X-ray Absorption Spectroscopy

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- Overview of XAFS Spectroscopy and its uses
- Theory (a little)
- Experiment (a little)
- Data Analysis (a little)
- Conclusion
Acknowledgements

~41 years and counting

Ed Stern, Dale Sayers, Farrel Lytle, John Rehr, Bruce Bunker, Steve Heald, Tim Elam, Gerd Rosenbaum, many others...

XAFS community that grew from it

Students and postdocs
Introduction to XAFS

X-ray absorption fine structure spectroscopy (XAFS) is a powerful and versatile technique for studying structures of materials in chemistry, physics, biology, and other fields. This textbook is a comprehensive, practical guide to carrying out and interpreting XAFS experiments.

Assuming only undergraduate-level physics and mathematics, the textbook is ideally suited for graduate students in physics and chemistry starting XAFS-based research. It contains concise executable example programs in Mathematica 7.

The textbook addresses experiment, theory, and data analysis, but is not tied to specific data analysis programs or philosophies. This makes it accessible to a broad audience in the sciences, and a useful guide for researchers entering the subject.

Supplementary material available at www.cambridge.org/9780521767750

- Mathematica code from the book
- Related Mathematica programs
- Worked data analysis examples

GRANT BUNKER is Professor of Physics at the Illinois Institute of Technology. He has over 30 years' experience in all aspects of XAFS spectroscopy, from technique development, instrumentation, and computation, to applications in biology, chemistry, and physics.
Many of the basic ideas re EXAFS were present since 1931, but specific key ideas to understanding were missing for decades. It all came together in Stern, Sayers, Lytle’s modern synthesis ~ 1970.

Subsequent progress in theory, experiment, and data analysis has been extensive.

XAFS has grown exponentially together with synchrotron radiation.
first xafs scan at SSRL
Sadly, Ed recently passed away at age 85 on May 17, 2016. Ed was an inspired physicist who always strived to get to the essence of things. He would invent or create whatever was necessary to do research. He also was a kind, thoughtful, person with a good sense of humor. He will be missed.

Dale Sayers, a gifted and energetic physicist and teacher, tragically passed away unexpectedly at age 60 on Nov 25, 2004.
raw data from Google Scholar July 2017 (excluding citations)

XAFS OR EXAFS OR XANES OR NEXAFS OR "X-RAY ABSORPTION SPECTR" OR "KRONIG STRUCTURE"
What is XAFS?

X-ray Absorption Fine Structure spectroscopy uses the x-ray photoelectric effect and the wave nature of the electron to determine local structures around selected atomic species in materials.

Unlike x-ray diffraction, it does not require long range translational order in the sample – it works equally well in amorphous materials, liquids, (poly)crystalline solids, and molecular gases.

XANES (near-edge structure) can be sensitive to charge transfer, orbital occupancy, and symmetry.
The X-ray absorption coefficient is the central quantity of interest. It is analogous to absorbance in UV-vis spectroscopy, and it is proportional to $f''(E)$. 

$$\frac{I}{I_0} = \exp(-\mu(E)x)$$

**Figure 1 – Schematic XAFS experiment**

The X-ray absorption coefficient is the central quantity of interest. It is analogous to absorbance in UV-vis spectroscopy, and it is proportional to $f''(E)$. 
Advanced Photon Source

Our Friendly Neighborhood Synchrotron Radiation Source at Argonne National Laboratory
absorption cross section, linear scale

Cross Section of Pt

- solid: photoelectric
- dotted: scattering
- dashed: photoelectric + scattering

Cross Section (cm$^2$/g) vs. Energy (KeV)
absorption cross section, log-log plot

Cross Section of Pt

Straight lines between edges implies power law:

\[ \sigma \propto \frac{1}{E^3} \] approximately

solid: photoelectric absorption

dashed: photoelectric + total scattering

dotted: elastic+inelastic scattering

expt: Molecular gases
GeH4, GeH3Cl, GeCl4
tetrahedral coordination

ZnS transmission

$\mu_X$ vs $E-E_0$ (eV)

