Coherent Imaging

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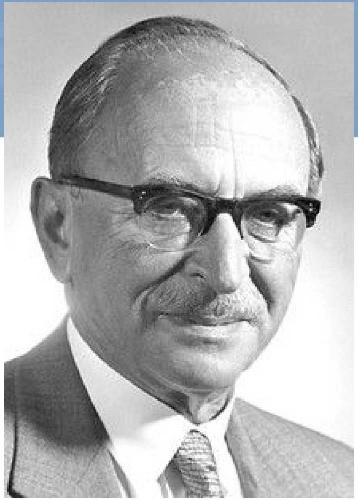
Physicist

18th National School on Neutron and X-ray Scattering
8 August 2015

"... the image that will be formed in a photographic camera --- i.e. the distribution of intensity on the sensitive layer --- is present in an invisible, mysterious way in the aperture of the lens, where the intensity is equal at all points"

Fritz Zernike

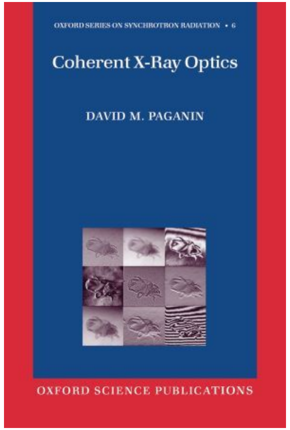
1953 Nobel Prize for Invention of Phase Contrast Microscope



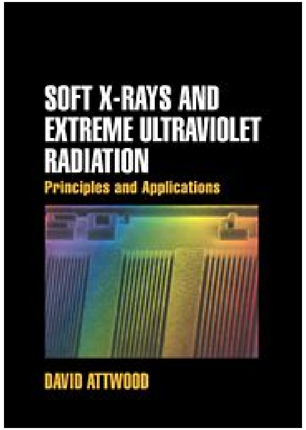
"... a solution suddenly dawned on me ...why not take a bad electron picture, but one which contains the whole information, and correct it by optical means?"

Dennis Gabor
1971 Nobel Prize for Invention of Holography

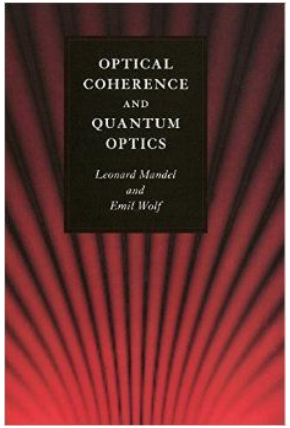
Further Reading



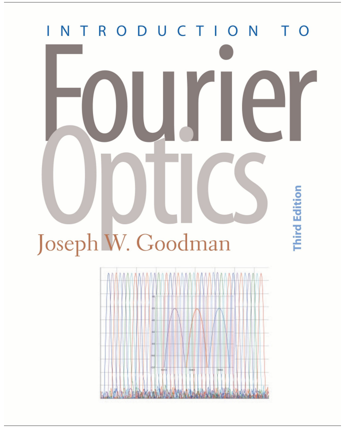
Paganin
“Coherent X-ray Optics”



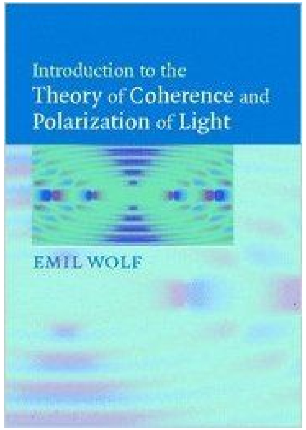
Attwood
“Soft X-rays and Extreme Ultraviolet Radiation”



Mandel & Wolf
“Optical Coherence & Quantum Optics”



Goodman
“Fourier Optics”



Wolf
“Introduction to the Theory of Partial Coherence and Polarization of Light”

Further Viewing

Richard Feynman at the Sir Douglas Robb Lectures, Uni. of Auckland, New Zealand

Part 1: https://youtu.be/eLQ2atfqk2c

Part 2: https://youtu.be/kMSgE62S6oo

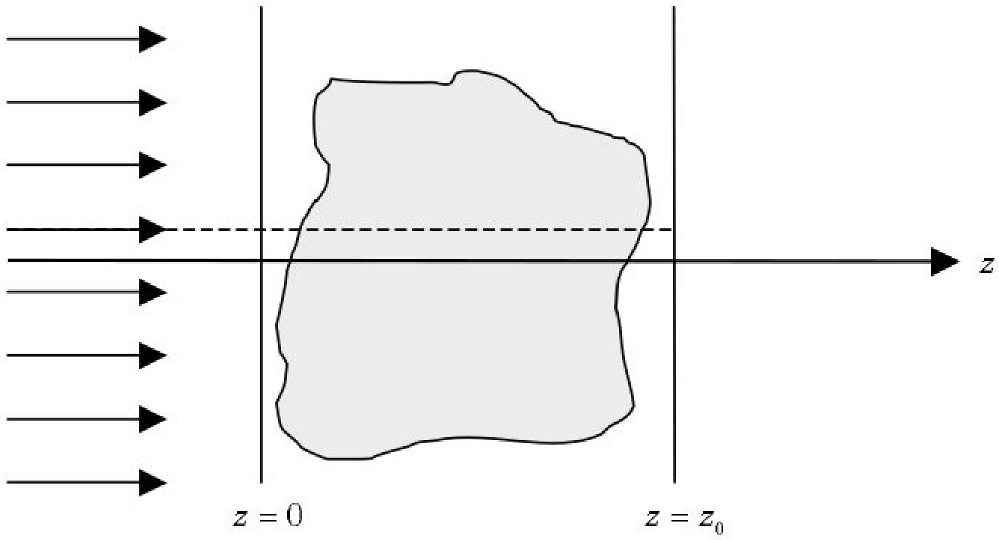
Part 3: https://youtu.be/jNNXD7fuE5E

Part 4: https://youtu.be/UigjOJm6F9o

Outline

- Phase sensitive imaging
  - Phase & magnitude of complex scalar wavefield
- Coherence & partial coherence
- Holography
- Phase contrast
- Coherent diffractive imaging
- Ptychography

An Imaging Gedanken Experiment



What do we see if we put a detector at z=z0?

Complex Scalar Wavefields in Matter

As wave traverses matter:

- magnitude decreases
- phase shift (relative to wave traveling same distance in vacuum) accumulates

We can describe this using the *complex amplitude*:

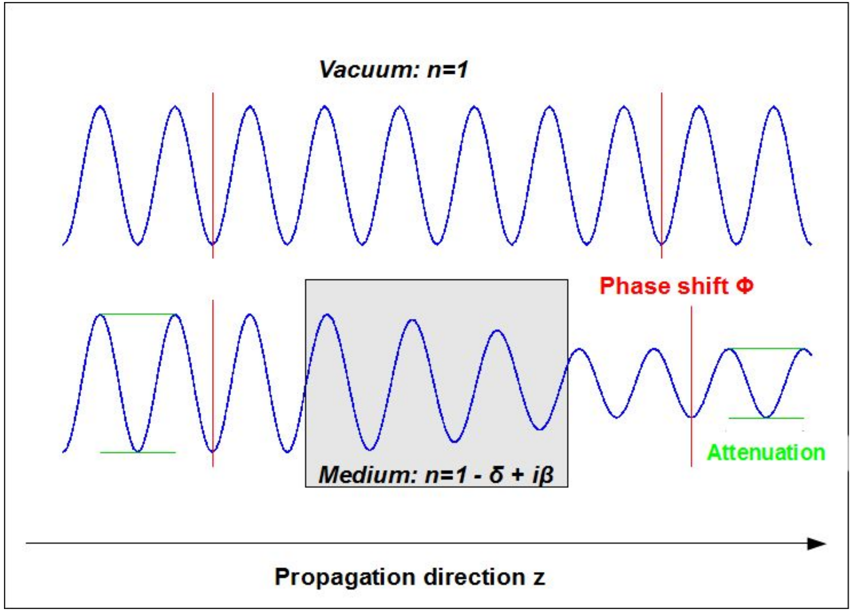
$$\psi(\boldsymbol{x}) = \exp(-in\boldsymbol{k}\boldsymbol{x})$$

Where

$$n(\boldsymbol{x}) = 1 - \delta(\boldsymbol{x}) + i\beta(\boldsymbol{x})$$

The total phase shift is

$$\phi(\boldsymbol{x}) = k\delta(\boldsymbol{x})$$



Measuring The Intensity

Wavefield oscillates at 10^{18}Hz which is much too fast to observe directly

Instead we measure intensity

$$I = |\psi|^2$$

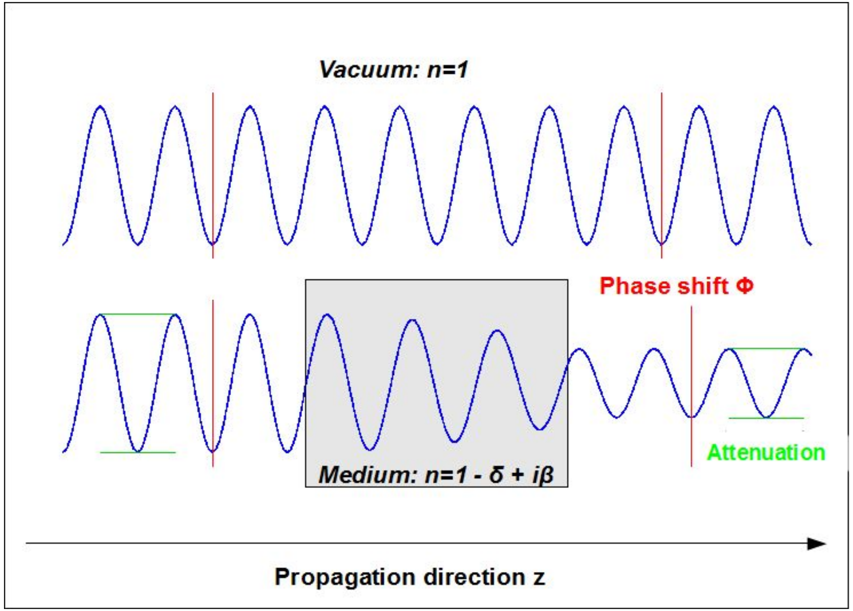
For our example above:

$$I(x) = |\exp[i\phi(x) - \beta(x)]|^2 = \exp[-\mu(x)]$$

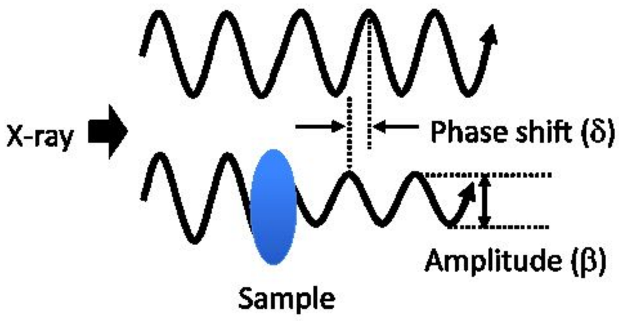
where $\mu(\boldsymbol{x}) = 2k\beta(\boldsymbol{x})$

which is the Beer-Lambert Law.

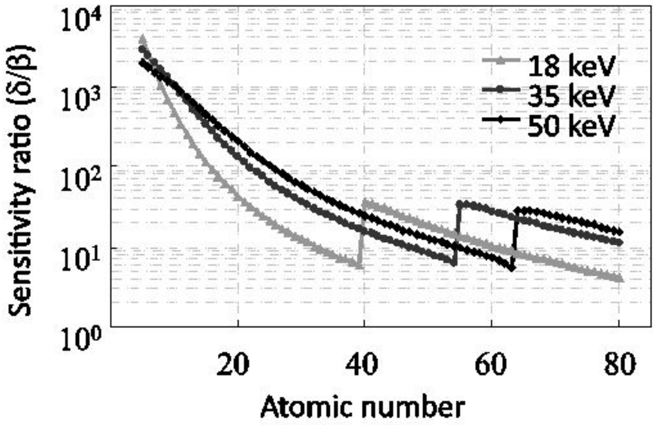
Phase information is lost upon measurement



X-ray Phase and Attenuation



(a)



(b)

Phase shift is

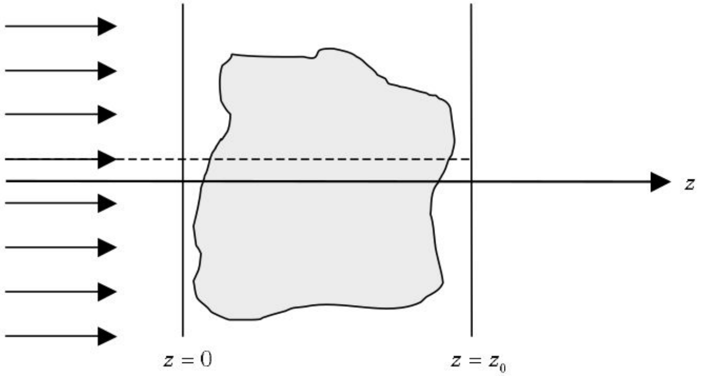
- 1000x larger than absorption for Z<20
- 10-100x larger for Z>20

Back To The Gedanken Experiment...

What do we see if we put a detector at z=z0?

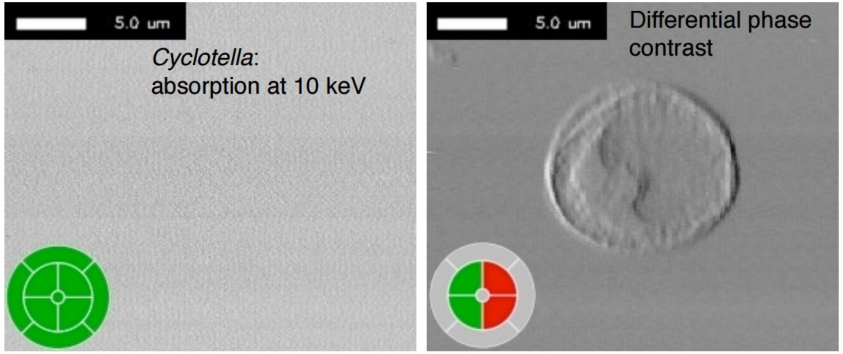
$$\psi(x) = \exp[i\phi(x))]$$

$$I(x) = |\psi(x)|^2 = 1$$



For an object which only phase shifts the beam (a good approximation for biological samples, polymers etc) our perfect imaging system produces no contrast!

