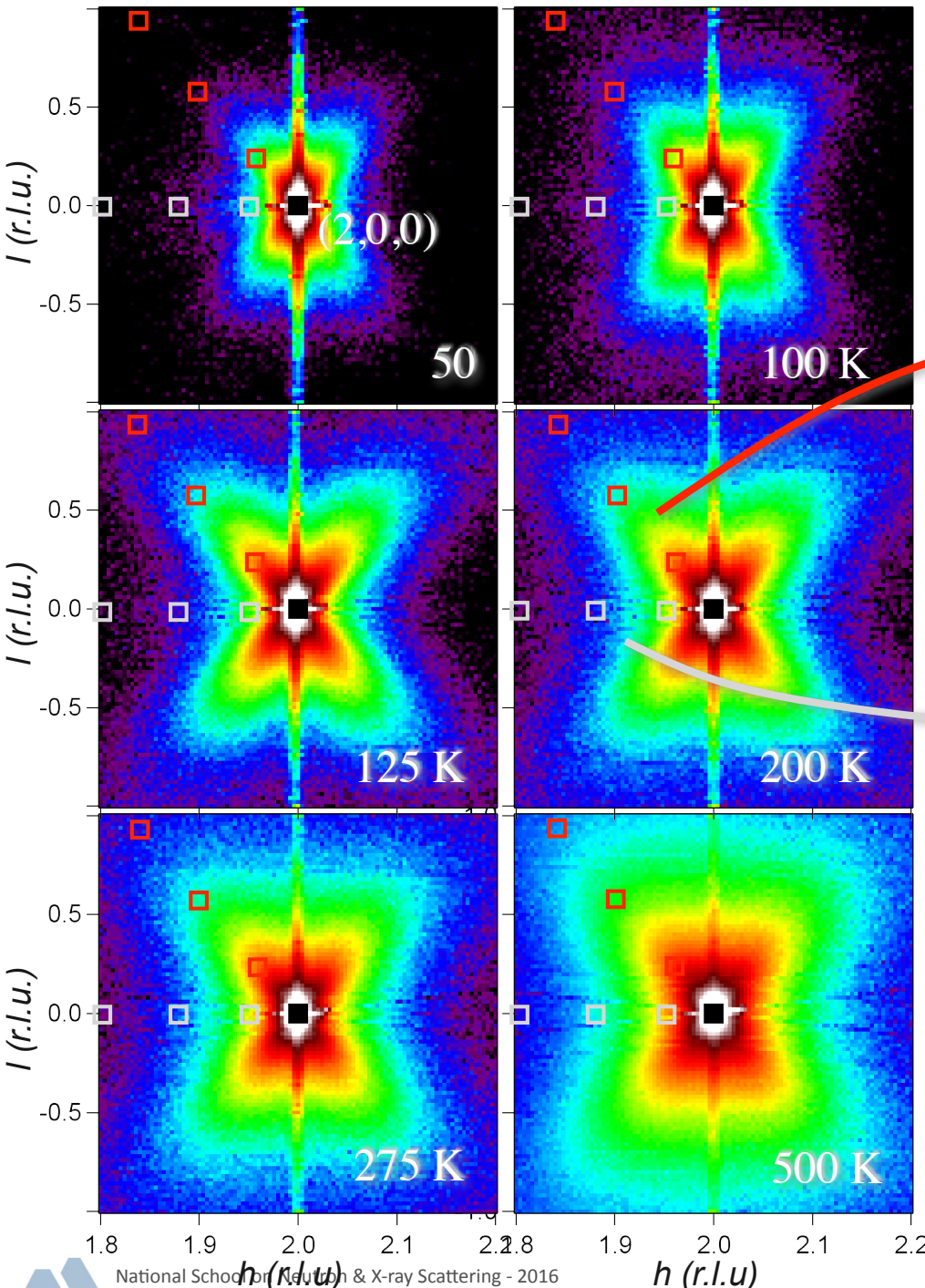
$$u_{m,\delta} = \int \frac{d^3q}{(2\pi)^3} \sum_{\beta} \left(\sum_j \frac{\epsilon_{\beta,\mathbf{q},j}^* \epsilon_{\delta,\mathbf{q},j}}{\omega_{\mathbf{q},j}^2} \right) \frac{1}{j} \sum_n \mathfrak{S}_{n,\beta} e^{i\mathbf{q}\cdot(\mathbf{R}_m - \mathbf{R}_n)}$$



B. Campbell et al Phys. Rev. B. **67**, 020409 (2003)

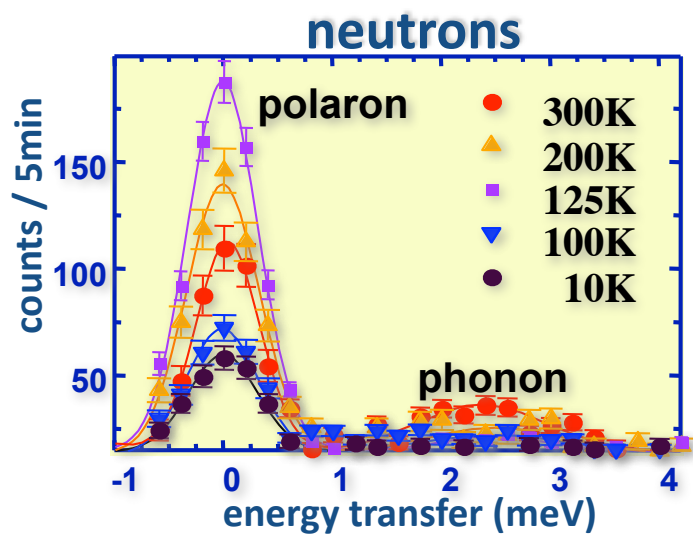
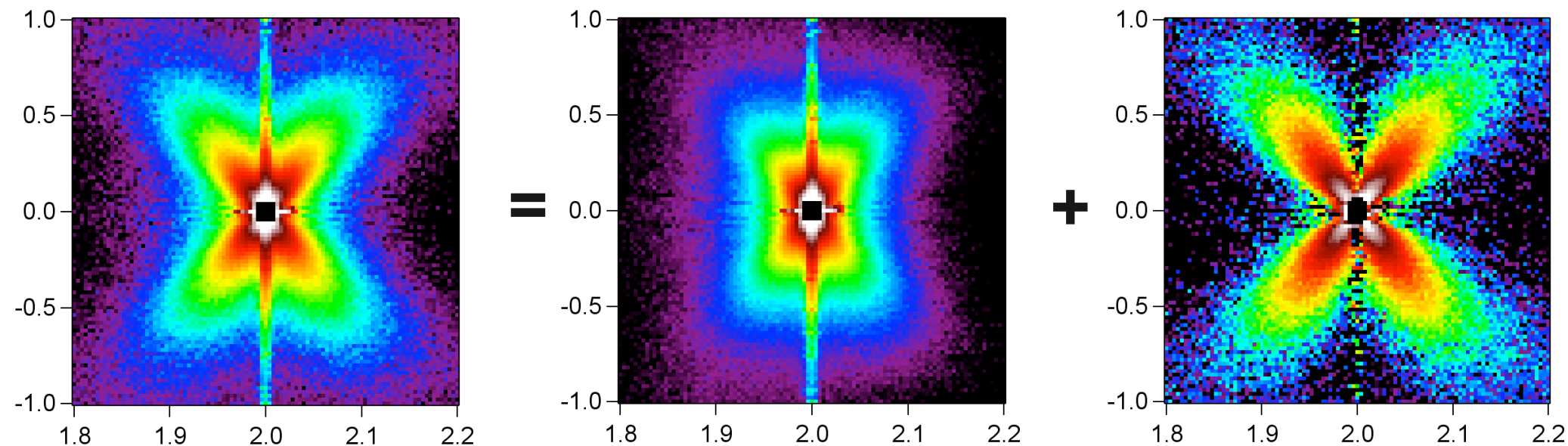


single crystal synchrotron x-ray scattering
11-ID-D, APS@Argonne



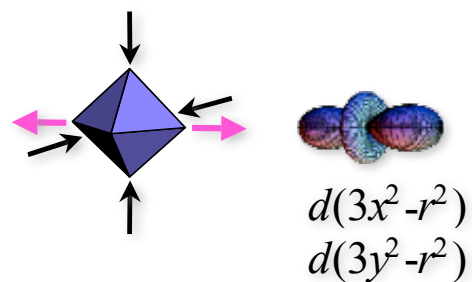
this allows us to subtract background from thermal diffuse scattering!