$XANES + EXAFS = XAFS$
Zn cys/his complexes: XAFS encodes structure

Koch models: spectra courtesy of J. Penner-Hahn
MnO
rock salt
structure
T=80K
XAFS is element selective

By choosing the energy of excitation you can “tune into” different elements in a complex sample.

\[ E_{K\text{edge}} \approx Z^{2.16} \]

Example:
Ca vs Mo

\[ \frac{20}{4} \approx \left(\frac{42}{20}\right)^{2.16} \]

It is usually feasible to work in a convenient energy range by choosing an appropriate edge.
X-ray Absorption Process

*K-edge*

X-ray photon causes transition from \( n=1, l=0 \) (1S) initial state to unfilled \( p \)-symmetry (\( l=1 \)) final state.

Absorption probability depends on dipole matrix element between initial and final quantum states of the electron, which are determined by local structure.
X-ray photon causes transition from inner level to unfilled final state of appropriate symmetry.

If photon energy exceeds binding energy $E_0$, electron has positive kinetic energy and propagates as spherical wave.

$$k = \frac{2\pi}{\lambda_e} = \sqrt{\frac{2m(E - E_0)}{\hbar^2}}$$
Electron wave emitted by central atom is scattered by neighboring atoms. The outgoing and scattered parts of the final state wavefunction interfere where the initial state is localized.

Interference is constructive or destructive depending on the distances and electron wavelength. Scanning the wavelength records an interferogram of distance distribution.
Outgoing p-symmetry electron wave

Isolated atom has no final state wavefunction interferences.

Absorption coefficient varies smoothly with electron wavelength.

This directionality can be useful for polarized XAFS.
Outgoing electron wave, with scatterers (animation)

Scattering from neighboring atoms modifies wavefunction near center of absorber, modulating the energy dependence of the transition matrix element.
Time Dependent Perturbation Theory
Fermi’s “Golden Rule” (from Dirac)
Transition matrix element

\[ \mu \propto \left| \int \psi_f^* \hat{e} \cdot \vec{r} \ e^{i\vec{k} \cdot \vec{r}} \ \psi_i \ d^3 r \right|^2 \]

dipole and quadrupole terms

\[ \approx \left| \int \psi_f^* \left( \hat{e} \cdot \vec{r} + i(\hat{e} \cdot \vec{r})(\vec{k} \cdot \vec{r}) \right) \ \psi_i \ d^3 r \right|^2 \]

Matrix element projects out the part of the final state that is of right symmetry (e.g. p-symmetry for K-edge & dipole selection rules)
<table>
<thead>
<tr>
<th>Rigorous rules</th>
<th>( \Delta J = 0, \pm 1 ) (except 0 ( \leftrightarrow ) 0)</th>
<th>( \Delta J = 0, \pm 1 ) (except 0 ( \leftrightarrow ) 0)</th>
<th>( \Delta J = 0, \pm 1, \pm 2 ) (except 0 ( \leftrightarrow ) 0, 1/2 ( \leftrightarrow ) 1/2, 0 ( \leftrightarrow ) 1)</th>
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<td>2. ( \Delta M = 0, \pm 1 ) (except 0 ( \leftrightarrow ) 0 when ( \Delta J = 0 ))</td>
<td>( \Delta M = 0, \pm 1 ) (except 0 ( \leftrightarrow ) 0 when ( \Delta J = 0 ))</td>
<td>( \Delta M = 0, \pm 1, \pm 2 )</td>
</tr>
<tr>
<td>Parity change</td>
<td>No parity change</td>
<td>No parity change</td>
<td>No parity change</td>
</tr>
<tr>
<td>With negligible configuration interaction</td>
<td>One electron jumping, with ( \Delta l = \pm 1 ), ( \Delta n ) arbitrary</td>
<td>No change in electron configuration; i.e., for all electrons, ( \Delta l = 0 ), ( \Delta n = 0 )</td>
<td>No change in electron configuration; or one electron jumping with ( \Delta l = 0, \pm 2 ), ( \Delta n ) arbitrary</td>
</tr>
<tr>
<td>For LS coupling only</td>
<td>5. ( \Delta S = 0 )</td>
<td>( \Delta S = 0 )</td>
<td>( \Delta S = 0 )</td>
</tr>
<tr>
<td></td>
<td>6. ( \Delta L = 0, \pm 1 ) (except 0 ( \leftrightarrow ) 0)</td>
<td>( \Delta L = 0 )</td>
<td>( \Delta L = 0, \pm 1, \pm 2 ) (except 0 ( \leftrightarrow ) 0, 0 ( \leftrightarrow ) 1)</td>
</tr>
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</table>
**Final state symmetry**