We need phase sensitive imaging!



Hornberger *et al.*, *Ultramic.* **107**, 644 (2007); Feser *et al.*, *Nucl. Inst. Meth. A* **565**, 841 (2006); de Jonge *et al.*, *Phys. Rev. Lett.* **100**, 163902 (2008)

Coherence & Partial Coherence

Adapted from David Attwood (http://www.eecs.berkeley.edu/~attwood/)

Optical Coherence

Some degree of coherence is required to observe phase contrast

Optical coherence is

a persistent relationship in the phase of a wavefield between two spacetime points

Formally:

$$\Gamma(P_1, P_2, \tau) = \langle \psi^*(P_1, t)\psi(P_2, t+\tau)\rangle_t$$

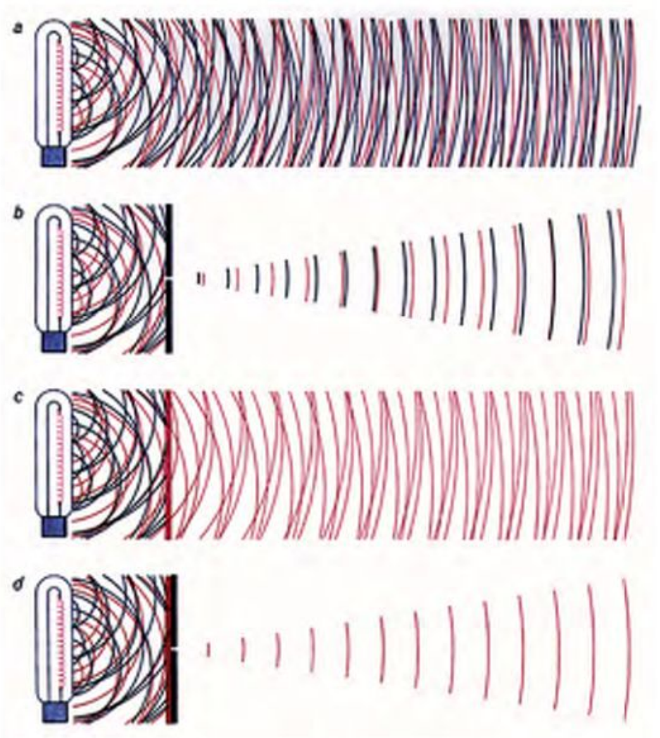
where

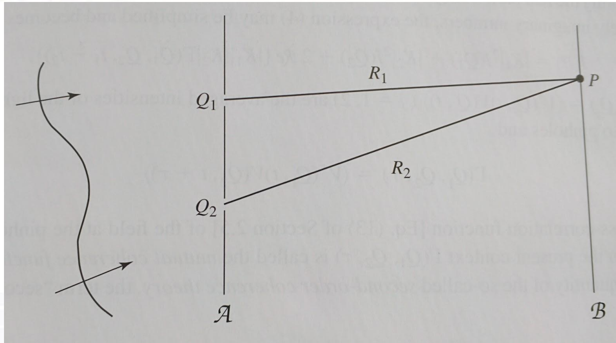
$$\tau = \frac{P_1 - P_2}{c}$$

The complex degree of coherence:

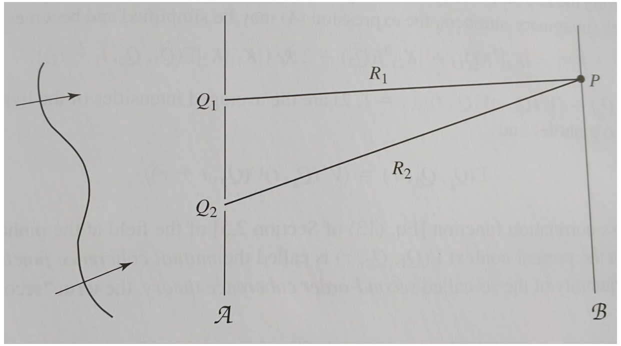
$$\gamma(P_1, P_2, \tau) = \frac{\Gamma(P_1, P_2, \tau)}{\sqrt{\Gamma(P_1, P_1, 0)}\sqrt{\Gamma(P_2, P_2, 0)}}$$

γ~1 is required to form high contrast fringes, γ~0 implies the radiation cannot form fringes. In practice all wavefields are 0<γ<1 or partially coherent





Fringe Visibility



$$V(P) = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$V(P) = |\gamma(Q_1, Q_2, \tau)|$$

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 47, NUMBER 10 OCTOBER, 1957

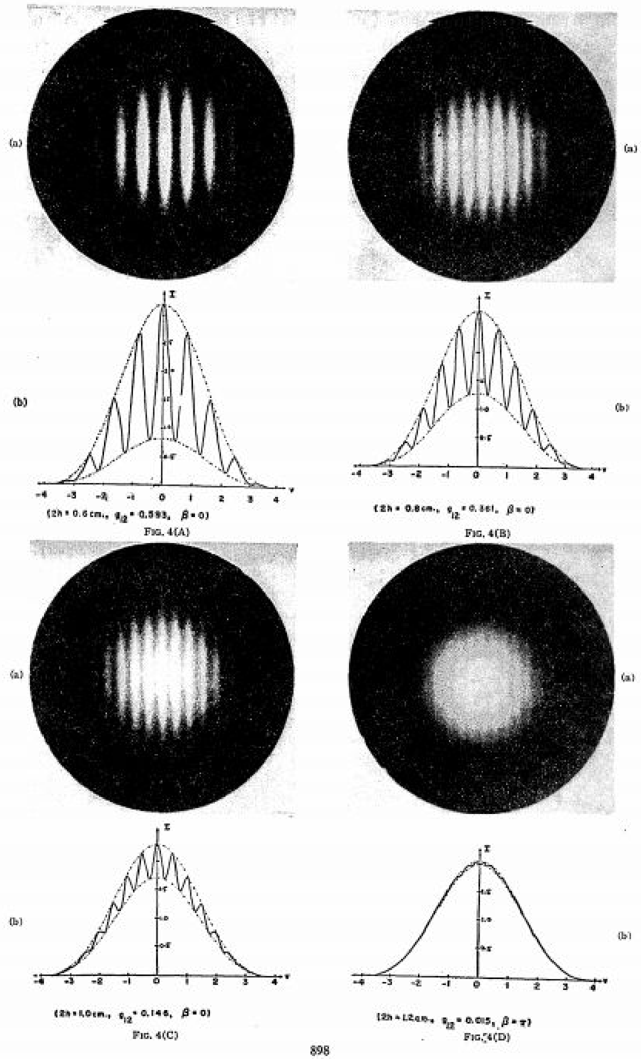
Two-Beam Interference with Partially Coherent Light

B. J. Thompson, *Physics Department, The College of Science and Technology, University of Manchester, Manchester, England*

AND

E. Wolf, *The Physical Laboratories, University of Manchester, Manchester, England**

(Received December 17, 1956)



Spatial & Temporal Coherence

Spatial coherence length is the lateral distance over which there is a well defined relationship in the phase

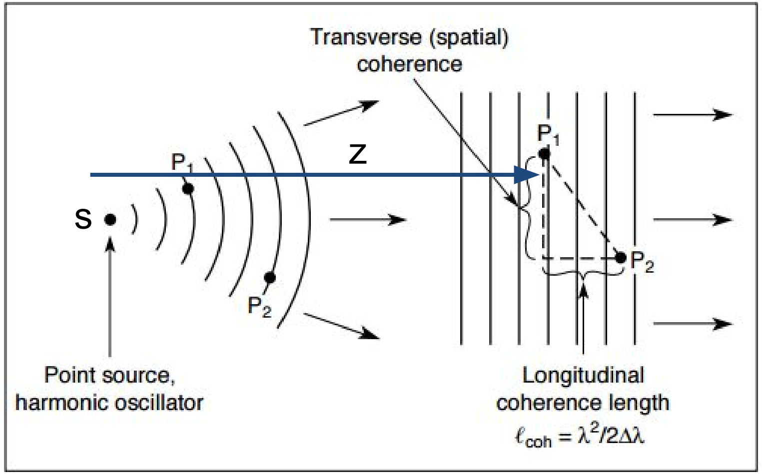
$$l_s = \frac{\lambda}{\Delta\omega}$$

$$l_s = \frac{\lambda z}{s}$$

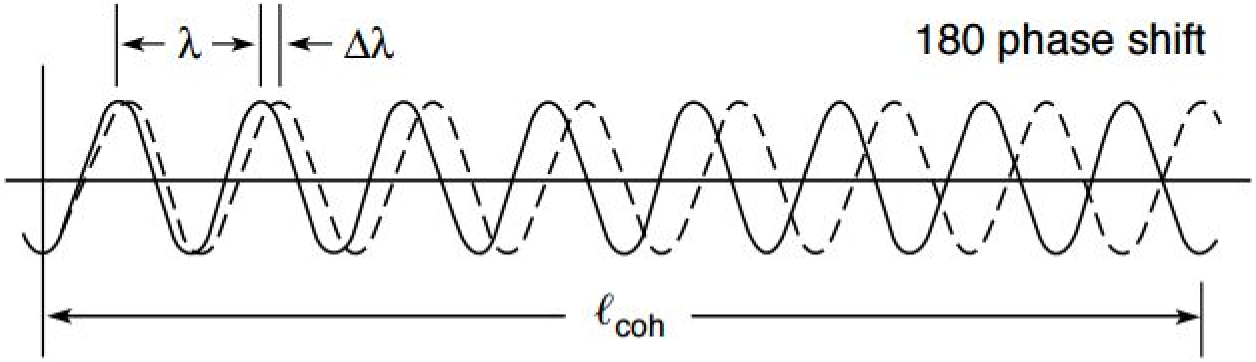
$$\Delta\omega = s/z$$

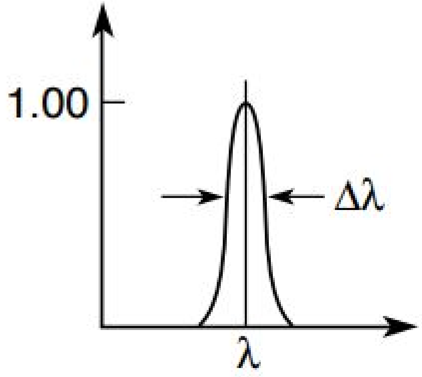
Temporal coherence length is the longitudinal distance over which there is a well defined relationship in the phase

$$l_t = \frac{\lambda^2}{\Delta\lambda}$$



Temporal Coherence





Distance at which λ and $\lambda + \Delta\lambda$ are 180 degrees out of phase

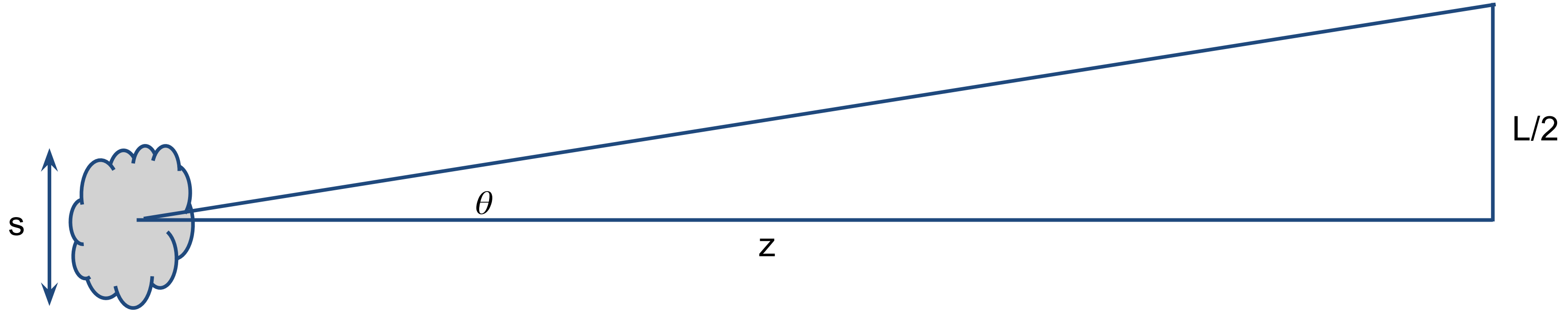
$$l_t = N\lambda = (N - 1/2)(\lambda + \Delta\lambda)$$

$$l_t = \frac{\lambda^2}{\Delta\lambda}$$

Spatial Coherence

Radiation from a single phase space cell is spatially coherent

$$\Delta x \Delta p = \hbar/2$$
$$\Delta x \Delta k = 1/2$$
$$\Delta x \Delta \theta = \lambda/4\pi$$



$$l_s = \frac{\lambda z}{s}$$

Coherence length depends on wavelength, distance from source and source size
Any source can be coherent by adjusting these parameters

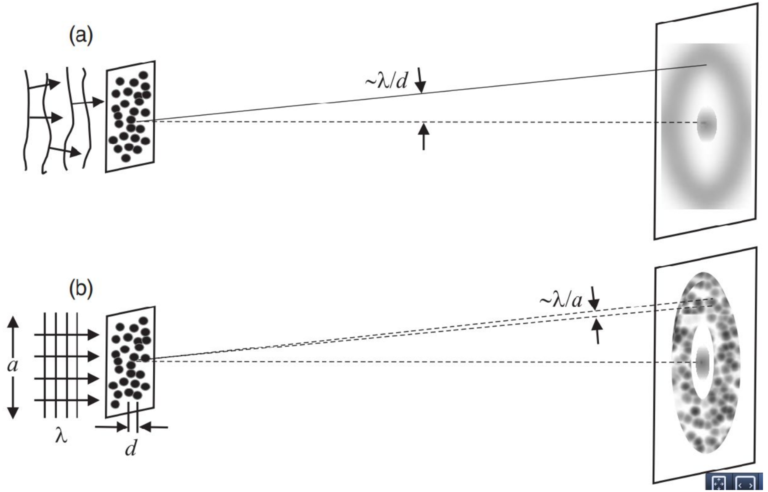
Coherent Imaging Systems

In the analysis of coherent imaging systems we add *amplitudes*,

$$I_{coh} = |\psi_1(x) + \psi_2(x)|^2$$

for incoherent imaging systems we add *intensities*

$$I_{incoh} = |\psi_1(x)|^2 + |\psi_2(x)|^2$$



Coherence of APS Source

Sector 2

Horizontal source size: 280 micron

Vertical Source Size: 10 micron

Coherence length @ 8keV & 60m

H: 26 micron

V: 750 micron

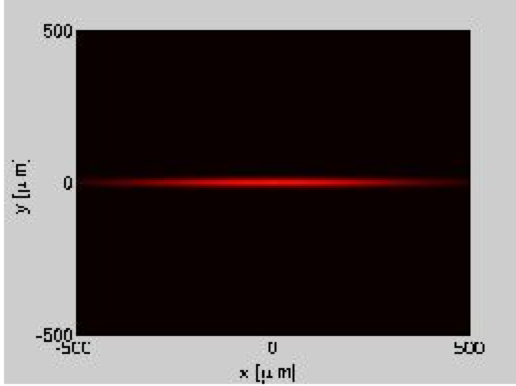
Longitudinal coherence length

pink beam: 5nm

mono: 1 micron

Table 1: Source Point Data Generated Tue Apr 28 21:16:05 CDT 2015 for I=101.60 mA, along with nominal (0) values.