TDS + Huang scattering

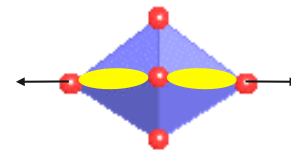
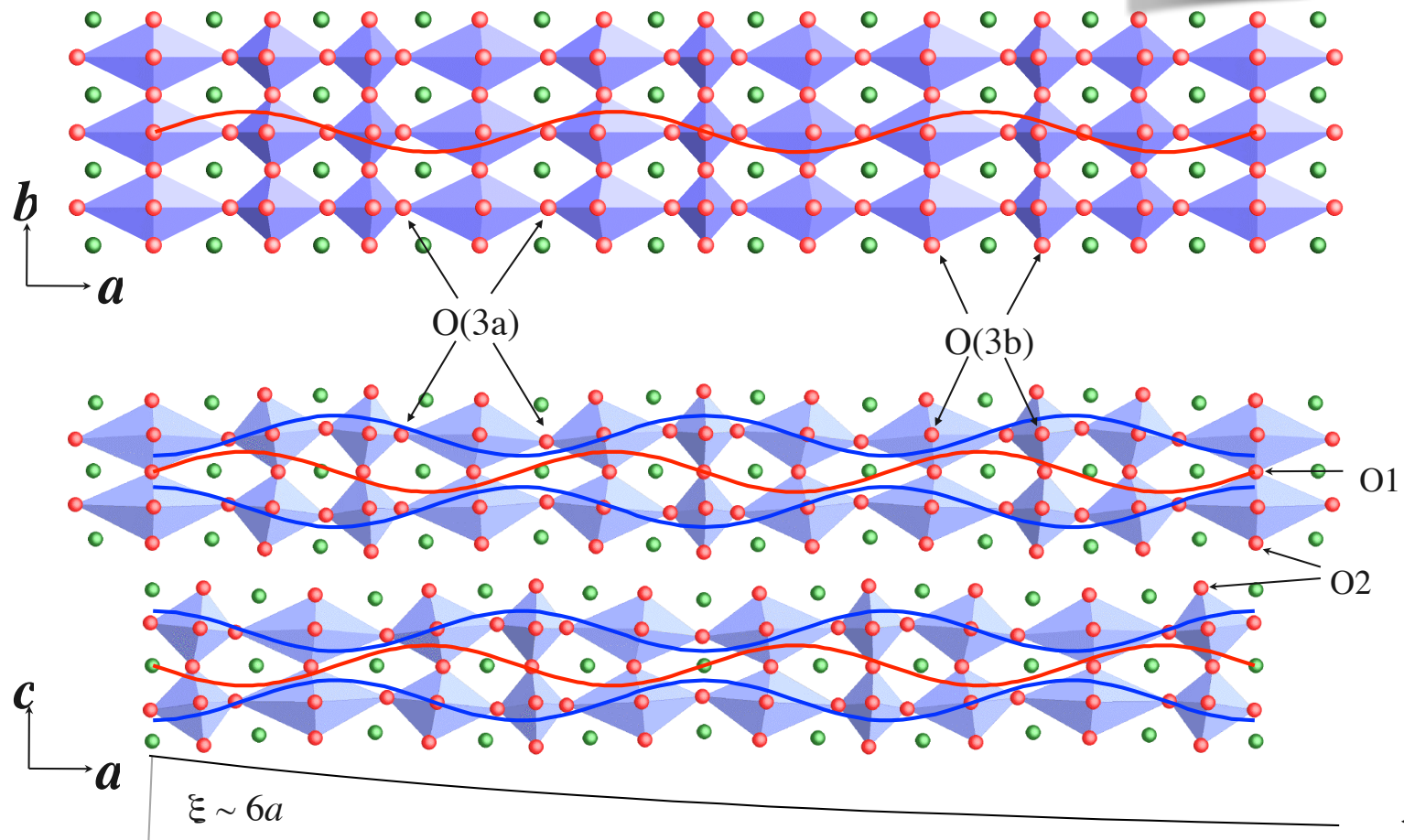
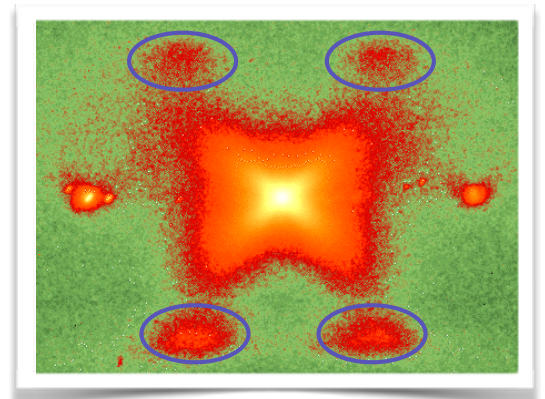


TDS

polaron



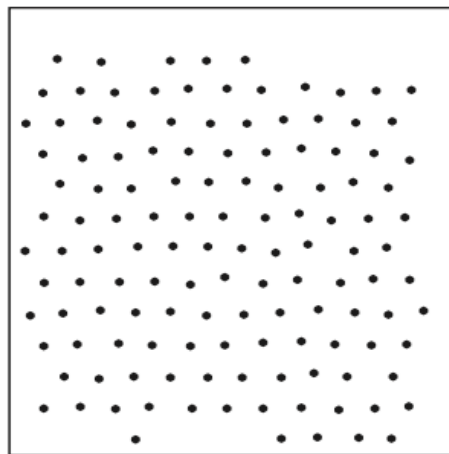
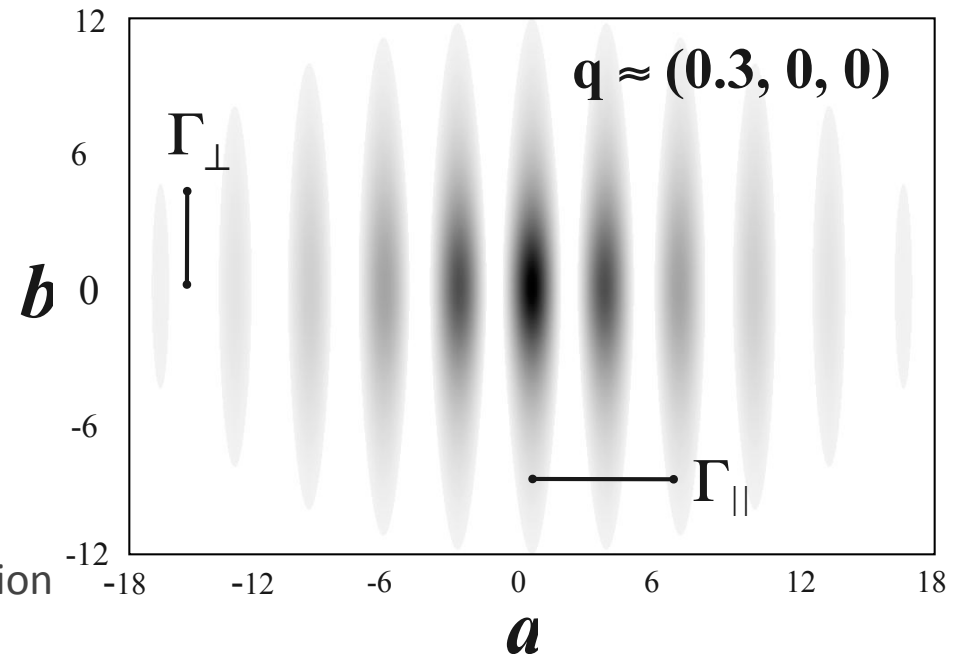
Cooperative Jahn-Teller Distortions



B. J. Campbell, R. Osborn, D. N. Argyriou, L. Vasiliu-Doloc, J. F. Mitchell, S. K. Sinha, U. Ruett, C. D. Ling, Z. Islam, and J. W. Lynn, *Physical Review B* **65**, 014427 (2001)

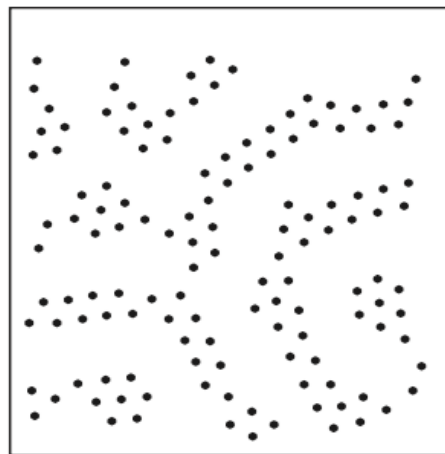
Origins of Stripe Formation

- ▶ Stripe formation is a very common motif of disordered systems
- ▶ It is the response of a system with interactions that compete on different length scales
 - *e.g.*, long-range repulsion vs short-range attraction