- **K-edge**: 1s initial state \((n=1, l=0, m=0)\)
- **L\(_1\)-edge**: 2s initial state \((n=2, l=0, m=0)\)
- **L\(_2\)-edge**: 2p \((j=1/2)\) initial state \((n=2, l=1)\)
- **L\(_3\)-edge**: 2p \((j=3/2)\) initial state \((n=2, l=1)\)

Dipole selection rules project out specific symmetry components of final state wavefunction.

- **K, L\(_1\)** edges probe p part of final states
- **L\(_2,3\)** edges probe d (and s) part of final states
The measured spectrum is an ensemble average of the “snapshot” spectra (~10^-15 sec) of all the atoms of the selected type that are probed by the x-ray beam.

In general, XAFS determines the statistical properties of the distribution of atoms relative to the central absorbers. In the case of single scattering, the pair correlation function is probed. Multiple scattering gives information on higher order correlations. This information is encoded in the chi function:

\[
\mu(E) = \mu_0(E)(1 + \chi(E)); \quad \chi(E) = \frac{\mu(E) - \mu_0(E)}{\mu_0(E)}
\]
EXAFS oscillations

\[
\mu(E) = \mu_0(E)(1 + \chi(E)); \quad \chi(E) = \frac{\mu(E) - \mu_0(E)}{\mu_0(E)}
\]

Modulations in chi encode information about the local structure.

Chi function represents the fractional change in the absorption coefficient that is due to the presence of neighboring atoms.
XAFS spectroscopy provides:

- Precise local structural information (distances, numbers of atoms, types, disorder) in crystalline or noncrystalline systems e.g. metalloprotein active sites, liquids, amorphous materials

- All atoms of selected type are visible - there are no spectroscopically silent atoms for XAFS

- Information on charge state, orbital occupancy may be available by studying XANES depending on system and edge

  *in situ* experiments, under conditions similar to natural state, as well as crystals.

- XAFS probes effects of arbitrary experimental conditions on sample (high pressure, low temperature, pH, redox state, pump-probe, T-jump, p-jump...)

- Oriented samples provide more angular information
Complementary Structure Probes

- **X-ray and Neutron diffraction**
  - powerful and fast (x-ray), need good crystals, no solutions

- **2-D and higher dimensional NMR**
  - Atomic resolution structures in solution, no large molecules, slow

- **X-ray scattering**
  - SAXS gives only low resolution information
  - wide angle can be informative
  - PDF (pair distribution function)

- **XAFS**
  - Gives short range structure around metal atom. Not sensitive or fast.
  - XANES probes orbital matrix elements and occupancy.
Related techniques

- **XMCD**: X-ray Magnetic Circular Dichroism uses circularly polarized x-rays to probe magnetic structure.

- **IXS**: Inelastic X-ray Scattering analyzes the fluorescence radiation at high resolution, providing a 2-D excitation map. Provides a great deal of information in the near-edge region.

- **X-ray Raman**: essentially allows one to obtain XAFS-like information using high energy x-rays.