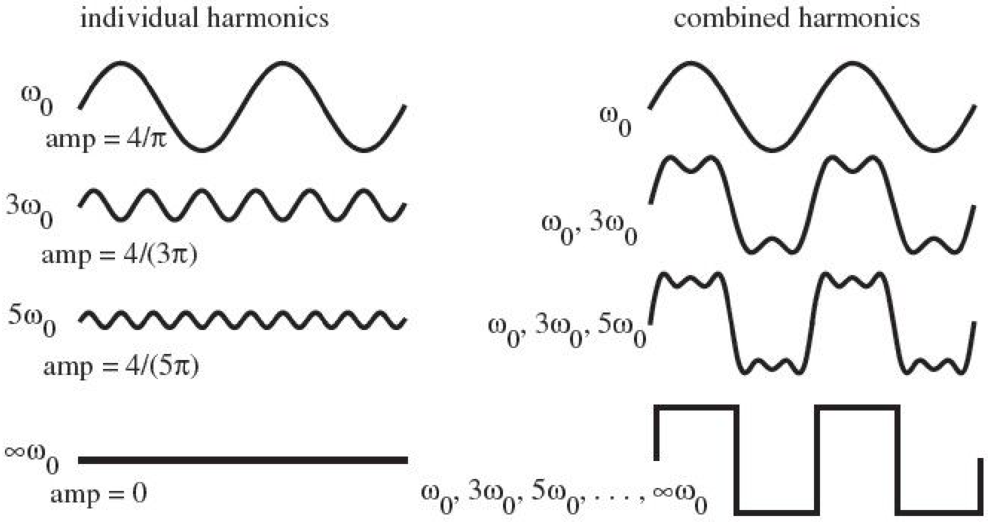
Name	exEff	CplngEff	Sx	Sxp	Sy	Syp	exEff0	CplngEff0	Sx0	Sxp0	Sy0	Syp0
	nm	%	um	urad	um	urad	nm	%	um	urad	um	urad
S1ID	3.30	1.2	279.8	11.8	10.5	3.8	3.15	0.9	274.6	11.5	8.9	3.3
S2ID	3.30	1.2	280.3	11.8	10.5	3.9	3.14	1.0	275.0	11.4	9.0	3.4
S3ID	3.30	1.3	280.9	11.7	11.4	3.8	3.14	1.0	275.7	11.4	10.0	3.4



Diffraction Through Free Space, Lenses & the Fourier Transform

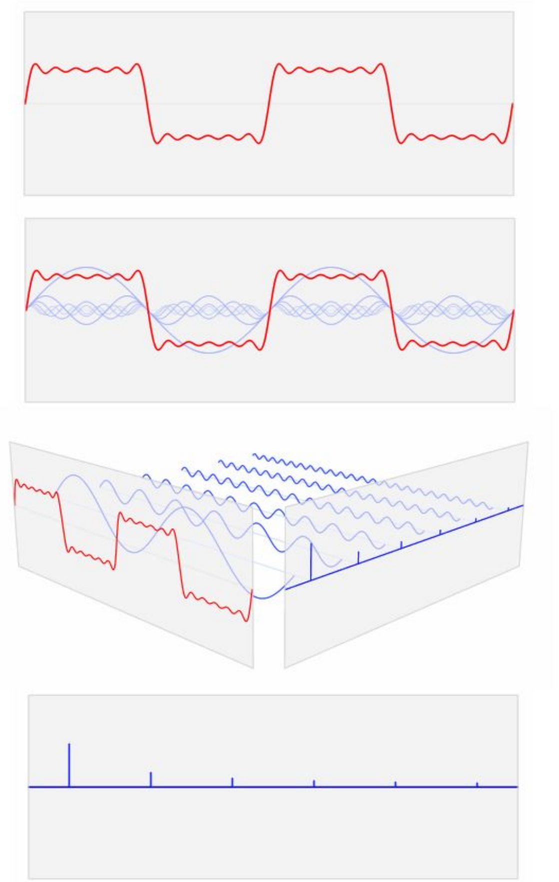
What is a Fourier Series/Transform?

A Fourier transform breaks a signal into its frequency components



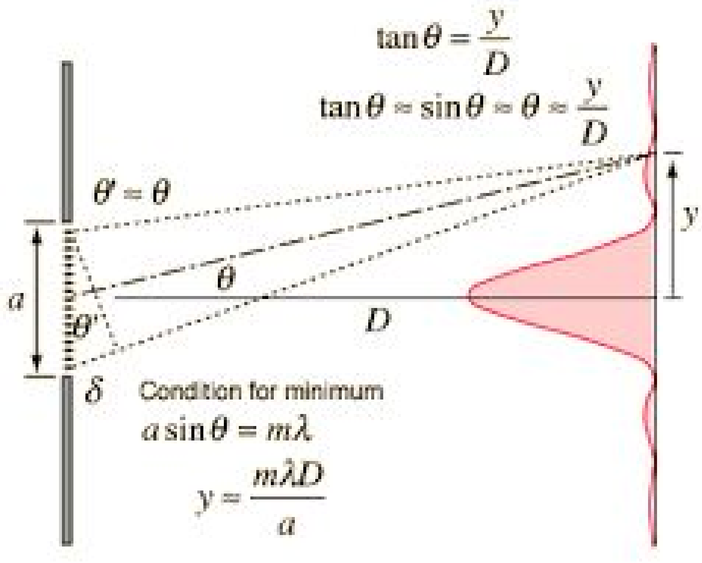
The machine in the background allows Kanye to adjust the volumes of certain frequency ranges in an attempt to make his music sound “good”

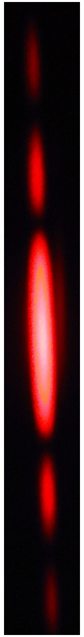
What is a Fourier Transform?



Single Slit Diffraction Is A Fourier Transform

$f(x)$





Fraunhofer Diffraction

By placing detector in the "far field" we record the Fraunhofer diffraction pattern - i.e. the modulus squared of the Fourier transform

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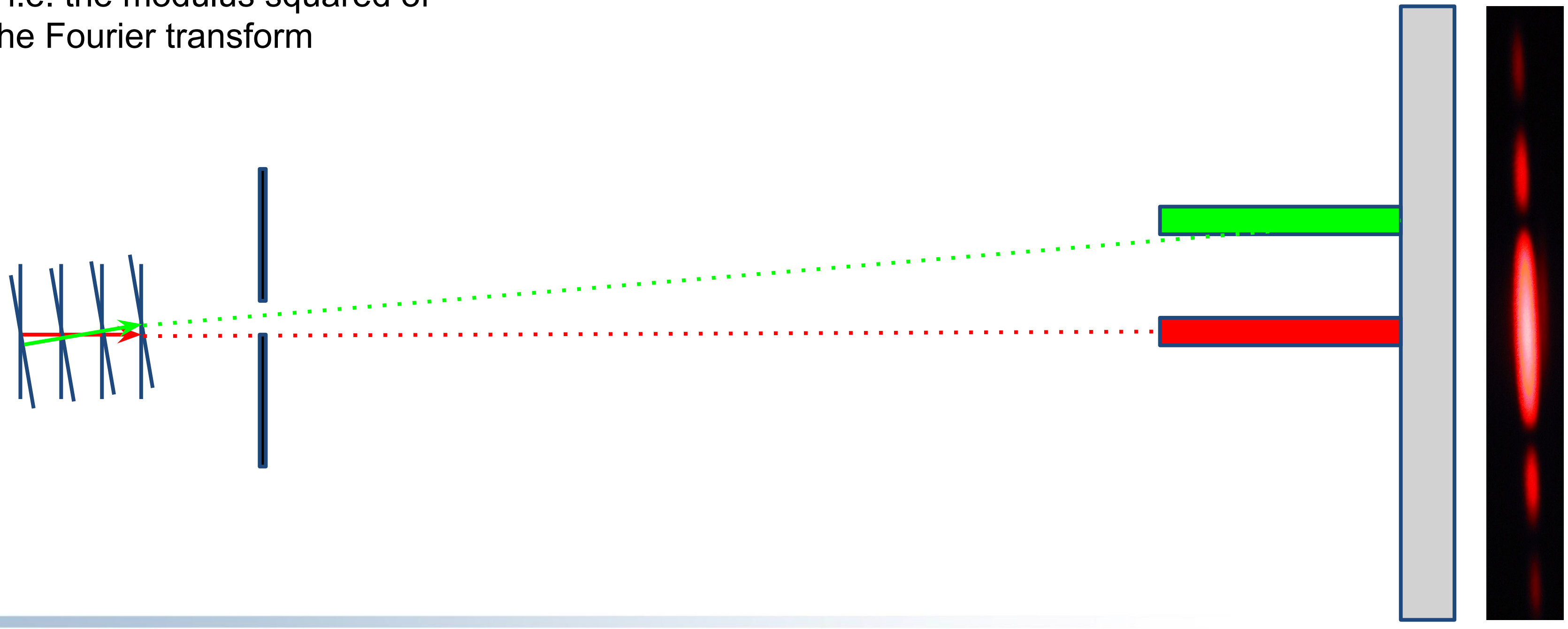
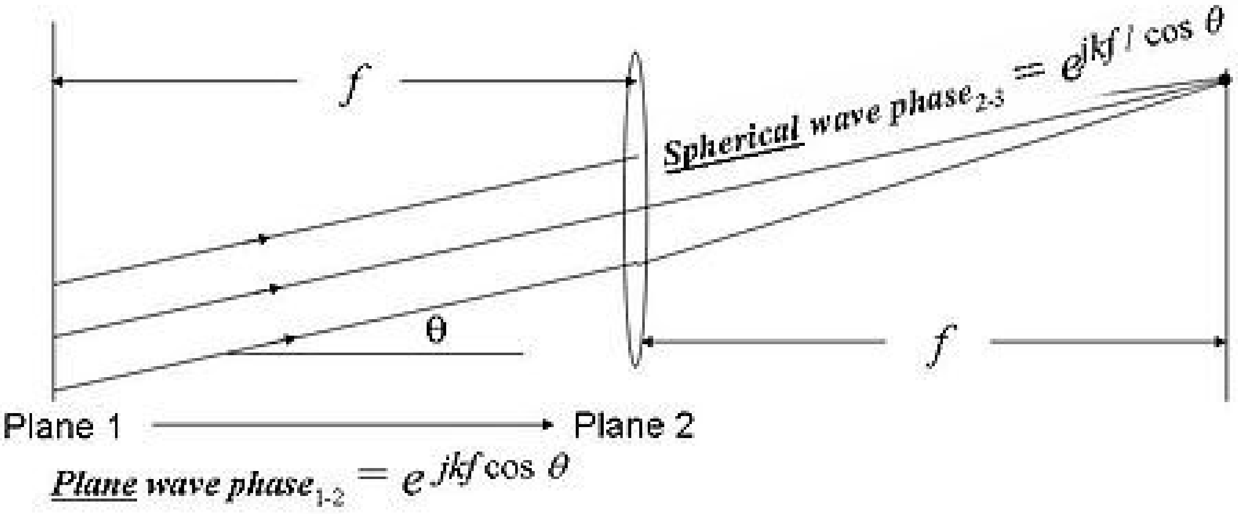


Lens as a Fourier Transform



Holography

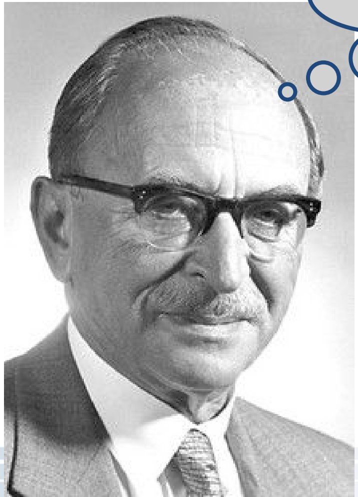
Invention of Holography

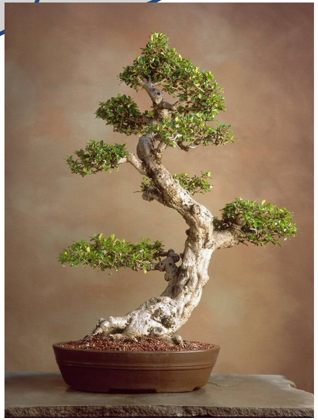
In 1948 Denis Gabor was trying to improve the quality of electron microscopy images

Aberrations in the electron optics limited the achievable spatial resolution to 1.2nm, far above the wavelength limit

Zen thought of the day:

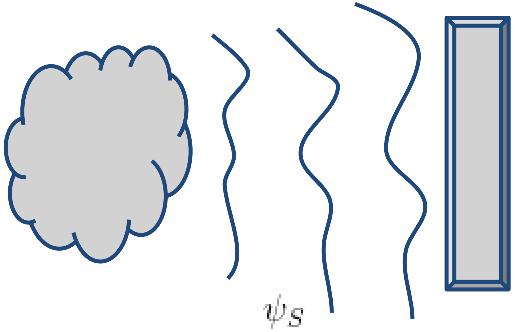
Maybe the best lens…
is no lens at all?