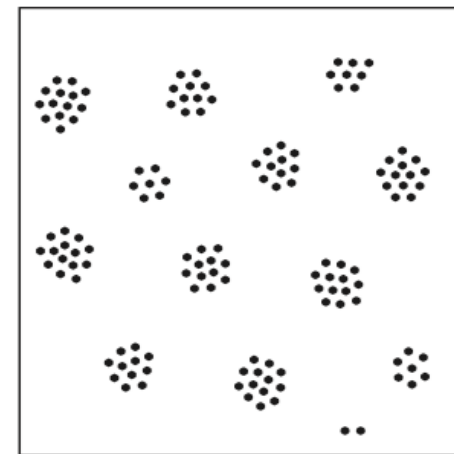
a

Wigner Lattice



b

Stripes

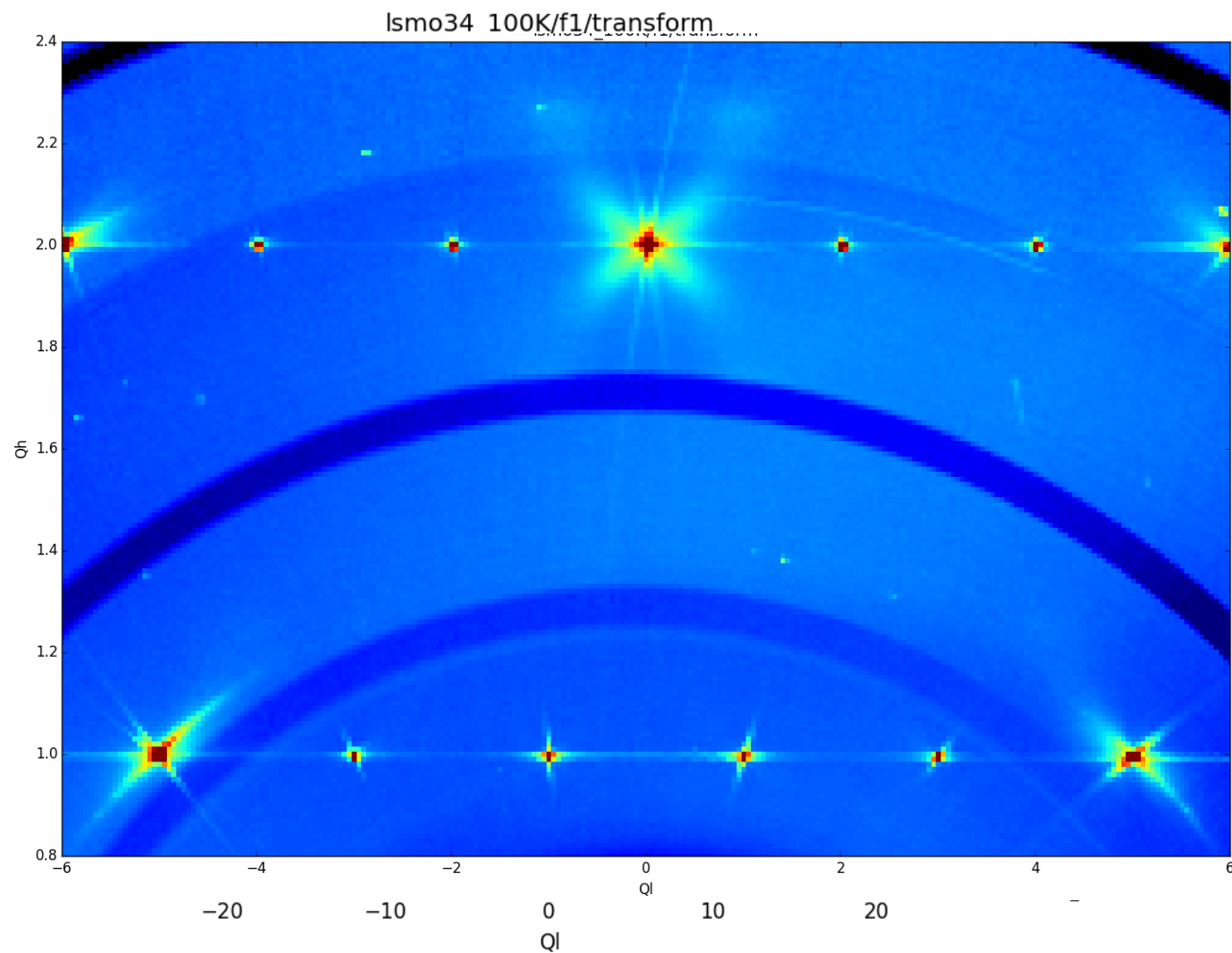


c

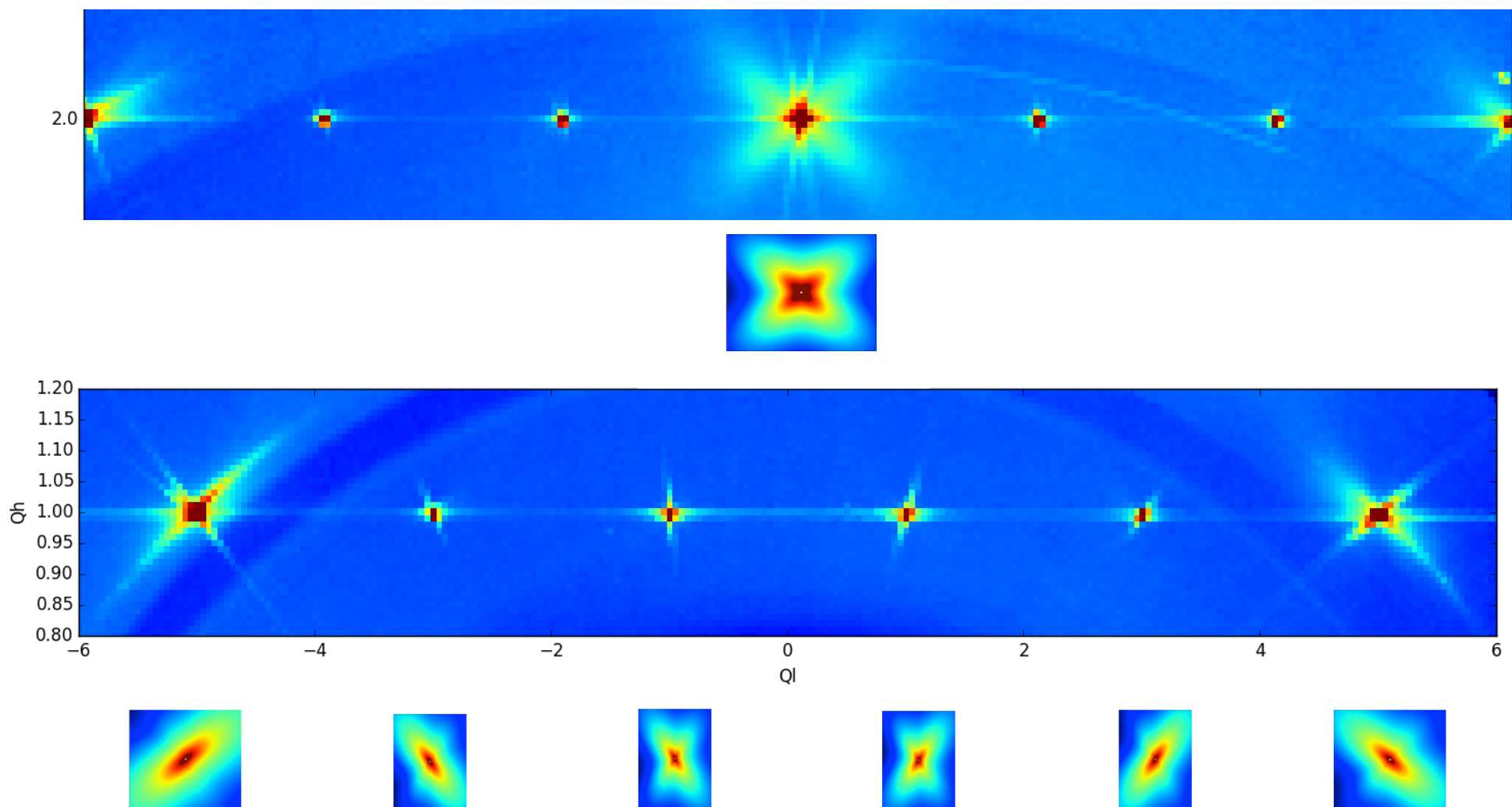
Phase Separation

C. Reichardt, C. J. Olsen, I. Martin & A. Bishop, EPL **61**, 221–227 (2003).

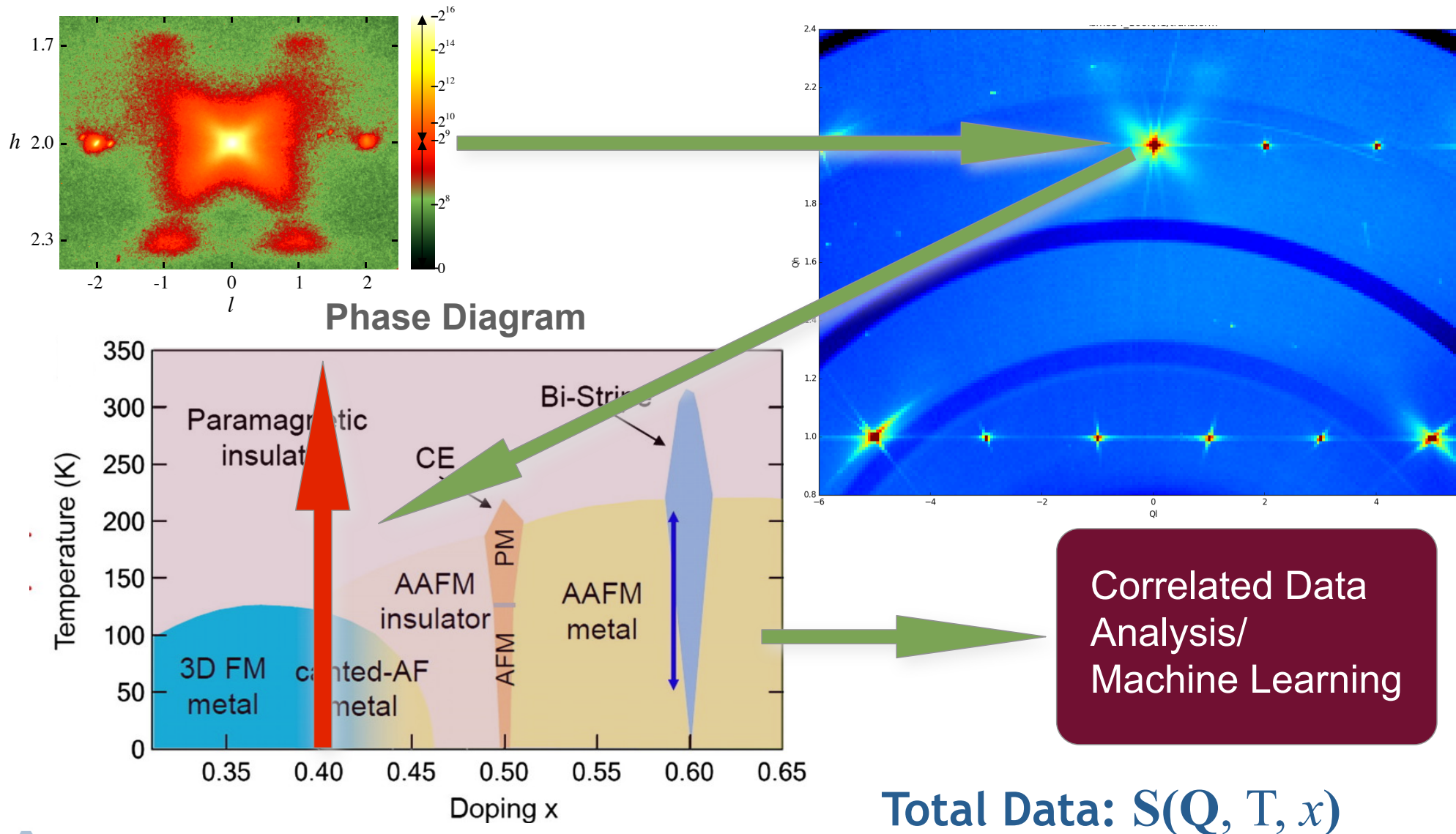
Bilayer Manganites Revisited



Huang Scattering as a Function of (Q_h , Q_k , Q_l)

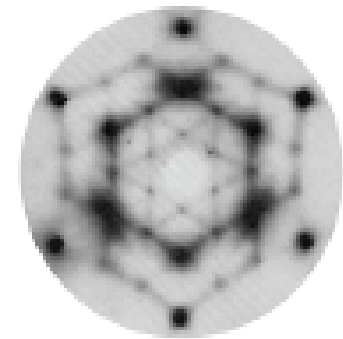


Expanding the Concept of a Data Set

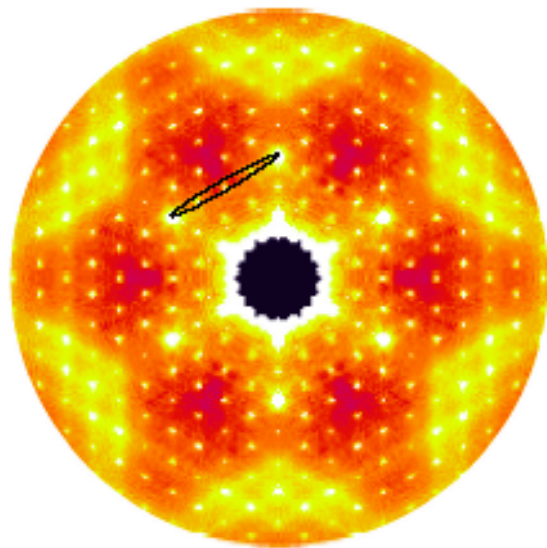


How do I look at static disorder?