- **DAFS**: hybrid diffraction/XAFS gives sensitivity to inequivalent sites in crystals and multilayers.

- **XPS, ARPEFS, fluorescence holography...**
Single Scattering EXAFS Equation

Stern, Sayers, Lytle

\[ \chi(k) = S_0^2 \sum_j \frac{N_j}{kR_j^2} e^{-2k^2\sigma_j^2} e^{-2R_j/\lambda(k)} f_j(k; R_j) \sin(2kR_j + \delta_j(k; R_j)) \]

Experimental data are fit using the EXAFS equation with theoretically calculated (or empirically measured) scattering functions to determine structural parameters.

The \( k \)-dependence of scattering amplitudes and phases helps distinguish types of backscatterers.

This equation is a bit too simple \{large disorder, multiple scattering [focussing effect]\}, but it can be generalized.
k-dependence of scattering amplitudes and phases helps identify scatterers.
common applications

- real-world systems, amorphous, disordered
- Condensed matter physics, materials science
- Catalysts, batteries
- Biology, metalloproteins
- Environmental science and geoscience
Simple example: Fx Fe-S protein
Structure of Fe-S cluster in Fx from Photosystem I

**XAFS fits for 4 Fe- 4 S cluster**

- Fe-S  N= 4.00  R=2.27(2)  SS= 0.007(1)
- Fe-Fe  N= 3.00  R=2.68(2)  SS= 0.007(1)

The figure shows a molecular model based on XAFS that is consistent with the determined distances. These require a distortion of the cubane-like box. Bunker and Carmeli, 2002

Protein solution only - no crystals!
The most basic form of the EXAFS equation is:

\[
\chi(k) = \left\langle S_0^2 \sum_i \frac{3 \cos^2(\theta_i)}{kr_i^2} |f_i(k; r)| e^{-2r_i/\lambda(k)} \sin(2kr_i + \delta_i(k; r)) \right\rangle
\]

where \( r_i \) is the distance to the \( i \)th neighbor; \( < \ldots > \) represents an average over all sites in the sample; \( \lambda \) is the electron mean free path, and \( S_0^2 \) is a loss factor; \( f_i \) and \( \delta_i \) are the scattering amplitude and phase shift of atom \( i \); \( \theta_i \) is the angle between the electric polarization vector of the x-ray beam \( \hat{e} \) and the vector \( \hat{r}_i \) from the center atom to neighboring atom \( i \). The \( r \)-dependence of \( f \) and \( \delta \) is weak.
EXAFS equation (isotropic average)

Averaging over angle and grouping atoms of the same atomic number and similar distances into “shells” we obtain:

$$\chi(k) = S_0^2 \sum_j \frac{N_j}{kR_j^2} e^{-2k^2\sigma_j^2} e^{-2R_j/\lambda(k)} f_j(k; R_j) \sin(2kR_j + \delta_j(k; R_j))$$

where $N_j, R_j, \sigma_j^2$ are the coordination number, average distance, and mean square variation in distance to atoms in shell $j$. These are the leading terms in the “cumulant expansion”. If $k\sigma$ is not $<< 1$, higher order terms should be considered.

EXAFS is basically a sum of damped sine waves

$\rightarrow$ Fourier Transform, beat analysis

EXAFS DWFs are comparable to, but distinct from, diffraction DWFS. There are both static and thermal contributions to $\sigma^2$.
Multiple Scattering Expansion

Multiple scattering is accounted for by summing over MS paths $\Gamma$, each of which can be written in the form [ref: Rehr, Rev. Mod. Phys., 2000]

$$\chi_\Gamma(p) = S_0^2 \text{Im} \left( \frac{e^{i(p_1+p_2+\cdots+p_N+2\delta)}}{\rho_1\rho_2\cdots\rho_N} e^{-2p^2\sigma_\Gamma^2} \times \text{Tr} M_{\Gamma} F^{N} \cdots F^2 F^1 \right)$$

where $p$ is the complex photoelectron momentum, $\rho_j$ are $p$ times the path lengths of the $i$th leg of the MS path $\Gamma$; the $F$ matrices describe the scattering from each atom in the path; $M$ is a termination matrix.