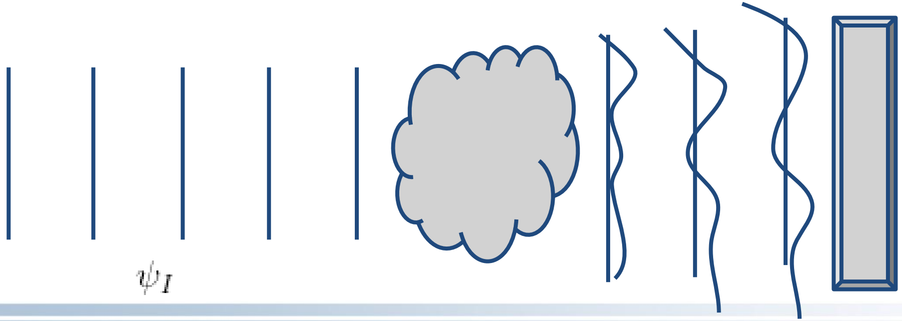


Inline Holography

Gabor's idea, the hologram, is a method for recording the "whole" wavefield information, including the phase that would otherwise be lost upon measurement



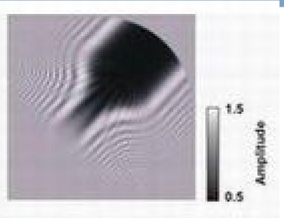
$I = |\psi_s|^2$ Phase information lost



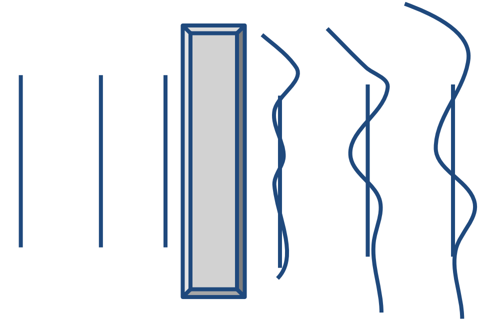
$$I = |\psi_I + \psi_s|^2$$
$$I = |\psi_I|^2 + |\psi_S^2| + \psi_I^*\psi_S + \psi_I\psi_S^*$$

Phase information recorded in the intensity

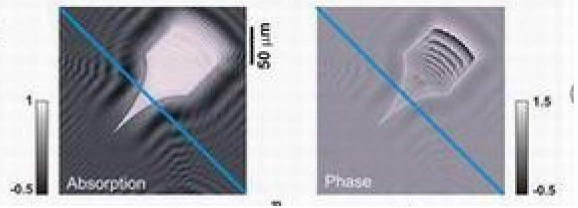
Holography Reconstruction



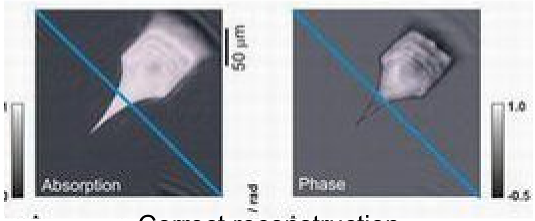
Hologram



$$\psi_I \times I = const + \psi_S + \psi_S^*$$



Reconstruction with twin image corruption



Correct reconstruction

$$I = |\psi_I + \psi_s|^2$$
$$I = |\psi_I|^2 + |\psi_S^2| + \psi_I^* \psi_S + \psi_I \psi_S^*$$

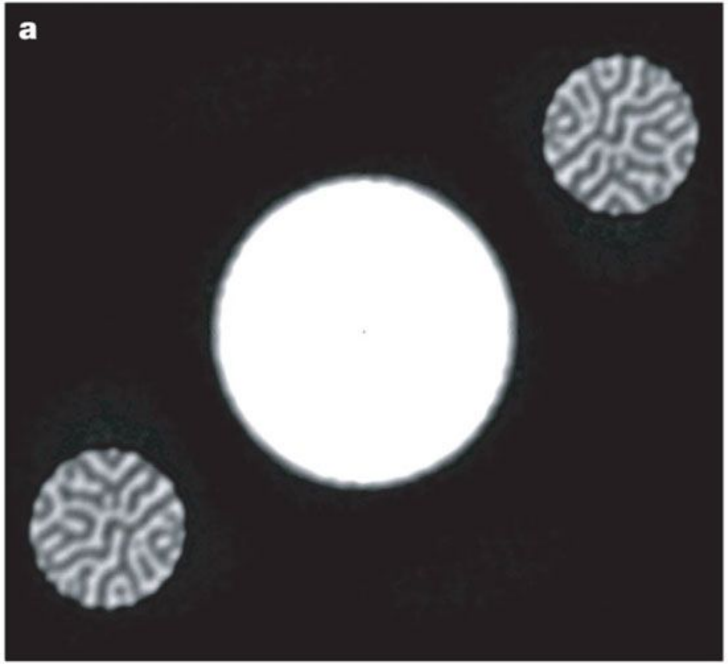
Gabor's approach was to make image formation a two step problem.
First record an image that is very aberrated, later illuminate the aberrated image to generate the corrected image

The complex conjugated term is the 'twin image' problem of inline holography and represents an out of focus copy of the wavefield

Phys. Rev. Lett. Vol. 98, 233901 (2007).

Fourier Holography

The twin image problem of Gabor (or inline) holography can be avoided using a different geometry for recording the hologram



$$\delta(r - r_0) + \psi_s$$

$$I = |\mathcal{F}\{\delta(r - r_0) + \psi_s\}|^2$$

$$I = 1 + |\mathcal{F}\{\psi_s\}|^2 + \exp(ikr_0)\psi_s^* + \exp(-ikr_0)\psi_s]$$

$$\mathcal{F}^{-1}\{I\} = const + \psi_s \otimes \psi_s^* + \psi_s(r - r_0) + \psi_s^*(r + r_0)$$

Lensless imaging of magnetic nanostructures by X-ray spectro-holography

S. Eisebitt[1], J. Lüning[2], W. F. Schlotter[2,3], M. Lörgen[1], O. Hellwig[1,4], W. Eberhardt[1] & J. Stöhr[2]

Nature **432**, 885-888 (16 December 2004)

Phase Contrast Imaging

Types of Phase Contrast Imaging

Holography adds a reference wave to encode the ‘whole’ information of the wavefield in the intensity image

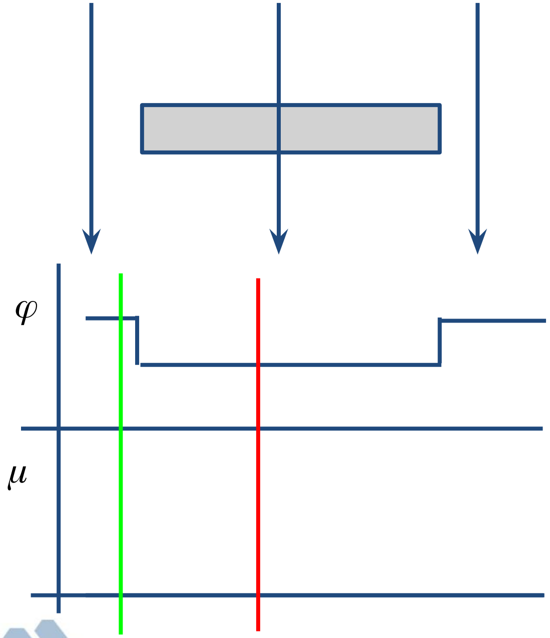
Phase contrast imaging systems aim to make phase shifts visible by modulating the intensity in a way that is somehow proportional to the phase. Unlike holographic imaging, phase contrast systems may not encode enough information to recreate the full complex wavefield

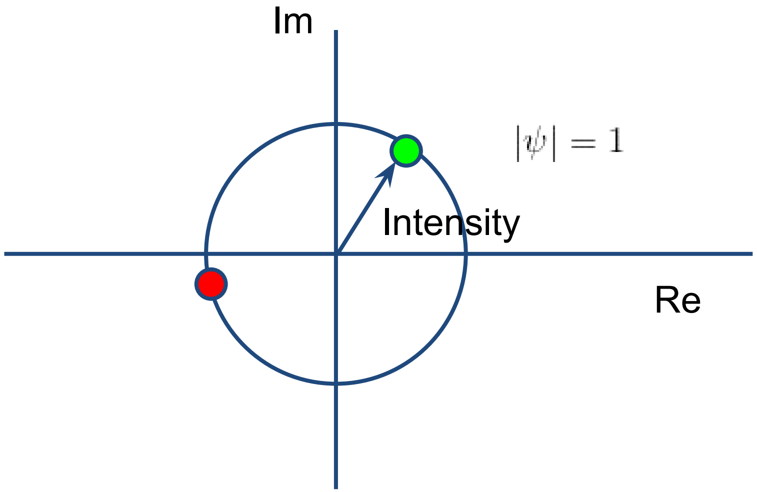
We’ll look at three types of phase contrast:

- Zernike phase contrast
- Schlieren phase contrast
- Crystal analyzed-based phase contrast
- Propagation-based phase contrast

Geometric Interpretation of Phase Contrast

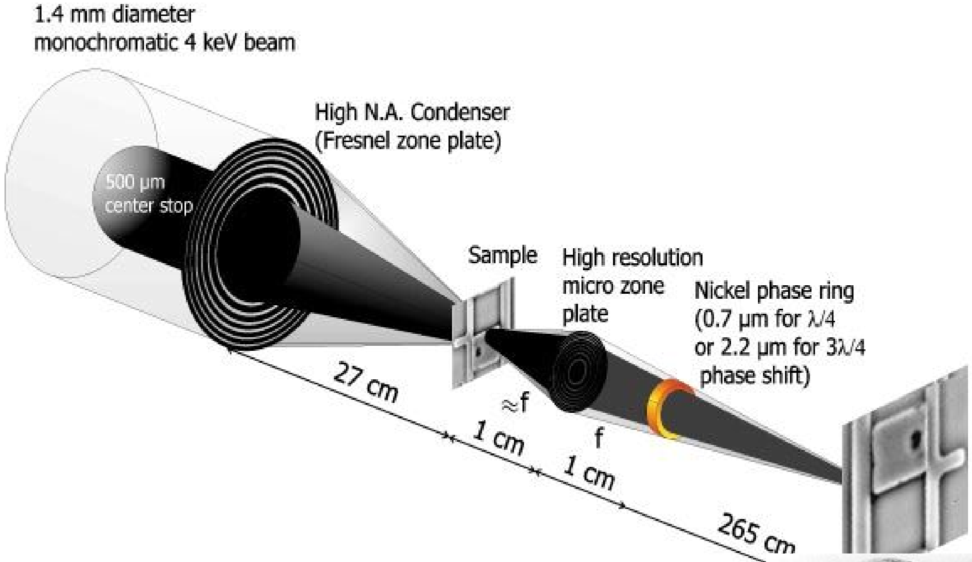
$$\psi = \exp(i\phi - \mu) = x + iy = \sqrt{x^2 + y^2}\measuredangle \tan^{-1}\frac{y}{x}$$

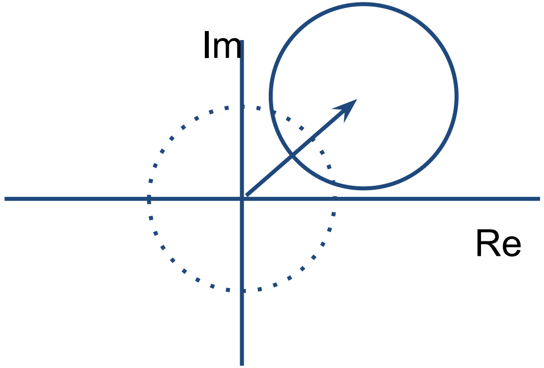




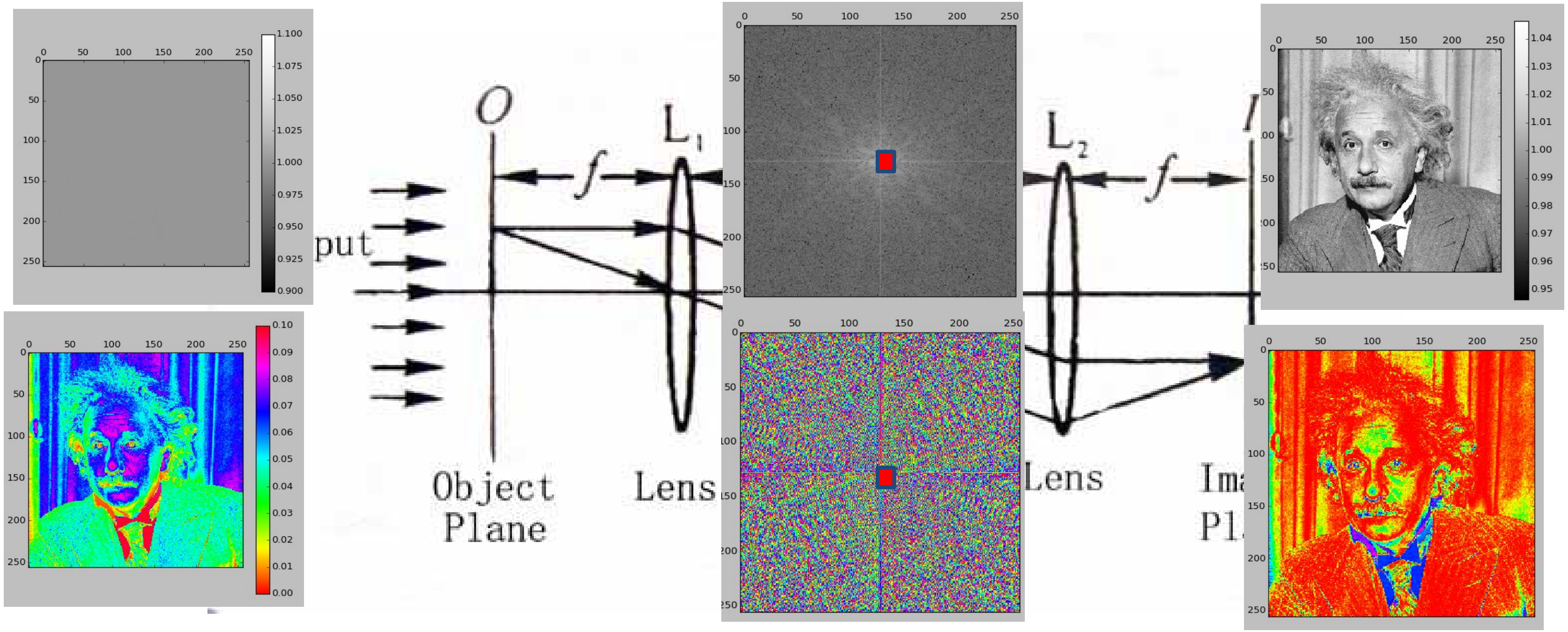
Zernike Phase Contrast

Zernike's idea was to phase shift the unscattered beam to make it interfere with the scattered radiation