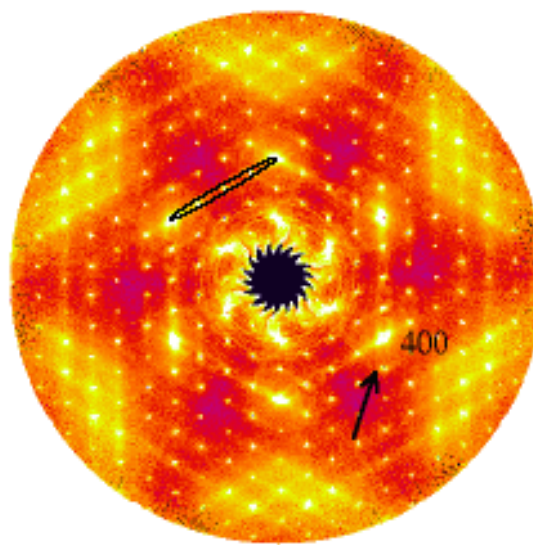
Importance of Elastic Discrimination



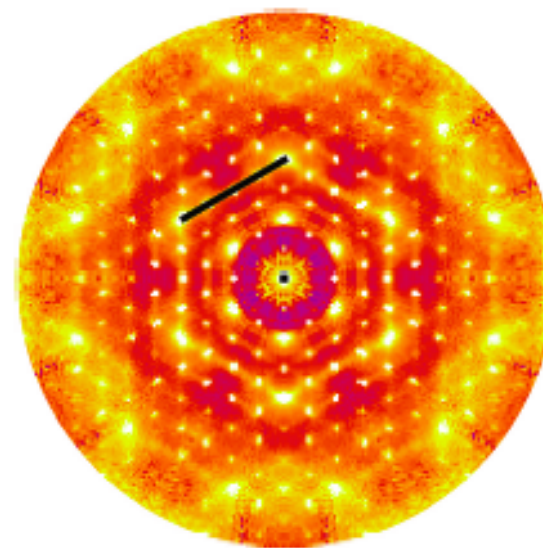
BENZIL



Detector 1
 $2\theta \sim 142.5^\circ$



Detector 2
 $2\theta \sim 90.0^\circ$



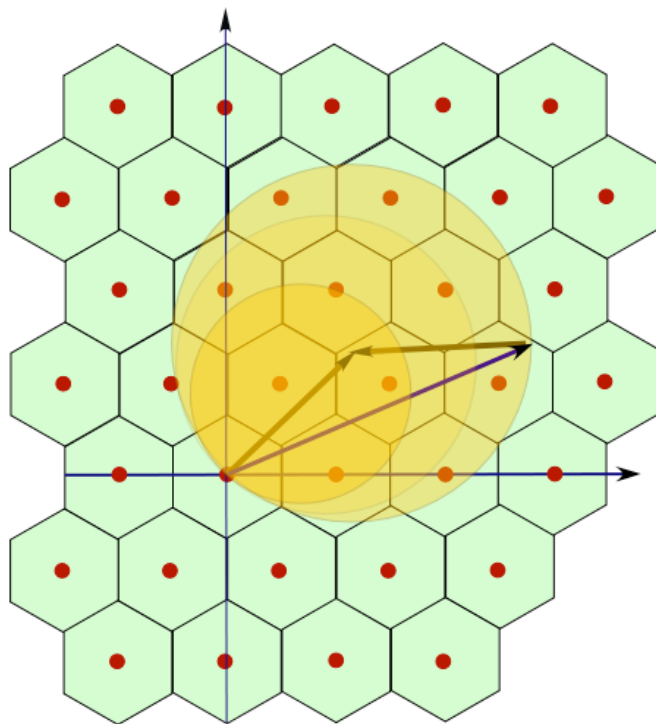
Detector 3
 $2\theta \sim 37.5^\circ$

T. R. Welberry *et al* J. Appl. Cryst. **36**, 1400 (2003)

Measuring Large Volumes of Reciprocal Space

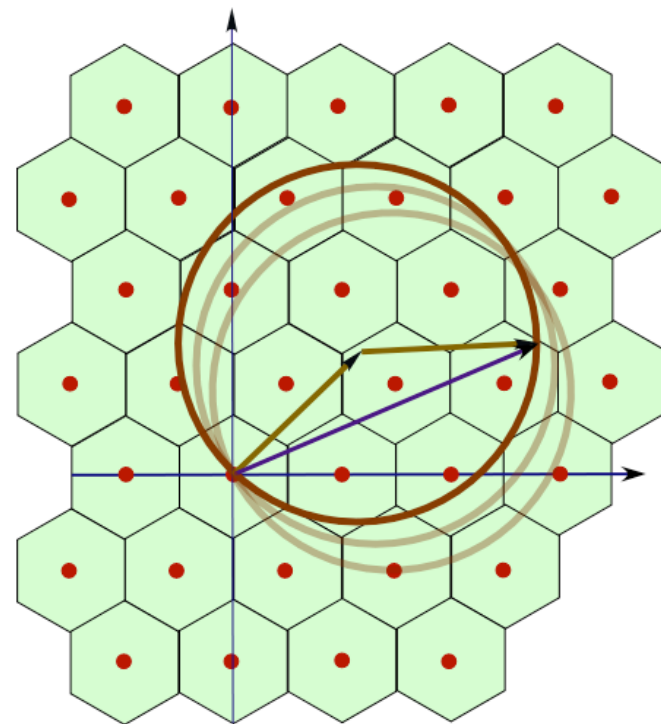
Conventional Time-of-Flight Neutron Methods