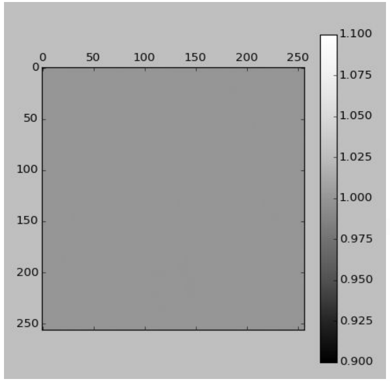


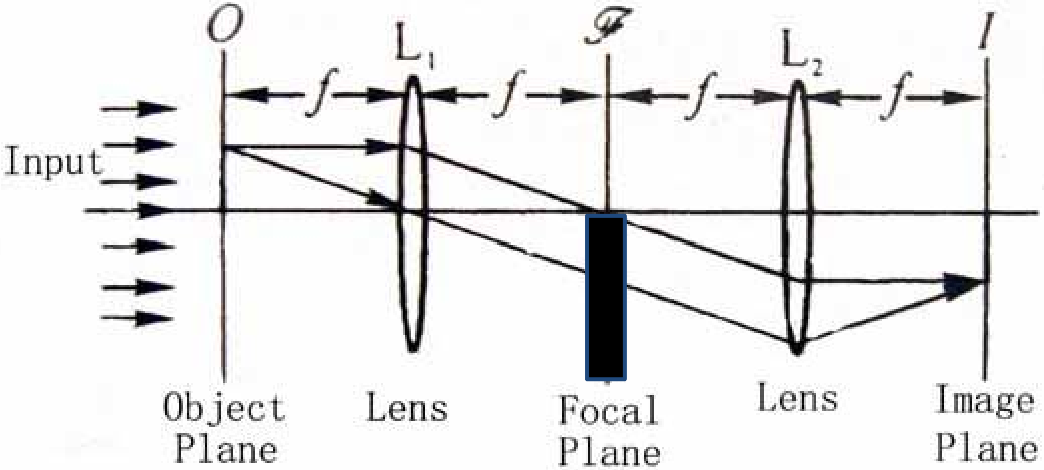
Zernike Phase Contrast As A Fourier Filter

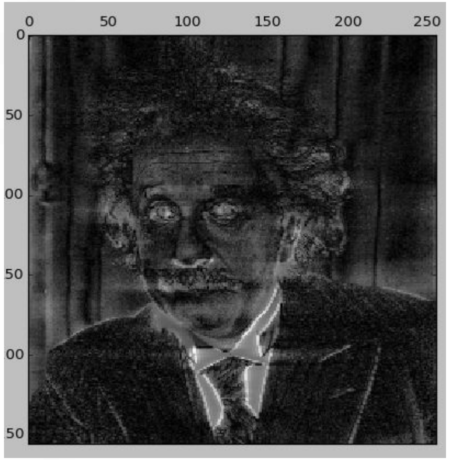


Schlieren Phase Contrast

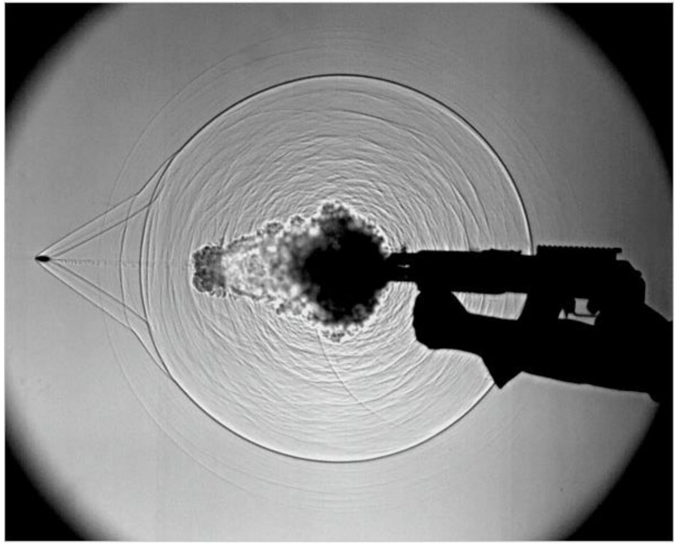
In Schlieren (or knife-edge) phase contrast a knife (or similar) is used to block half the light in the back focal plane







Schlieren Photography

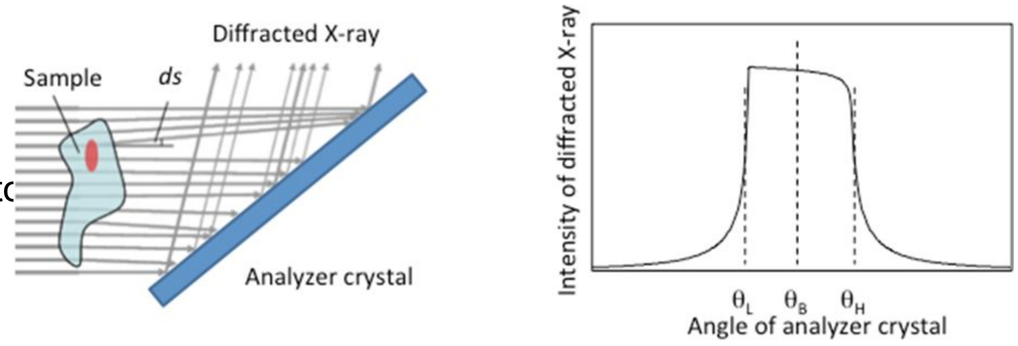


Crystal-analyzer Based Phase Contrast

The refraction angle for hard X-rays transmitted through thin, low-Z materials is on the order of micro-radian

The Darwin width of Bragg peak from single crystal of silicon is about that size

Can use the angular discrimination of crystal to convert phase shift to intensity variation



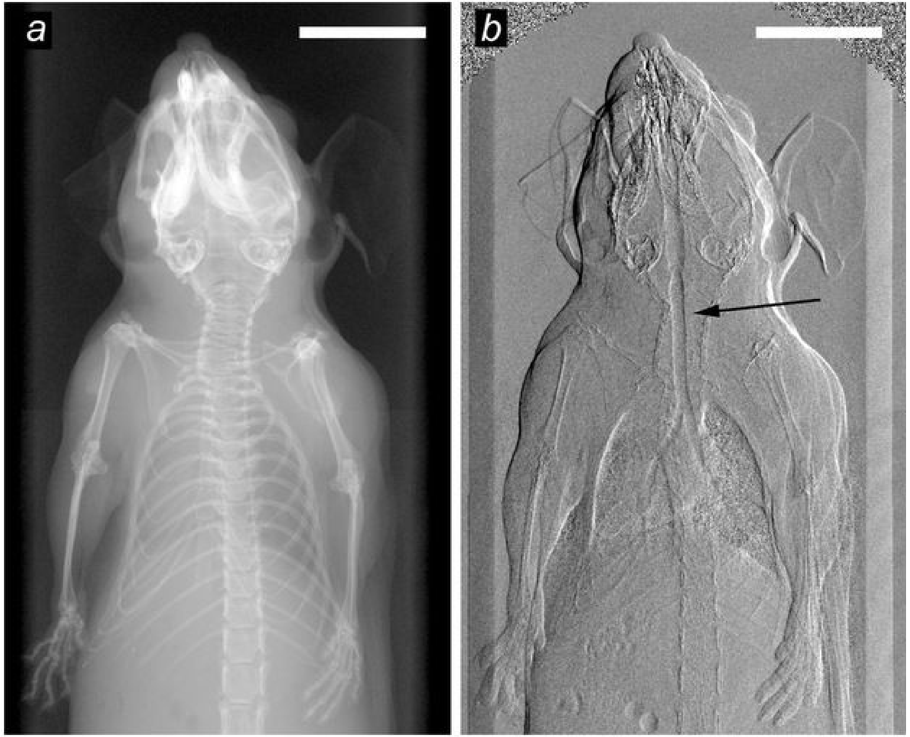
(a) (b)

Example of Analyzer Based Imaging

Conventional radiograph on left shows attenuation contrast

ABI image shows much more detail from soft material

Lungs and bubbles in gut show high contrast because of refractive index difference between air and tissue



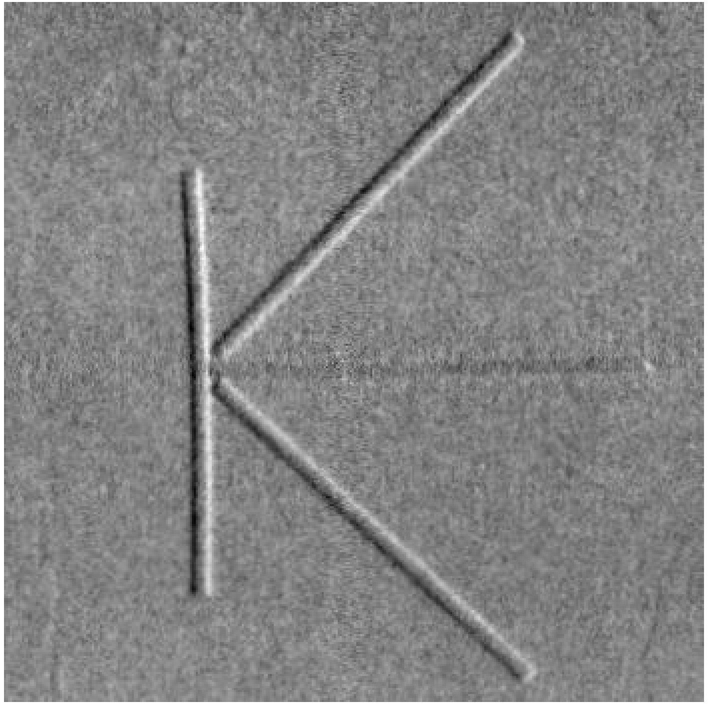
Bech et al., Scientific Reports 3 (2013)

ABI Contrast

ABI contrast is proportional to the phase derivative along the diffraction direction

$$I \propto \frac{d\phi}{dx}$$

Refraction perpendicular to the diffraction plane of the crystal gives no contrast

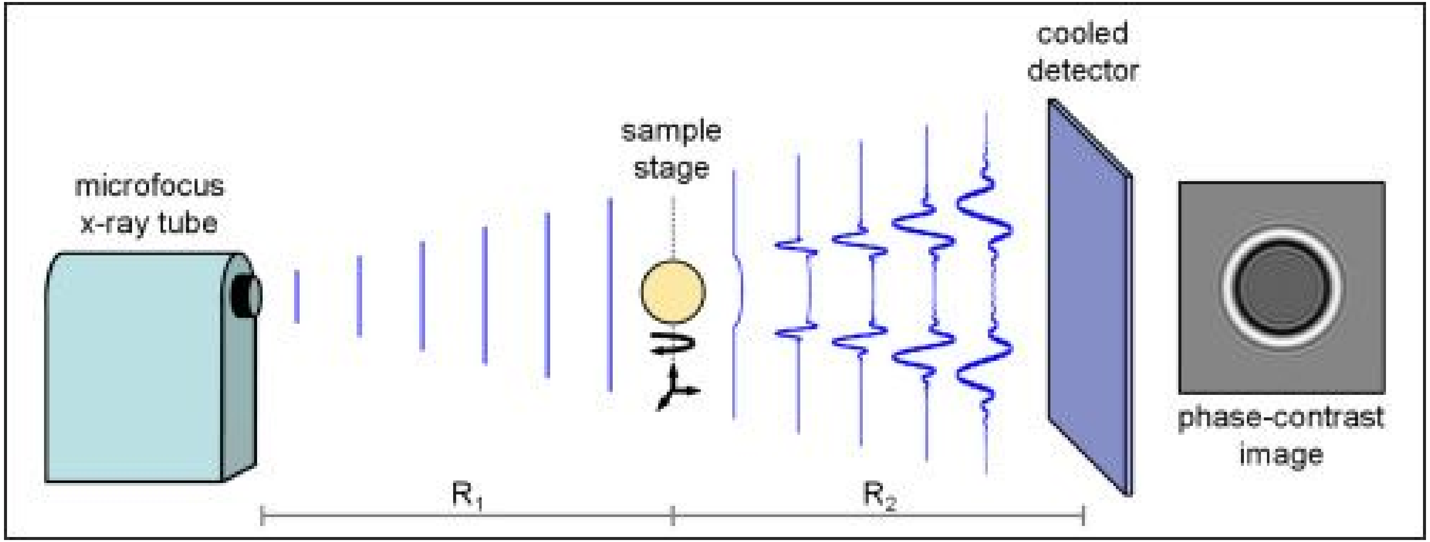


Paganin et al., Optics Communications Volume 234, Issues 1–6, 15 April 2004, Pages 87–105

Propagation-based Phase Contrast

The refracted X-rays will result in intensity variation if allowed to propagate

Contrast is proportional to second derivative of phase



Example of PBI

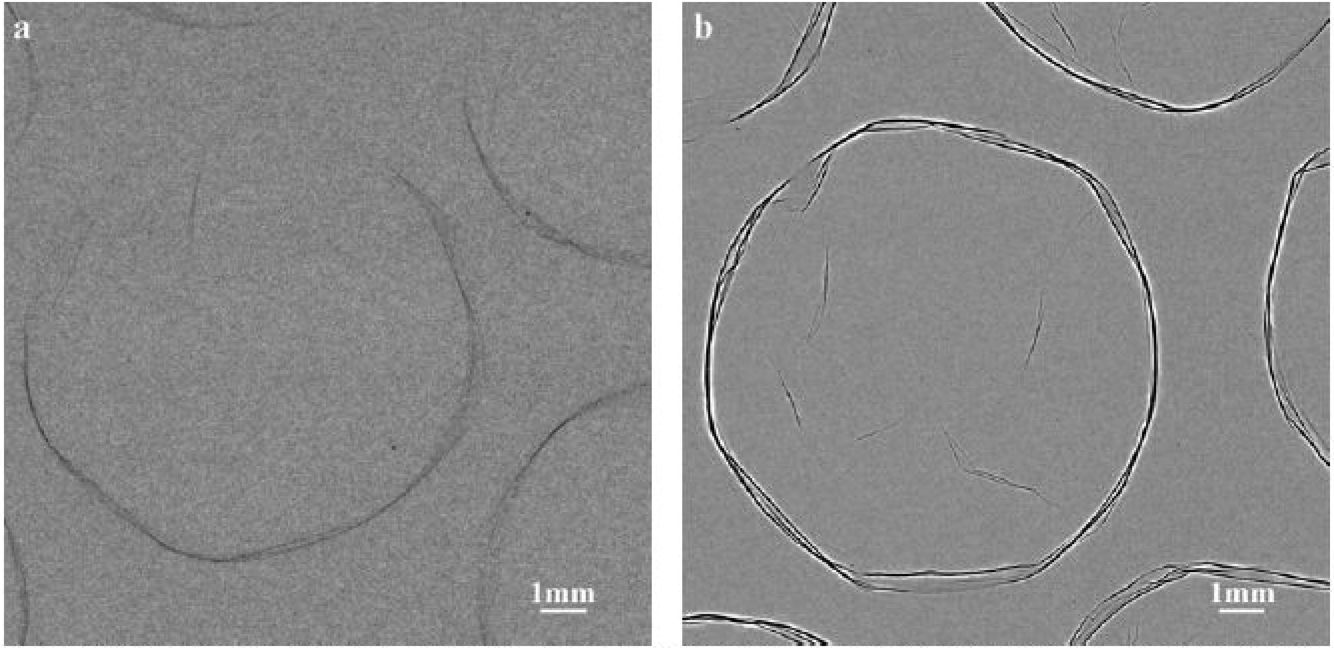
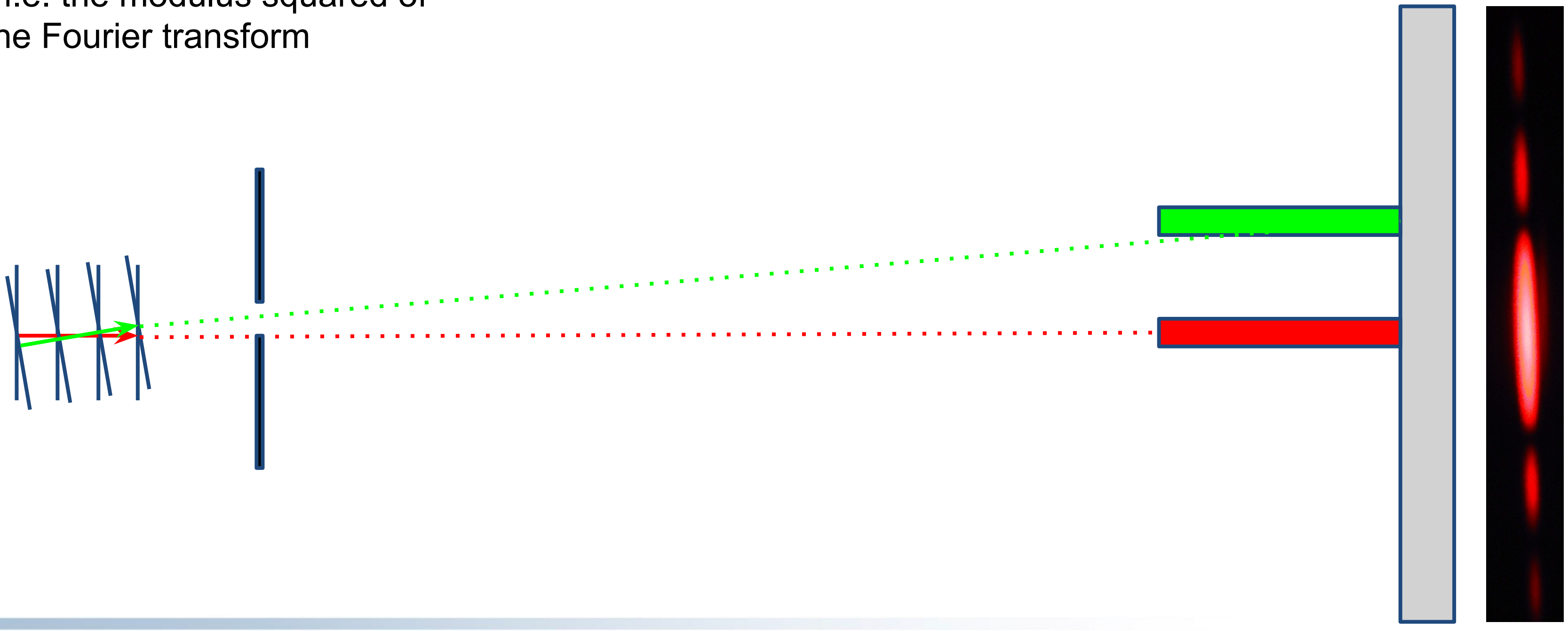


Figure 1 Images of a nylon air bubble wrap in: a) conventional (absorption) X-ray imaging and b) phase contrast imaging, edge detection regime. The edge-enhancement in b) allows the visualization of details not visible in a).

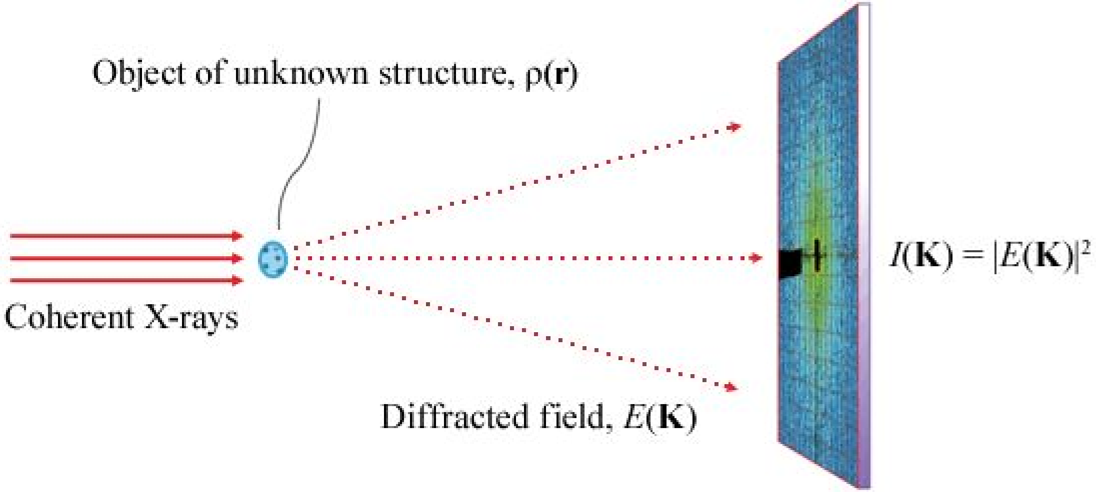
Coherent Diffractive Imaging

Fraunhofer Diffraction

By placing detector in the "far field" we record the Fraunhofer diffraction pattern - i.e. the modulus squared of the Fourier transform



Coherent Diffractive Imaging



If we measure the Fourier transform directly, can we simply invert it to get an image of the sample?

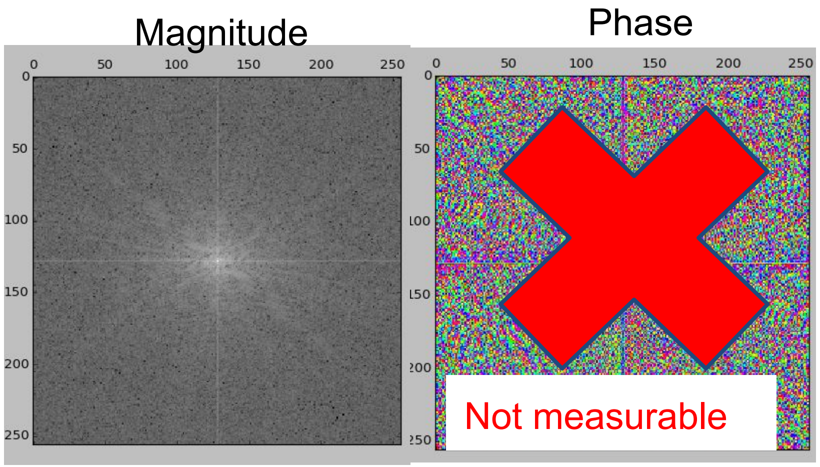
Coherent Diffractive Imaging

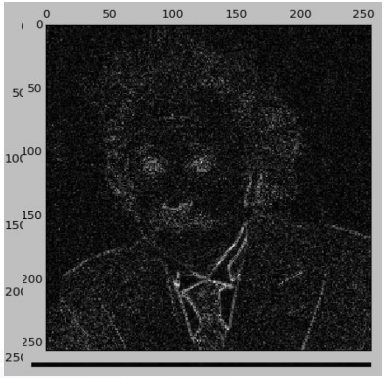
Previously we learnt that the wavefield in the far-field of a sample is related by a simple Fourier transform

If we record the Fourier transform directly can we just invert to get an image of the sample?

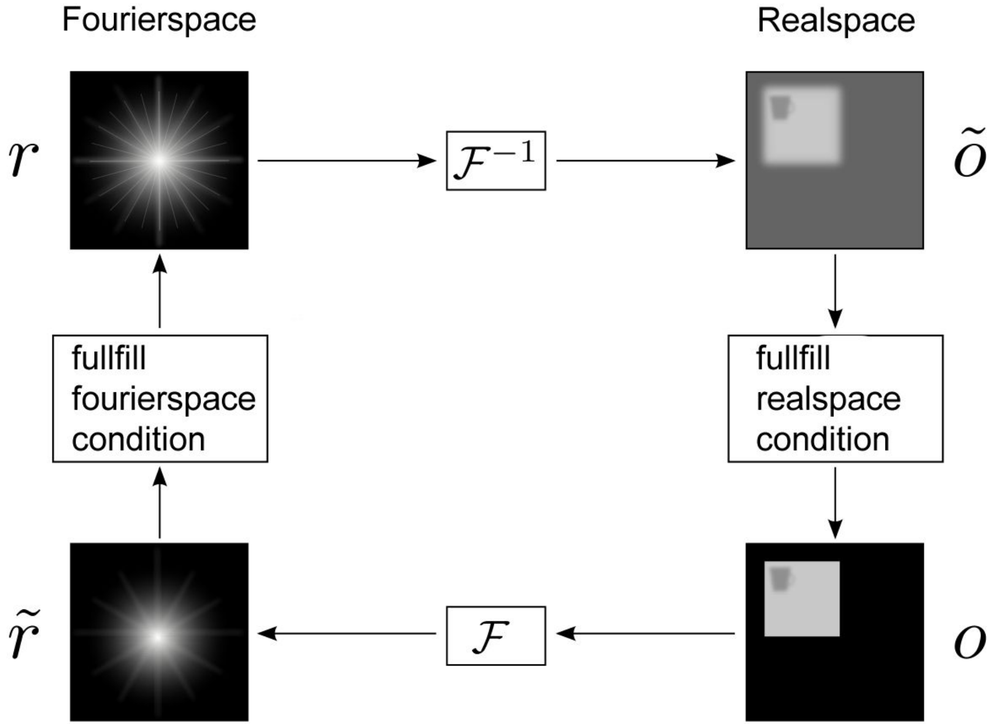
Almost…

We record the squared magnitude of the Fourier transform - phase information is lost - and so we are missing half the data required to invert the diffraction pattern





Iterative Phase Retrieval



Gerchberg & Saxton 1972

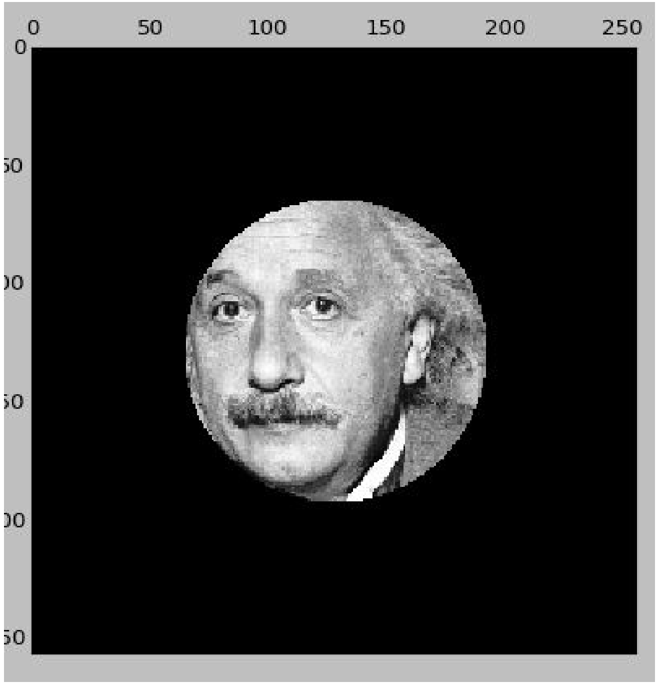
Iterative Phase Retrieval

We can recover the phase iteratively under certain conditions

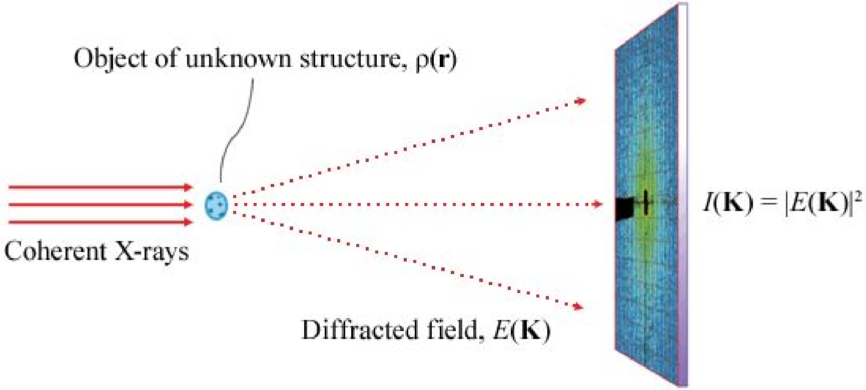
Half of the information (phase values) is lost upon measurement so we need to halve the number of 'unknowns' in the system

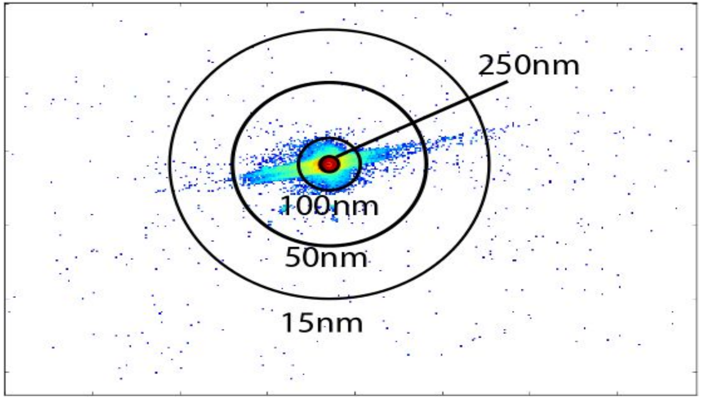
We call this the "finite support" constraint - it means the specimen must be surrounded by an area with a known value

In practice it means the sample must be isolated i.e. sitting by itself



Spatial Resolution in CDI

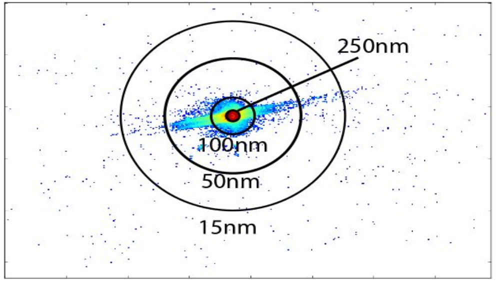




Spatial resolution limited by wavelength only

In practice, limited by wavelength, dose and dose tolerance

Coherent Flux=Spatial Resolution



- Spatial resolution limited by decay of intensity in diffraction pattern
- $I \sim q^{-4}$
- Increasing spatial resolution by 1 order requires 4 orders more flux!