White Beam:
efficient



**NO energy
discrimination**

Fixed k_i :
energy resolved



**NOT
efficient**

Cross Correlation Chopper

TOF Laue Diffractometer

- highly efficient data collection
- wide dynamic range in Q

Statistical Chopper

- elastic energy discrimination
- optimum use of white beam

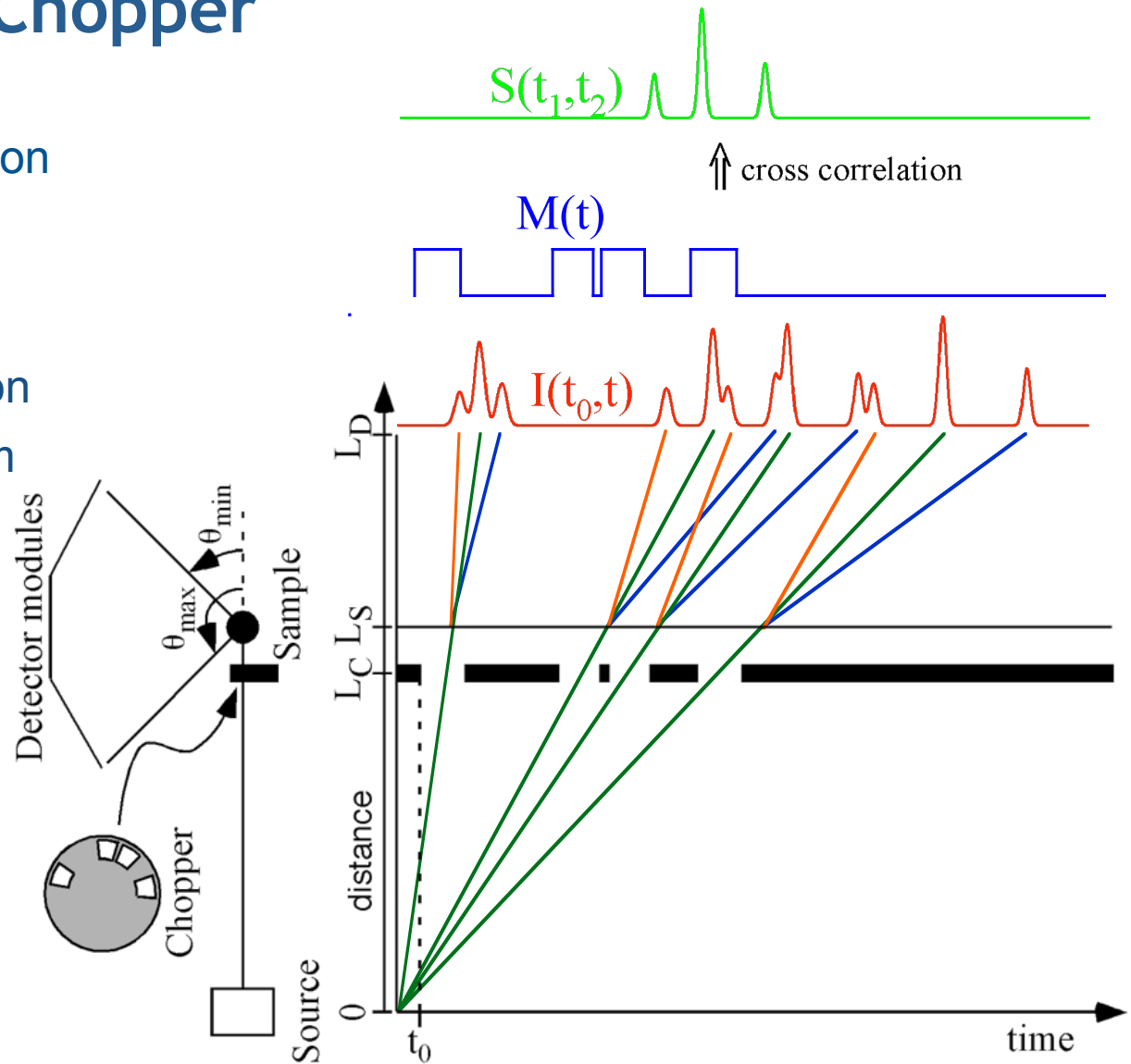
Sample with :
elastic scattering

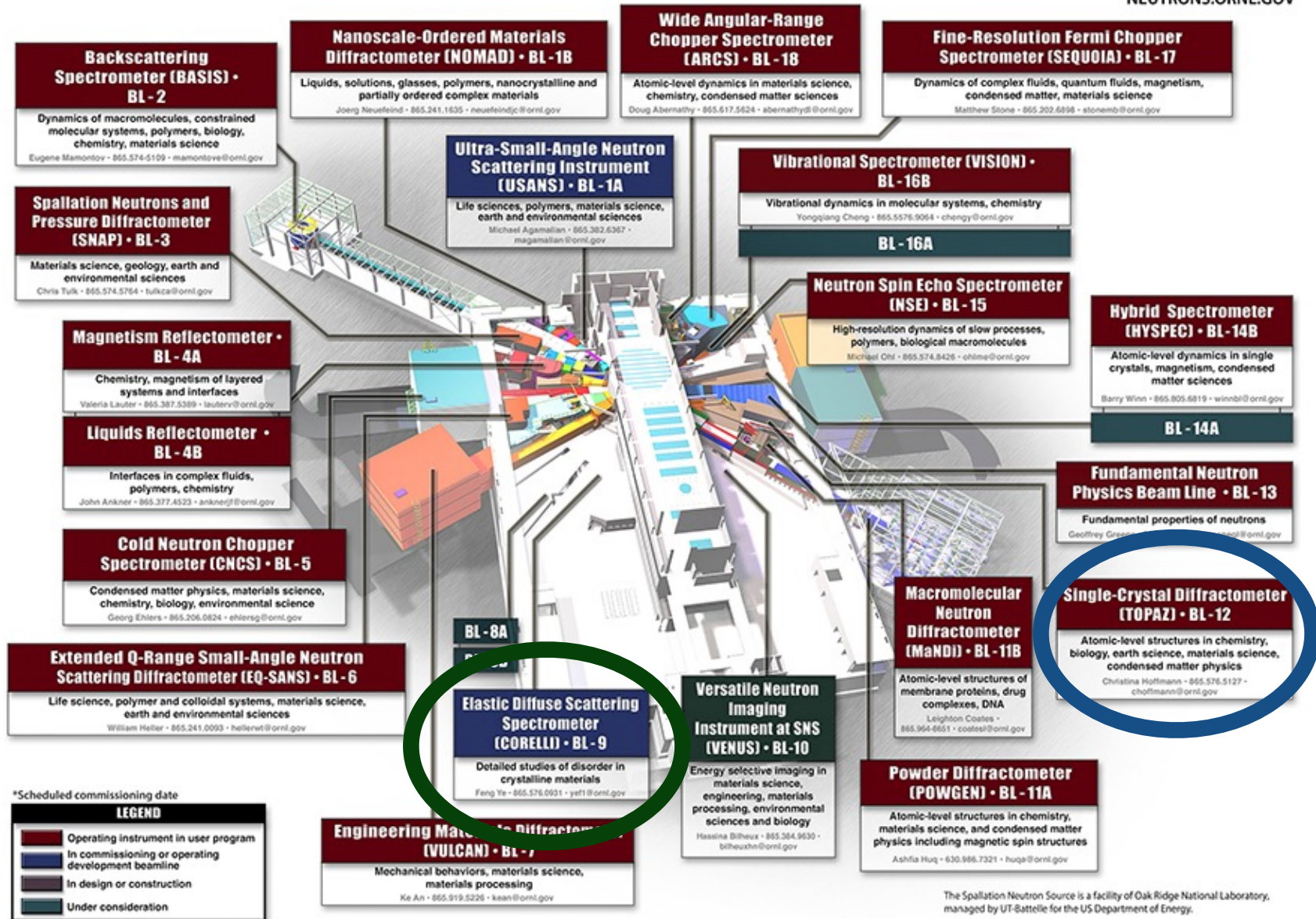
$$\hbar\omega = 0$$

inelastic excitations

$$\hbar\omega = +E_0$$

$$\hbar\omega = -E_0$$





14-G00875A/gjm

The Spallation Neutron Source is a facility of Oak Ridge National Laboratory, managed by UT-Battelle for the US Department of Energy.

Arcangelo Corelli (1653-1713)



Arcangelo Corelli was the greatest violinist of his age and an influential composer who became known as the "Father of the Concerto Grosso". This musical form contrasts music from a small ensemble of solo musicians with the full orchestra. Similarly, the properties of many materials are enriched by the interactions between both short and long-range ordering motifs that the *Corelli* instrument is designed to explore.

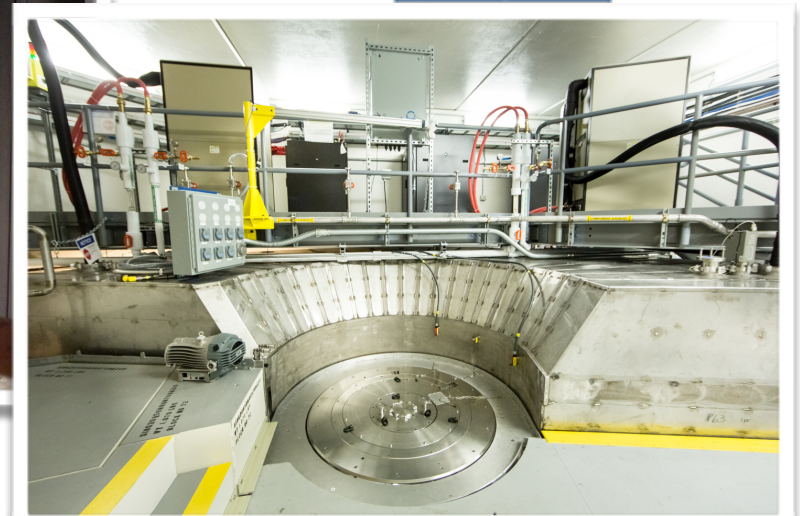
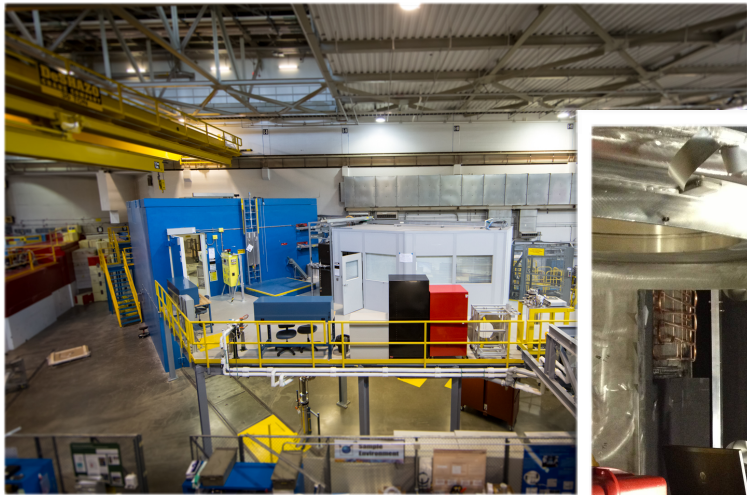
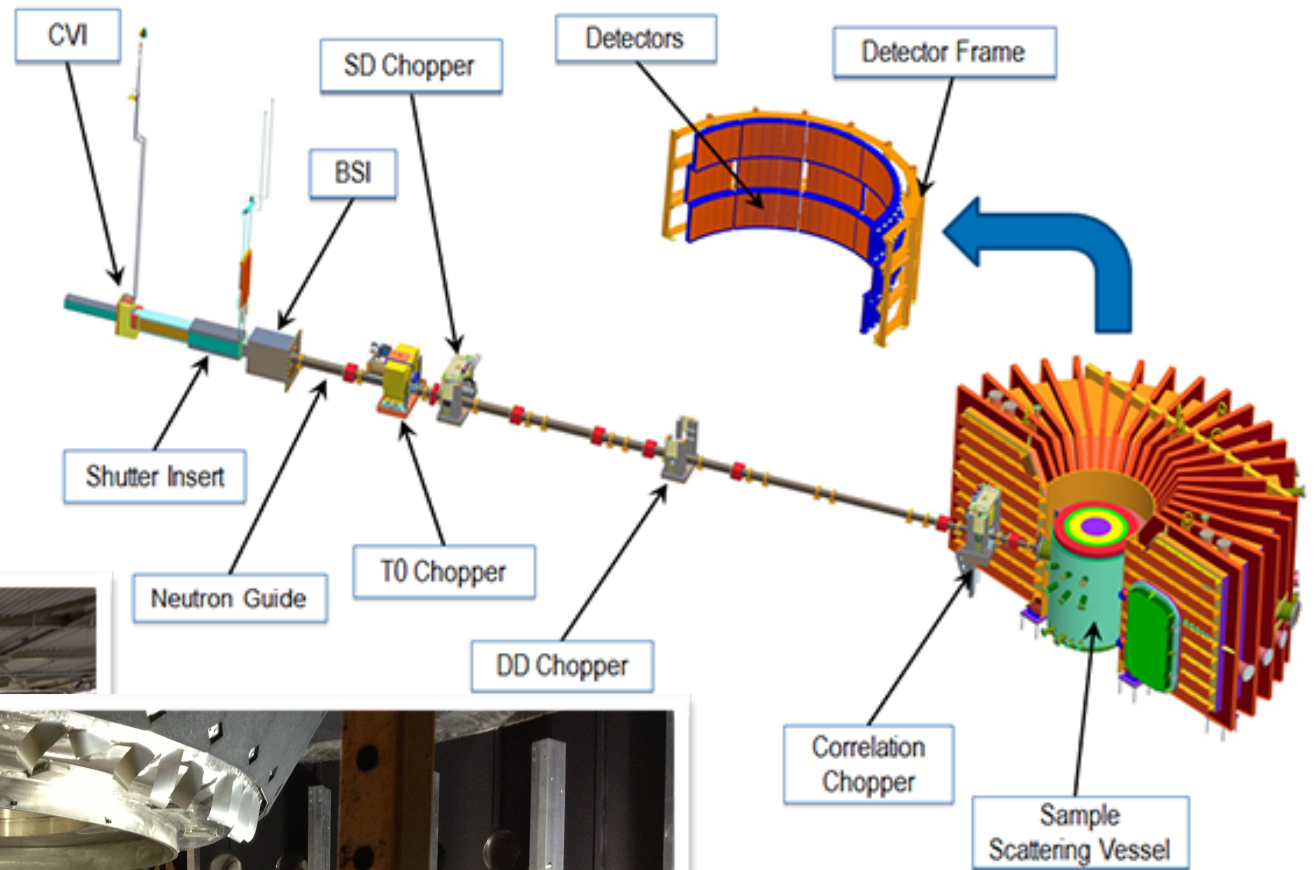


Corelli

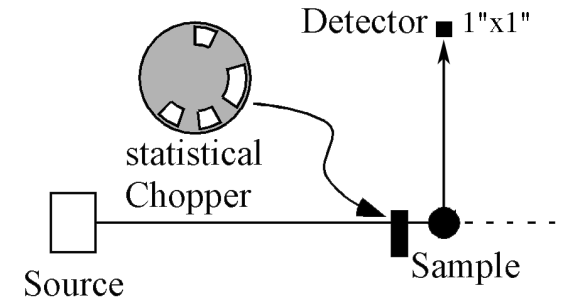
Instrument Scientists

Feng Ye

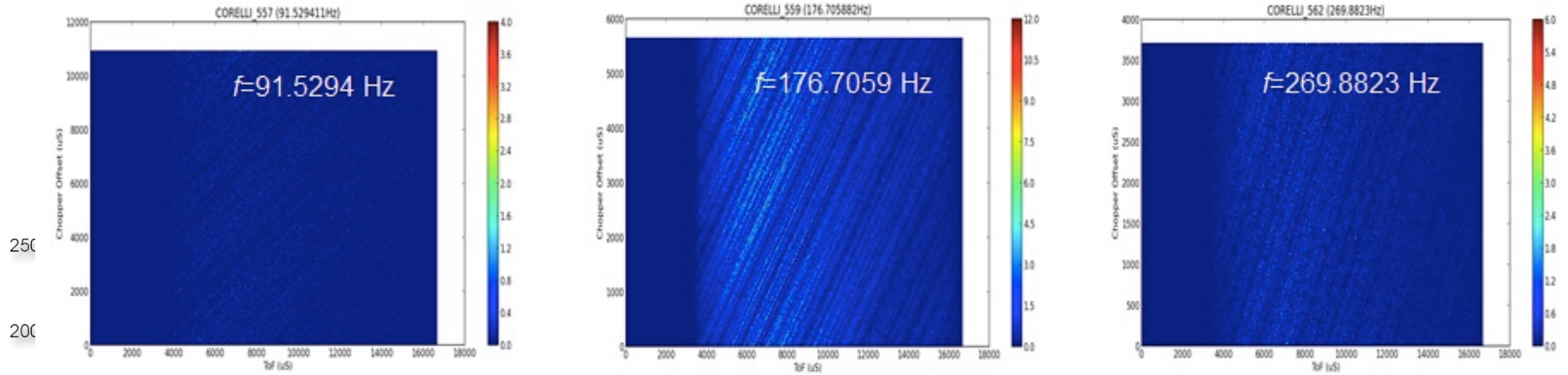
Yaohua Liu



Cross Correlation in Action

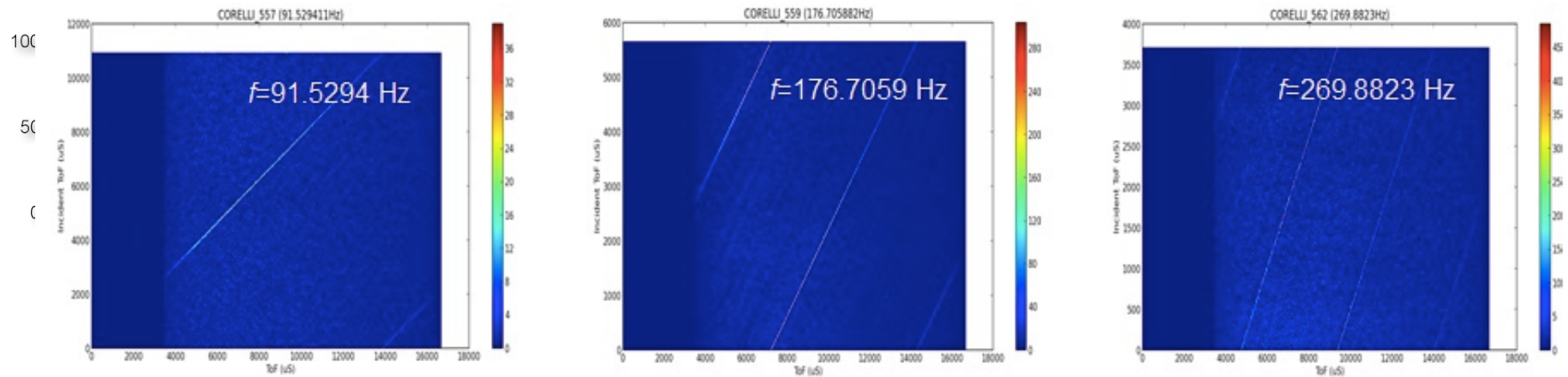


Before cross-correlation



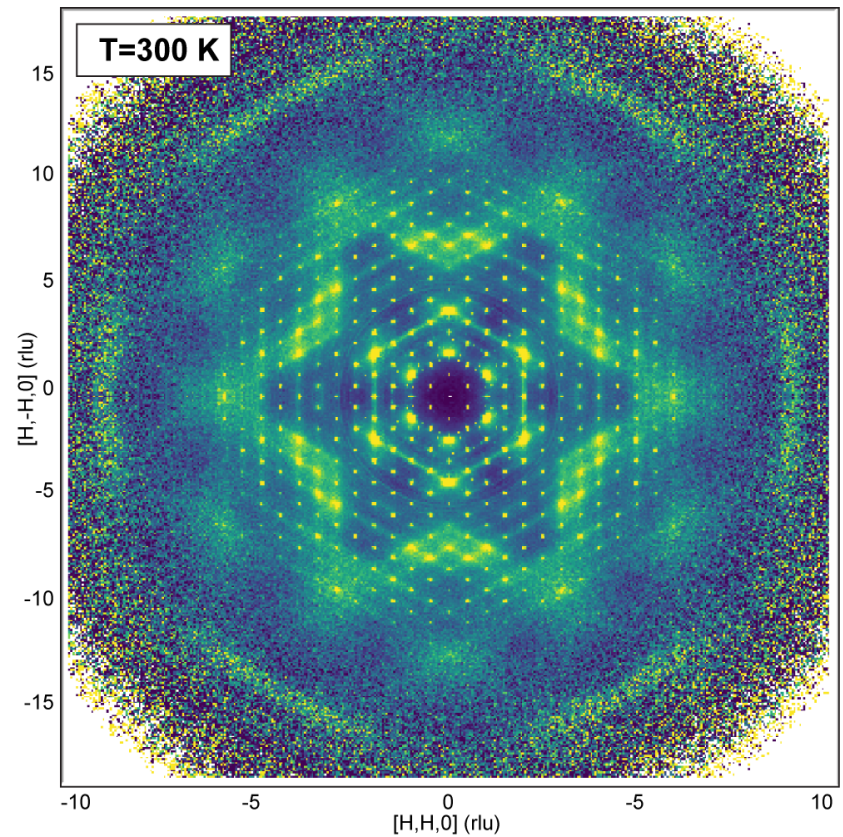
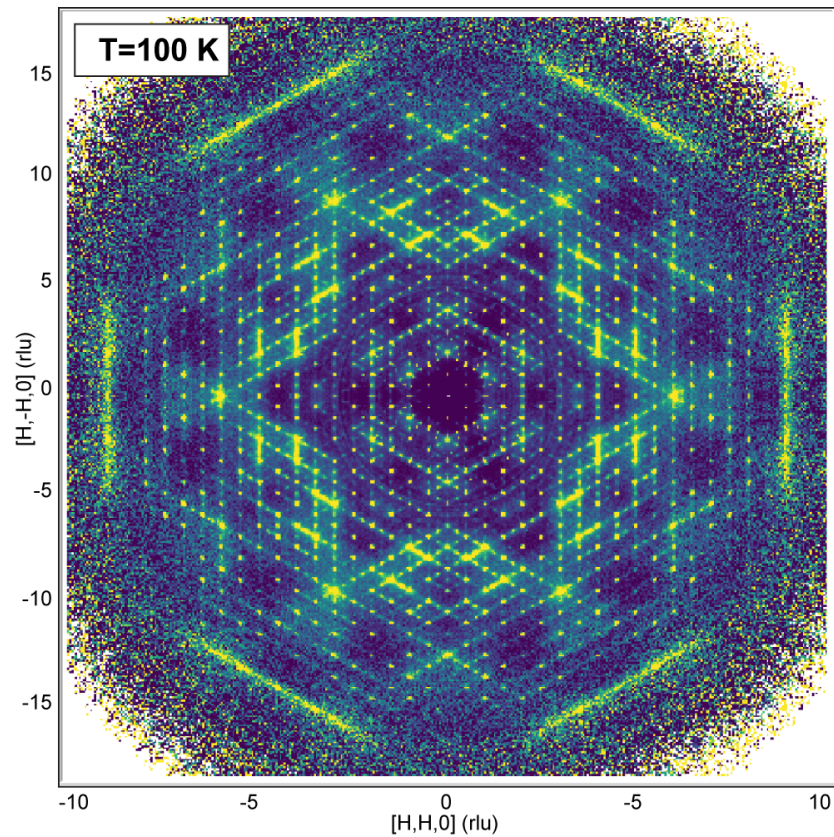
chopper phase

After cross-correlation



First Results

Benzil $C_{14}H_{10}O_2$

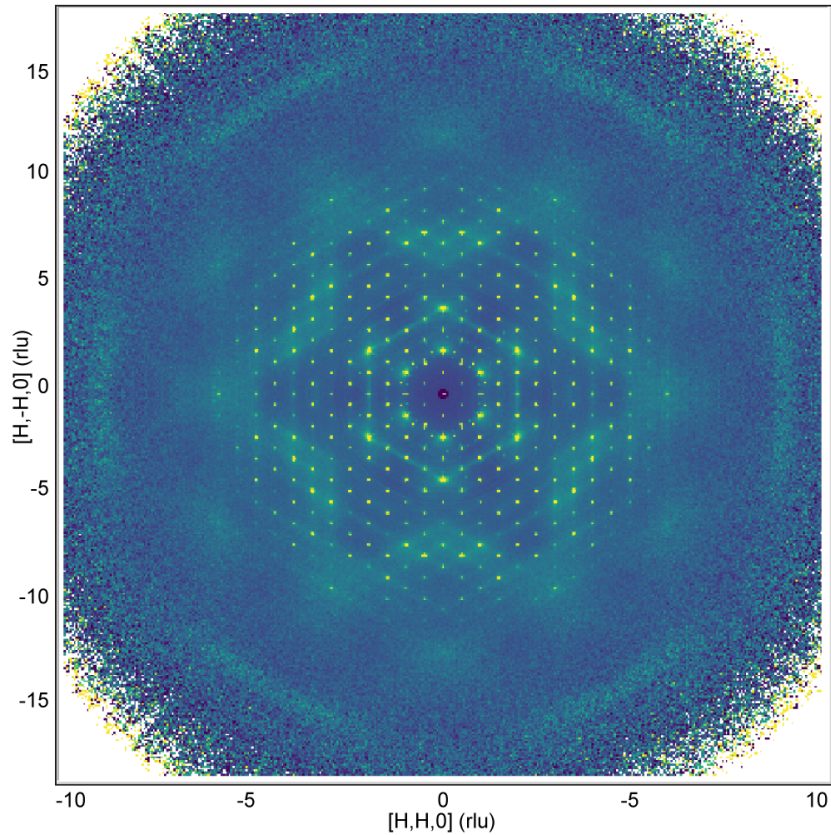


Acknowledgement: Richard Welberry (PI) and Christina Hoffmann

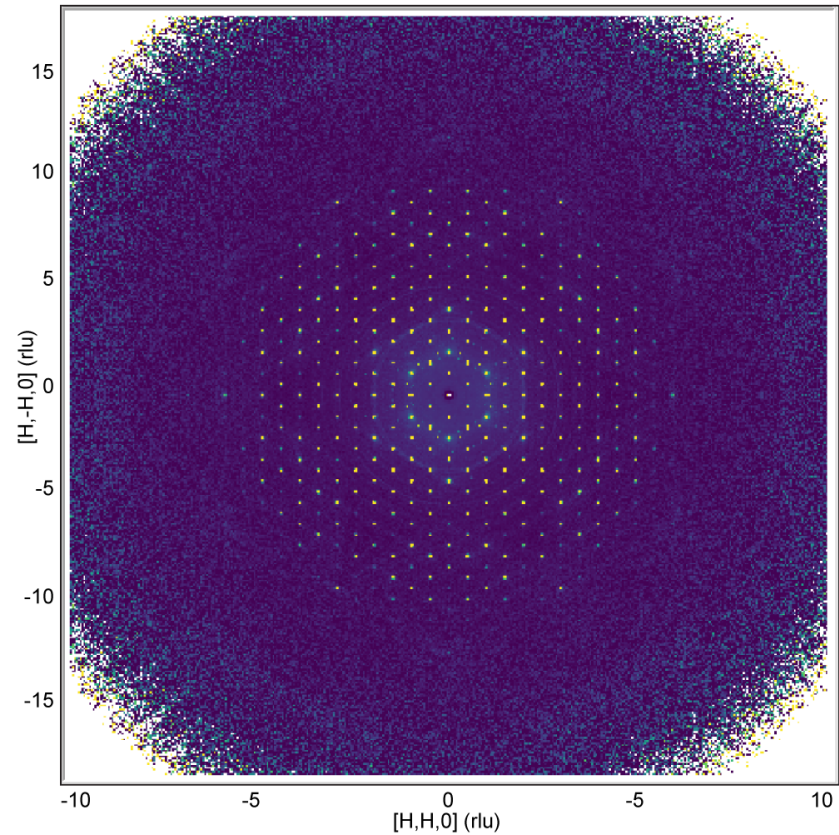
Does Cross Correlation Work?