- $I_C = F_C/A \propto Br.NA^2.(\Delta E/E).T$ where $A \propto (\lambda/NA)^2$

- Highest spatial resolution is achieved with:
  - Highest brilliance source
  - Highest NA optics

Schroer et al., PRL 101, 090801 (2008)
Schropp et al., APL 100, 253112 (2012)
Schropp et al., New J. Phys. 12 (2010) 035016

X-ray CDI

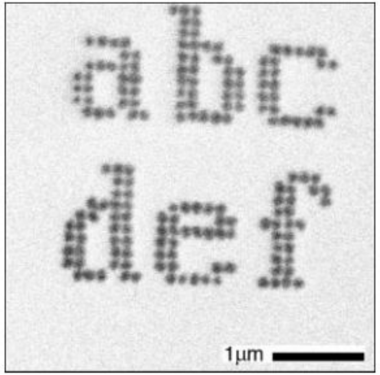


Figure 1 A scanning electron microscope image of the specimen. The specimen was fabricated by depositing gold dots, each ~100 nm in diameter and 80 nm thick, on a silicon nitride membrane.

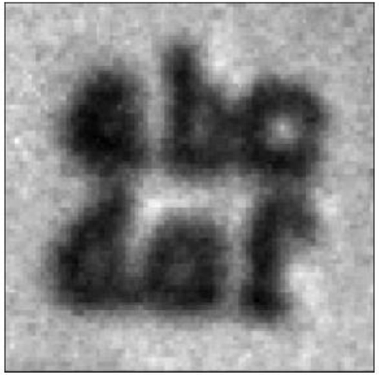


Figure 3 An optical microscope image of the specimen.

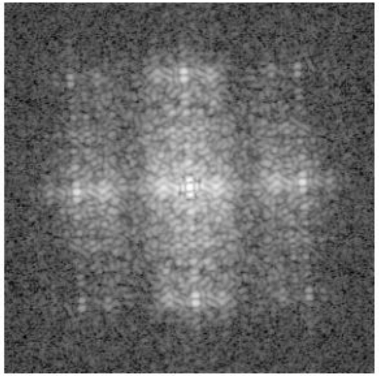


Figure 2 A diffraction pattern of the specimen (using a logarithmic intensity scale).

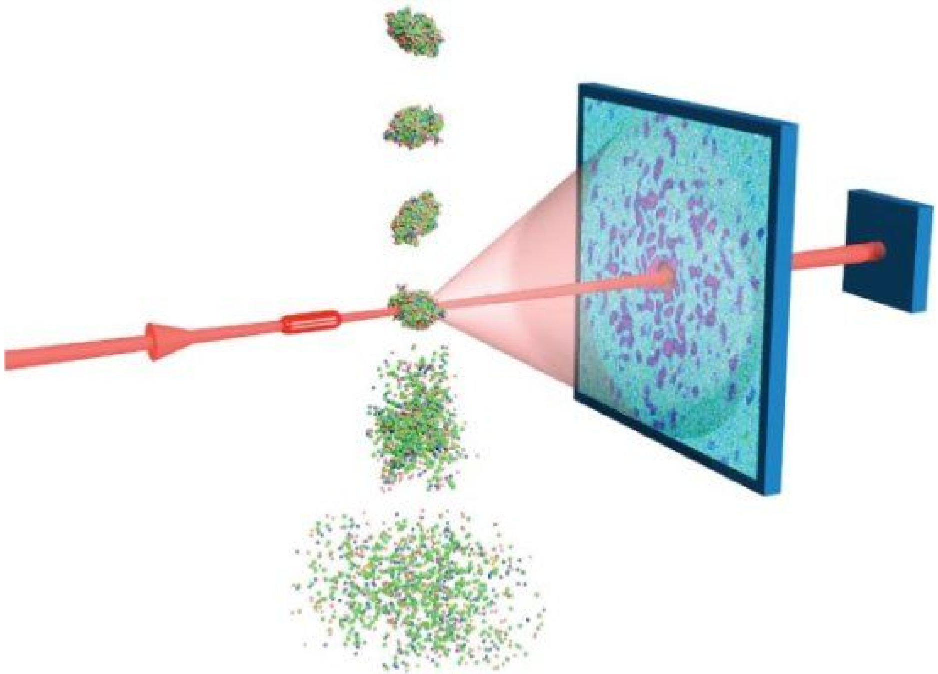
Miao et al Nature 1999

Diffract Before Destroy - FEL CDI

Avoid damage limitation to spatial resolution using femtosecond pulses of X-rays

The ultrashort pulses have passed through the specimen before damage can occur

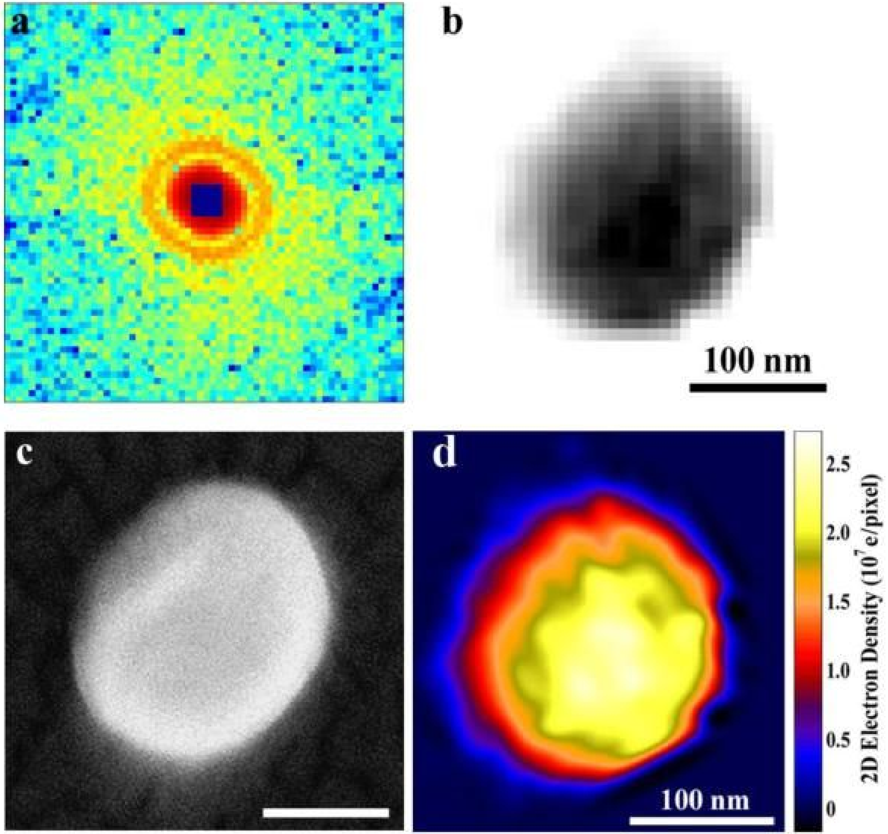
Can be used to image small crystals (serial femtosecond crystallography) or single particles



Single Particle FEL CDI

The ultimate goal is to realize the highest spatial resolution using the ultrashort pulses to outrun the damage process

Shown here are single particle imaging of herpesvirus at 22nm spatial resolution. Achieving higher resolution requires more flux



Song C, Phys Rev Lett.2008;101:158101.

Benefits of CDI

- Computational lens not limited by the ability to fabricate a high quality lens
- Spatial resolution limited by wavelength only (in practice can be limited by dose tolerance of sample)
- More dose efficient than zone-plate lens based imaging

Limitations of CDI

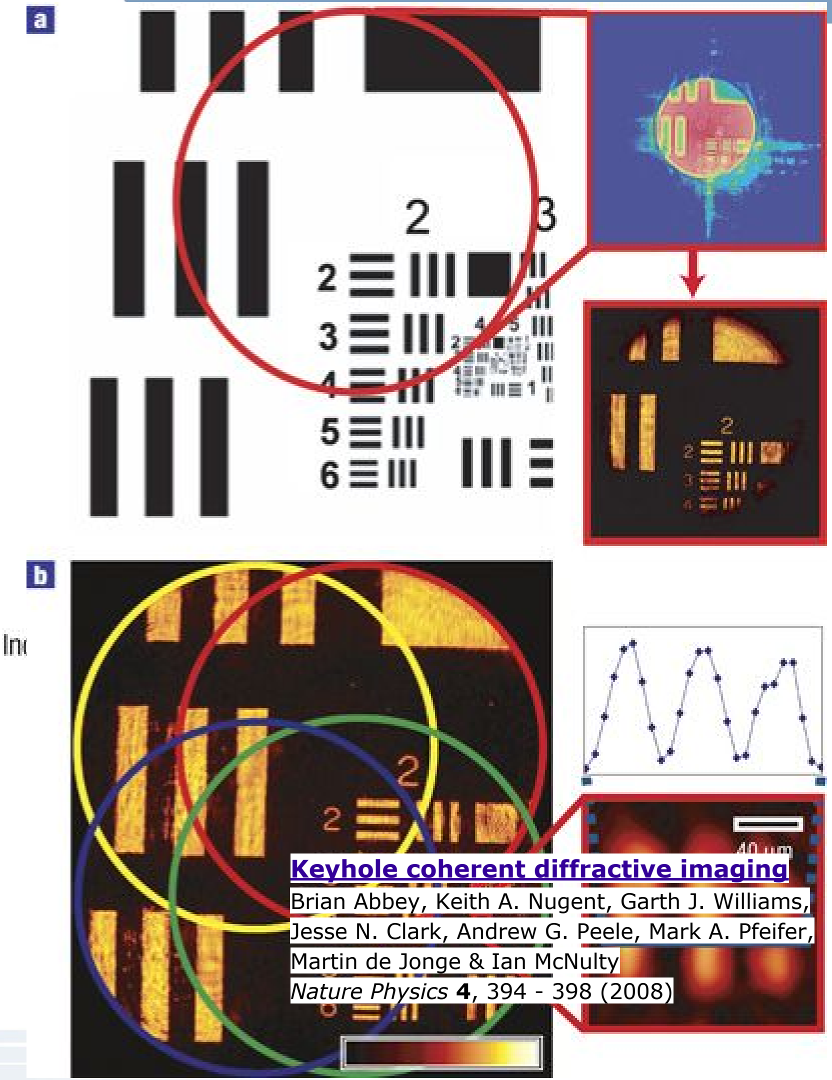
- Sample must be isolated
- Slow convergence
- Failure to converge

Keyhole CDI

We can use the beam as the object support

This removes the need to have an isolated object

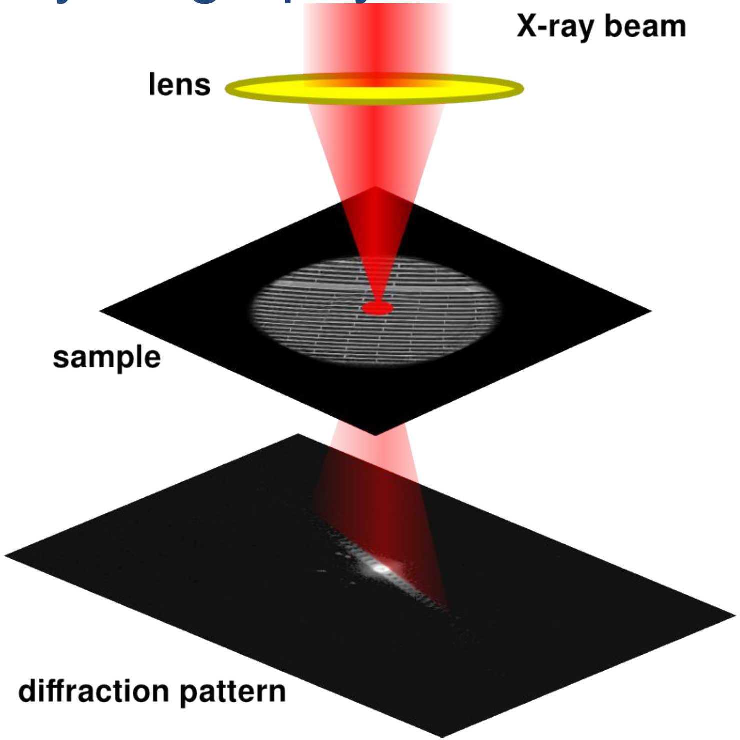
Adding phase curvature to incident beam improves convergence