Benzil $C_{14}H_{10}O_2$

T=300 K, no Cross-Correlation



T=300 K, Cross-Correlation

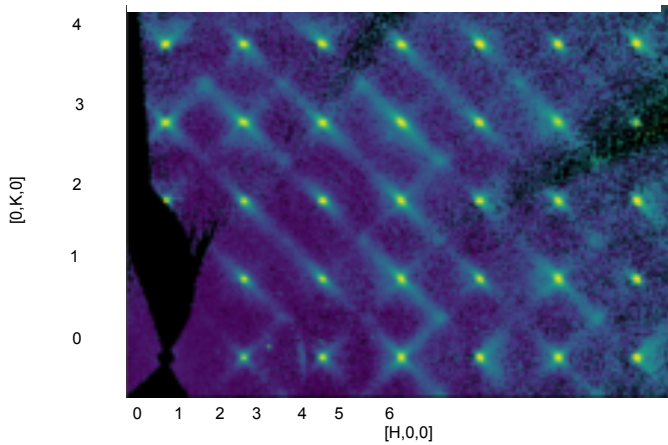


Acknowledgement: Richard Welberry (PI) and Christina Hoffmann

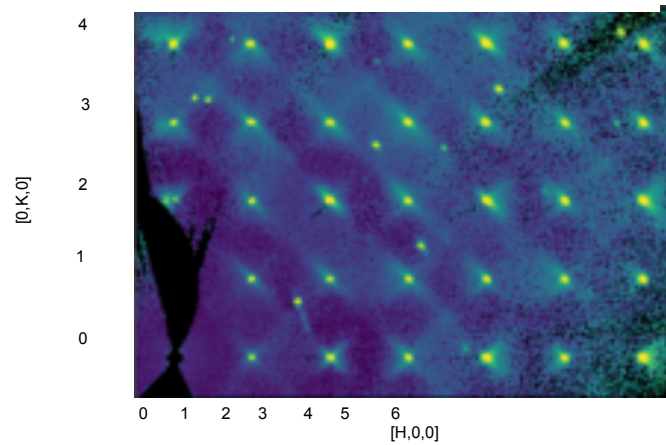
First Results

Relaxor Ferroelectrics - $\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3$ -30% PbTiO_3

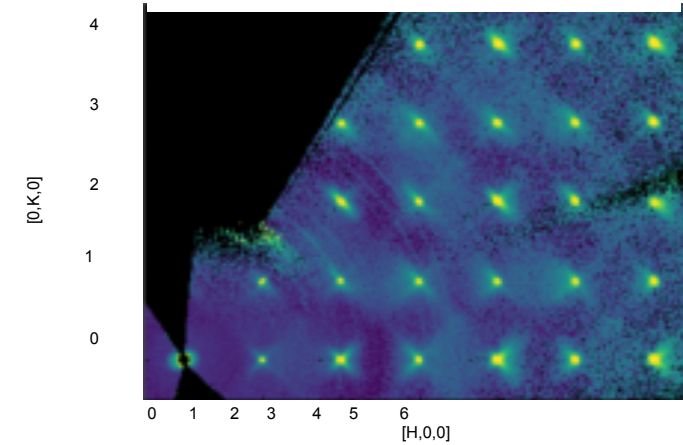
PMN



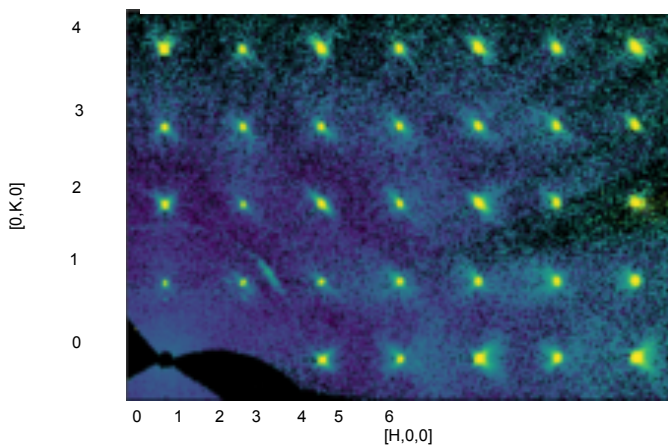
PMN-20PT



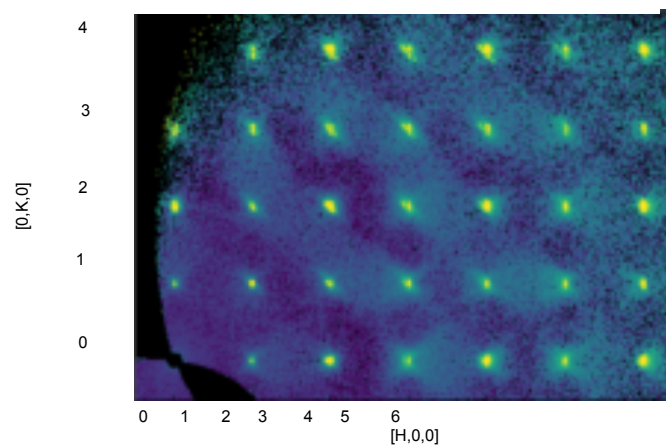
PMN-30PT



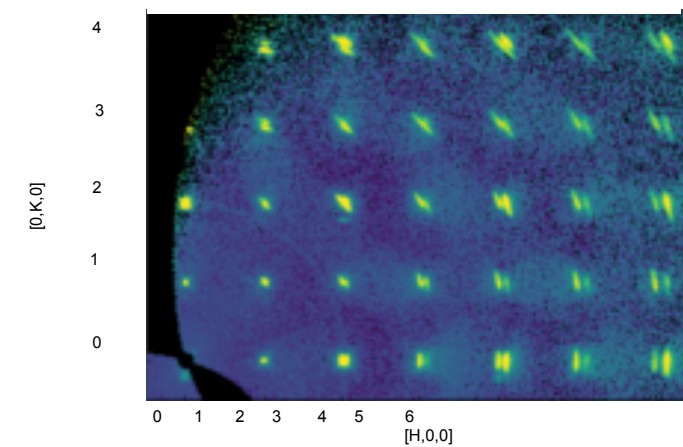
PMN-35PT



PMN-40PT



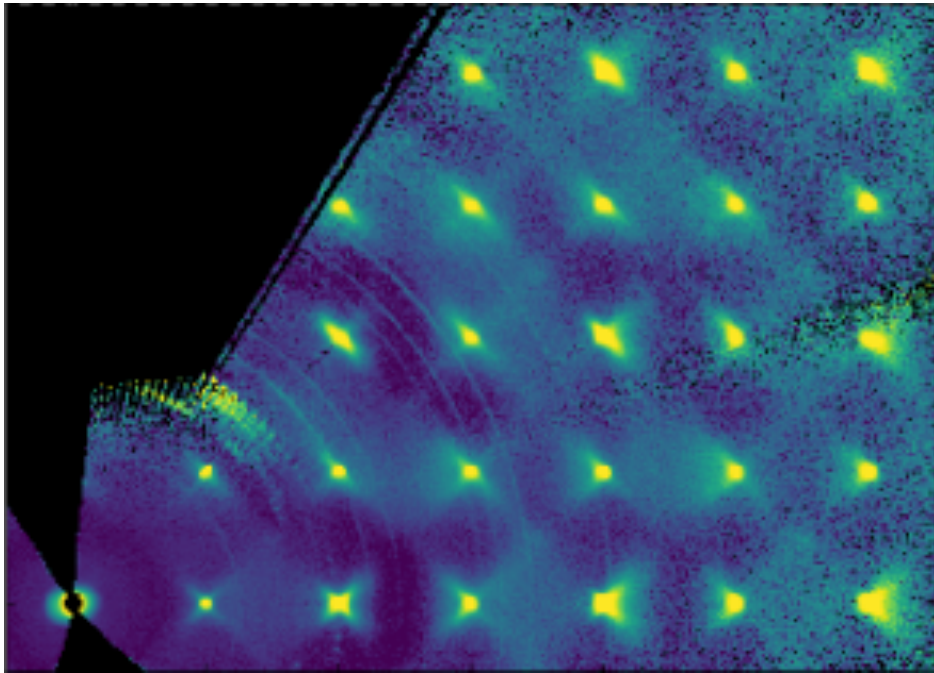
PMN-50PT



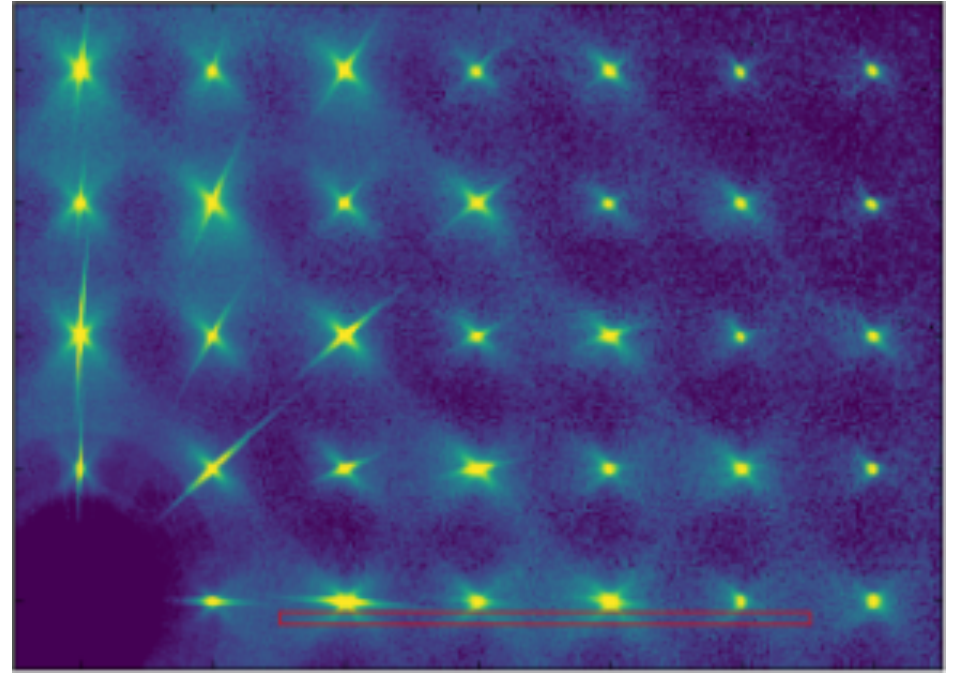
Acknowledgement: Matt Krogstad, Daniel Phelan, Stephan Rosenkranz

Complementarity of Neutrons and X-rays

$\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3\text{-}30\%\text{PbTiO}_3$



Corelli Neutrons



CHES 55keV X-rays

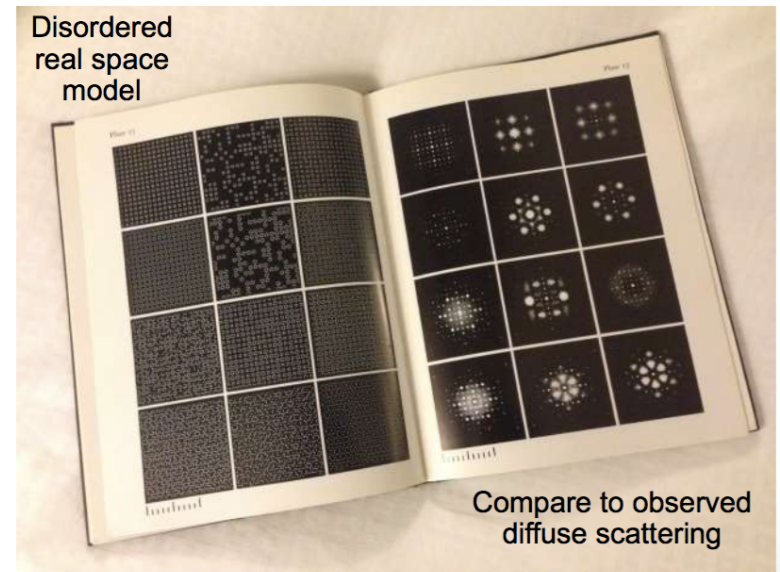
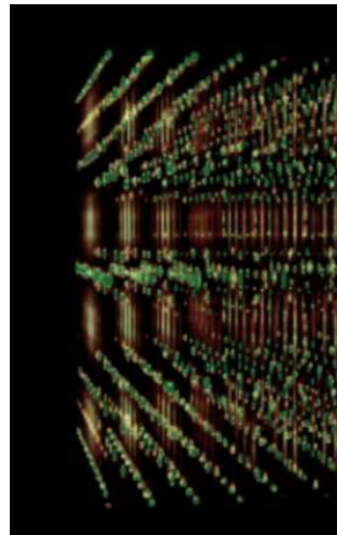
Acknowledgement: Matt Krogstad, Daniel Phelan, Stephan Rosenkranz

D3: Defects, Distortions, and Dynamics, June 27-28, 2016



The Future

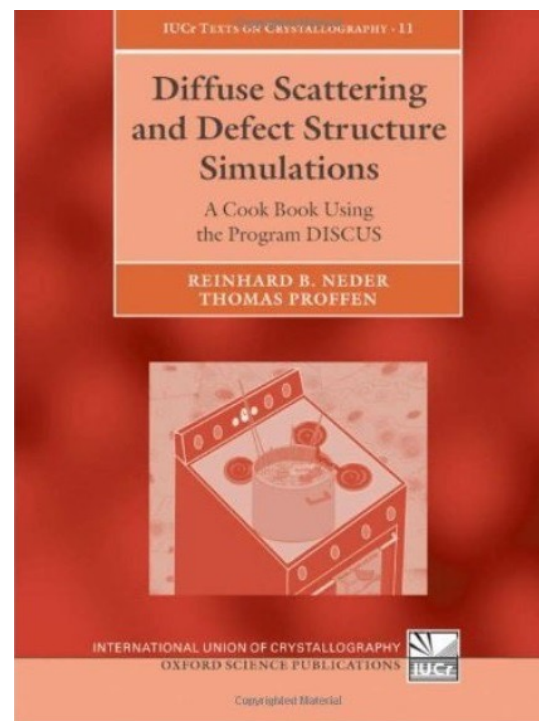
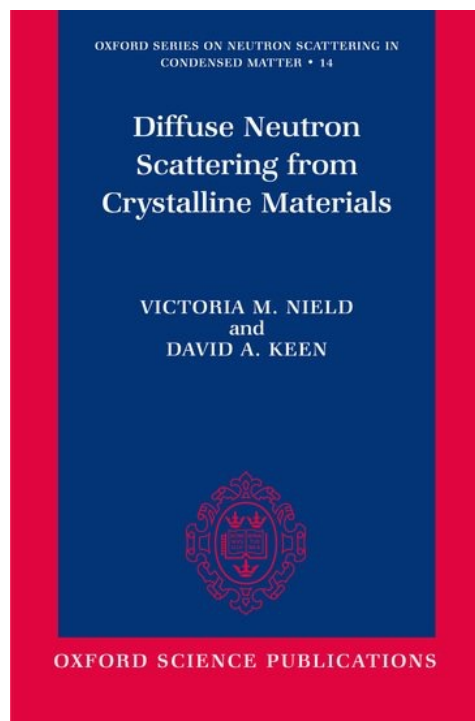
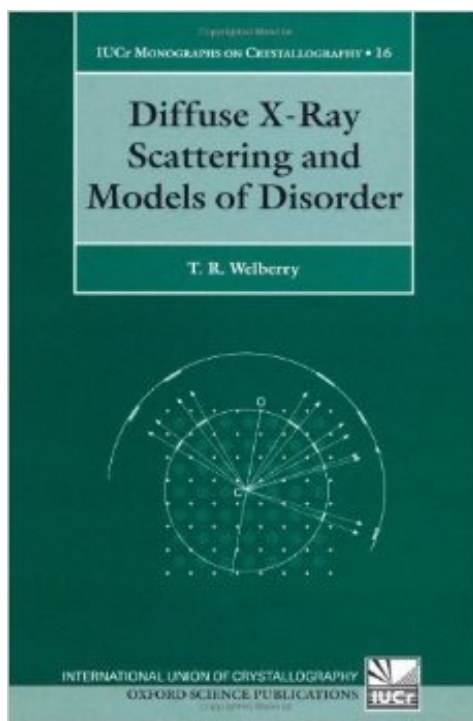
- ▶ High-Energy X-rays
 - Absorption lengths similar to neutrons
 - Most existing detectors have low efficiency but alternatives exist, *e.g.* CdTe
- ▶ Micro-diffuse scattering
 - Benefiting from increased brightness of, *e.g.*, APS Upgrade
- ▶ Increasing use of *ab initio* computational modeling
 - Allowing more complex systems to be investigated
 - Less dependence on intuition in modeling
- ▶ Enhanced analysis tools
 - Machine learning
 - Correlated data analysis
 - Easier co-refinement of neutrons and x-rays



Atlas of Optical Transforms, Harburn, Taylor and Welberry (1975)

A Few References

- ▶ T. R. Welberry & B. Butler, Chem Rev **95**, 2369–2403 (1995).
- ▶ F. Frey, Acta Cryst B **51**, 592–603 (1995).
- ▶ T. R. Welberry & D. J. Goossens, Acta Cryst A **64**, 23–32 (2007).
- ▶ D. A. Keen & A. L. Goodwin, Nature News **521**, 303–309 (2015).

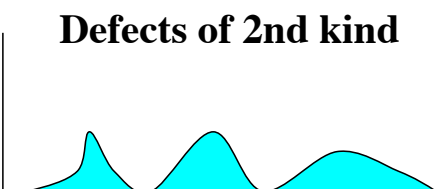
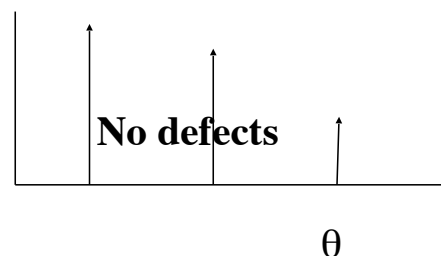
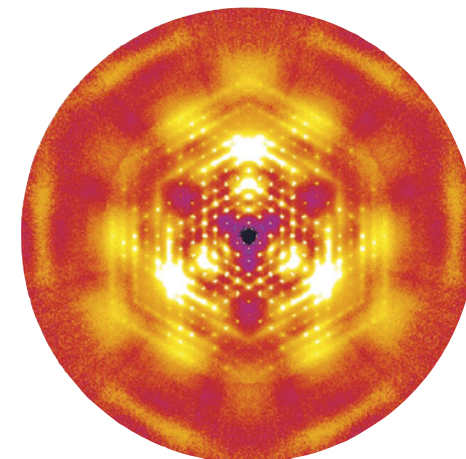


Diffuse Scattering Song

- ▶ Come eager young scholars - so tender and new
I'll teach you diffraction - what I says mostly true
Between the Bragg Peaks lies a world where you see
Fluctuations and defects- they stand out plane-ly
- ▶ *Chorus*
For its dark as a dungeon between the Bragg peaks
But here in the darkness - each defect speaks
It gathers- from throughout- reciprocal space
And re-distributes all over the place.
- ▶ Between the Bragg peaks - one thing that we see
Is TDS on our CCD
Intensity totals are conserved- you can't win
It steals from the Bragg peaks that stay very thin
- ▶ Substitutional alloys can cause quite a stir
The shorter the length scale the greater the blur
With care you can find out the bond length between
Each atom pair type-the measurements clean
- ▶ Dislocations and other- type 2 defects
Destroy the Bragg peaks -they turn them to wrecks
But near the Bragg peaks- you still can see
Intense diffraction continuously
- ▶ Many -are- the defects you find
Between the Bragg peaks where others are blind
So go tell your friends and impress your boss
You've new understanding -with one hours loss



Gene Ice



Krivoglaž Classifications