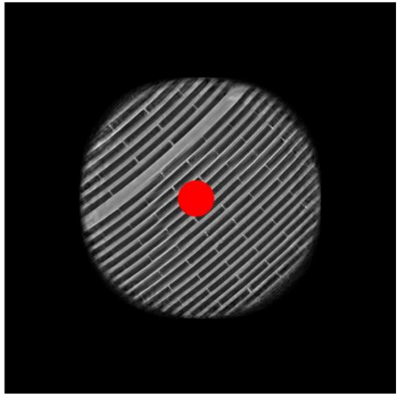


[Keyhole coherent diffractive imaging](#)
Brian Abbey, Keith A. Nugent, Garth J. Williams, Jesse N. Clark, Andrew G. Peele, Mark A. Pfeifer, Martin de Jonge & Ian McNulty
Nature Physics **4**, 394 - 398 (2008)

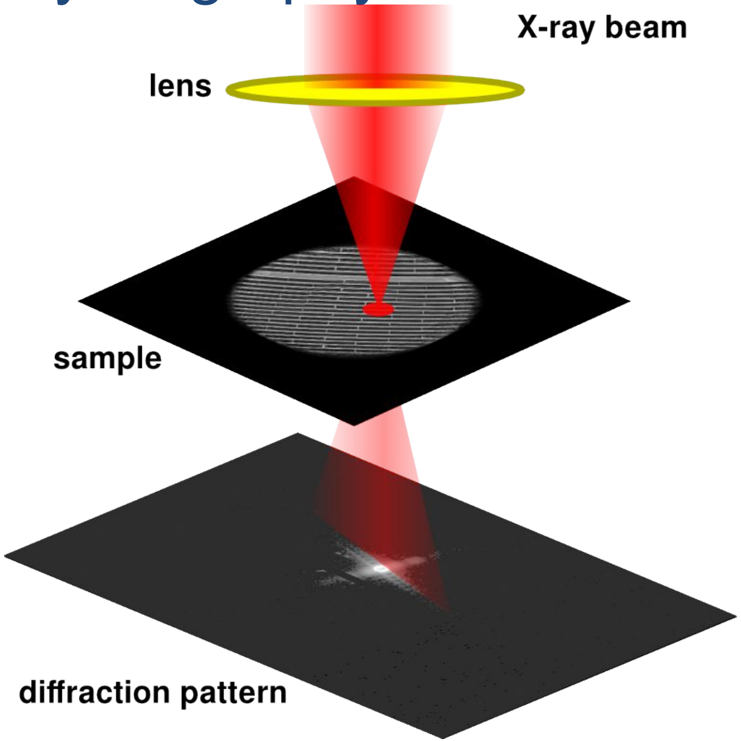
Ptychography

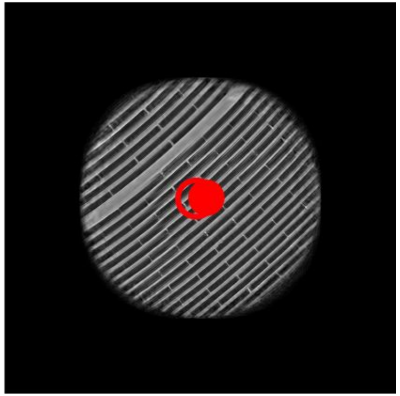
What is Ptychography?



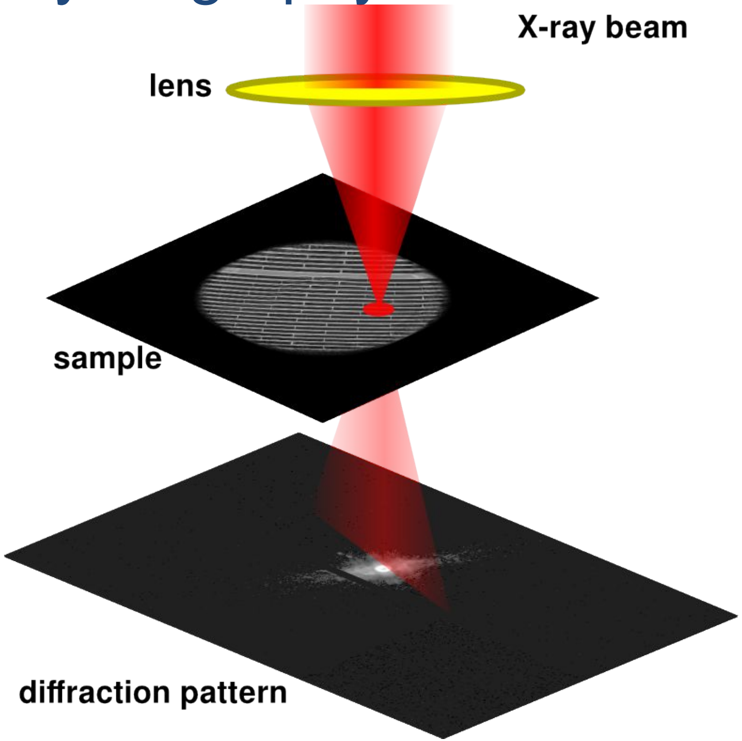


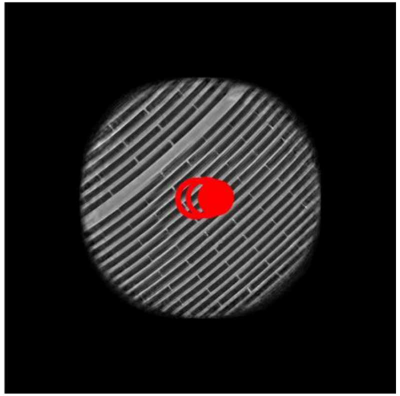
What is Ptychography?



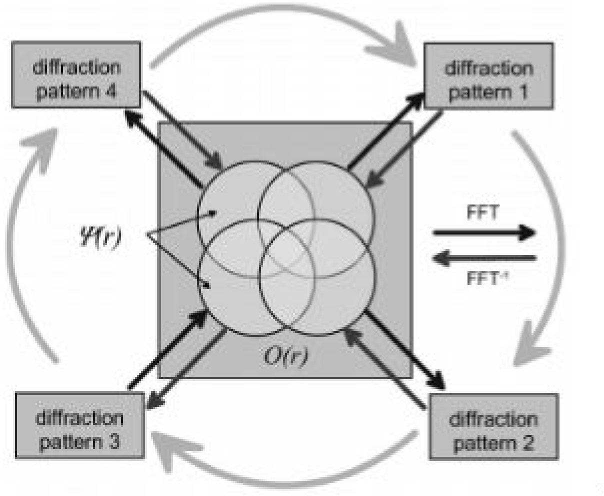


What is Ptychography?

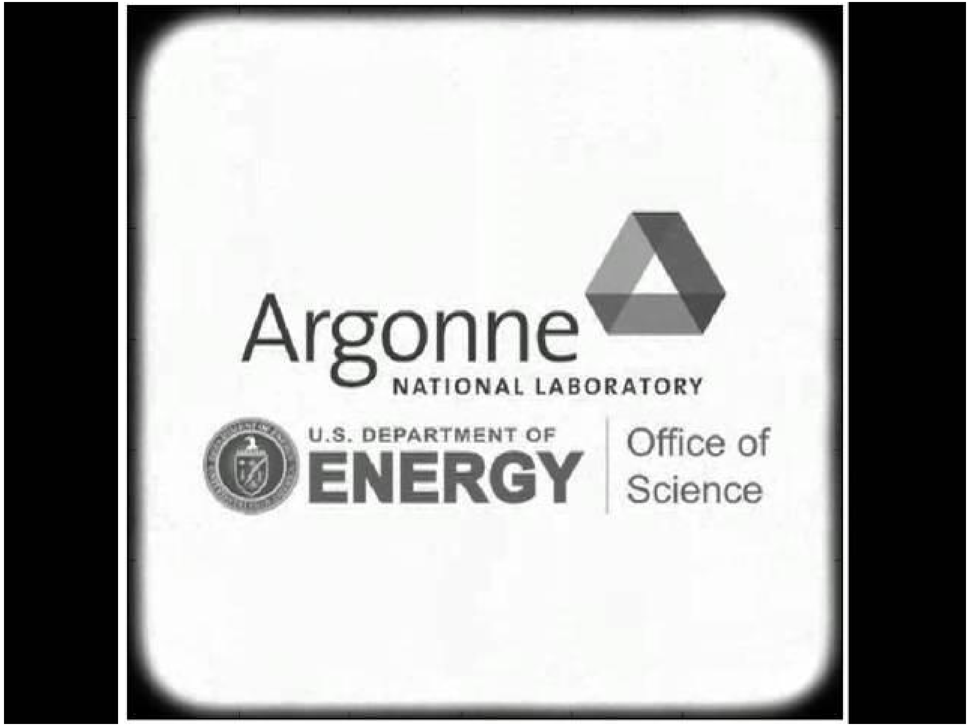




From Diffraction Pattern to Image: Phase Retrieval



- Iterate between real & reciprocal space
- Reconstruct sample, beam (composed of coherent modes)

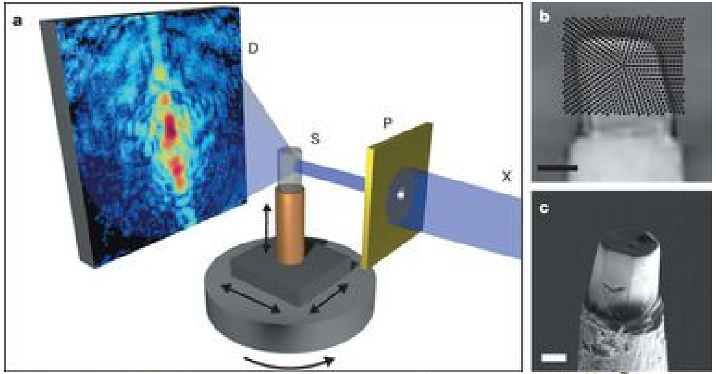


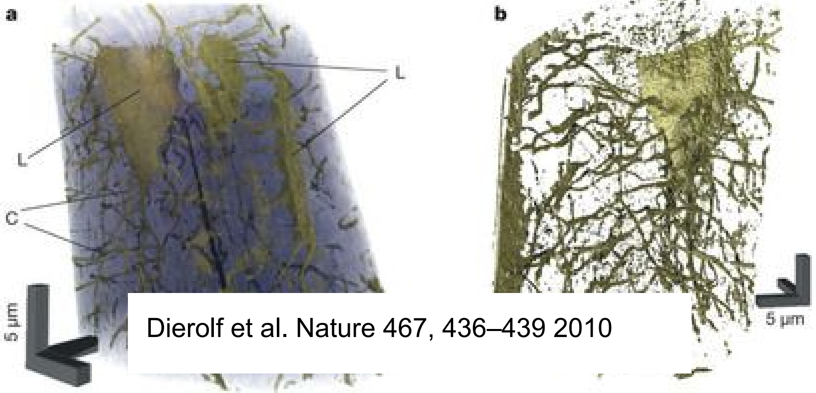
Ptychography

5nm spatial resolution soft X-ray ptychography demonstrated

16nm three-dimensional imaging using hard X-ray ptychography

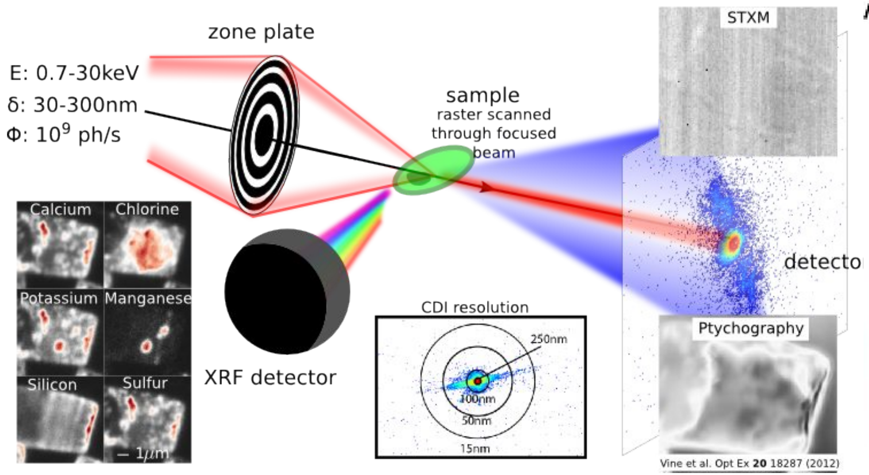
Shapiro, et al. Nature Photonics 8, 765–769 (2014)

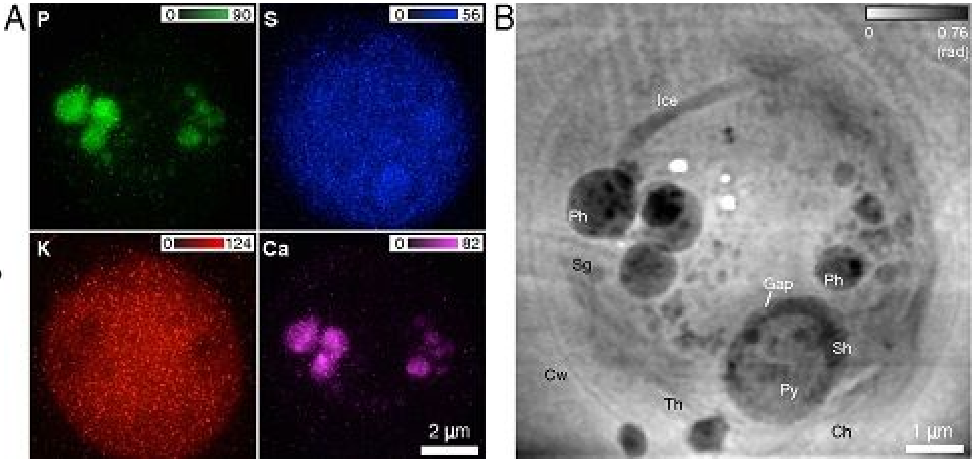




Dierolf et al. Nature 467, 436–439 2010

Ptychography & X-ray Fluorescence





Cryogenically plunge-frozen samples

Cryo-Ptychography