This can be expressed similarly to SS form

$$\chi_\Gamma(p) = S_0^2 \text{Im} \left( \frac{f_{\text{eff}}}{k R^2} e^{2ikR+2i\delta} e^{-2p^2 \sigma_\Gamma^2} \right)$$

whence “Feff”
focussing effect

- multiple scattering is always important if the central atom and two or more other atoms are in a line, or nearly so
- the intervening atom literally focusses the electron wave on the “shadowed” atom and enhances its effect ~2x
Leading MS paths tetrahedral MnO₄

2 legs
reff = 1.9399

3 legs
reff = 3.52382

4 legs
reff = 3.87979

4 legs
reff = 3.87979

4 legs
reff = 5.10774

5 legs
reff = 5.46371
MnO$_4$ tetrahedral cluster

$r = 1.63, 1.73, 1.84, 1.94$ Å, $\text{f}_{\text{eff}} 8.2$ SCF/FMS

The shorter the distance, the higher the edge energy and the more intense the pre-edge peak.
MnO$_4$ tetrahedral cluster

$r=1.63, 1.73, 1.84, 1.94\text{Å}$  
$\text{feff} 8.2$  
$\text{SCF/FMS}$

Energy rescaled as $1/r^2$

Extended continuum $-17\text{eV}$
Edge shifts: chemical *correlation* of bond length with formal charge explains the large K-edge shifts associated with oxidation state because edge shift primarily depends on average bond distance.

KMnO₄, MnO₂, Mn₂O₃, Mn₃O₄, MnO

formal charge $\sim 1/R^4$
Solid KMnO₄ at 80K and 300K experimental data*

the temperature sensitive fine structure over edge is single scattering from atoms beyond first shell with very large DWFs

* G Bunker thesis 1984
XANES landscape is from SS+MS among nearest neighbor tetrahedron. SS from distant atoms adds temp dependent fine structure.

FIG. 2. (a) The two multiple-scattering paths that contribute to the second peak in the Fourier transform of KMnO$_4$, denoted by solid and dashed lines. The black dot is Mn, and the open dots are O atoms. (b) The logarithm of the ratio of the $k$ dependence of the amplitudes of the first (SS) peak to the second (MS) peak of Fig. 1(b), plotted vs energy from the edge.

Bunker and Stern
PRL 52, 22 (1984)
now for something completely different
XAFS experimental requirements

- suitable sample (depends on measurement mode)
- intense broad-band or scannable source
- monochromatic (~ 1 eV bandwidth), scannable beam, energy suitable for elements of interest
- suitable detectors (depends on mode)
- special equipment (cryostats, goniometers..)
Basic Beamline Components for XAFS

source (mirror) monochromator (mirror) slits detectors

Collimating mirror is sometimes used to match source to acceptance of mono mirror following mono is often used for harmonic rejection or focussing

graphic courtesy of SER-CAT
**Experimental modes**

**Detection**
- Transmission mode
- Fluorescence mode
- Electron yield
- Inelastic X-ray Scattering

**Geometry**
- Oriented e.g. single crystal
- Magic Angle Spinning
- Total External Reflection
- Grazing (glancing) incidence

*Figure 1 – Schematic XAFS experiment*
Which mode to use?

- concentrated, not too thick: -> use transmission
  want edge step ~1.0 (>0.1, <2.0)

- concentrated, thick: -> use electron yield, total external
  reflection fluorescence, or apply fluorescence corrections
  numerically, IFPY?

- dilute samples: (< .1 absorption length edge step) use
  fluorescence detection

- microbeams can used to measure small grains which may
  be concentrated even if sample is dilute on average (still
  must worry about particle size effects though)
Checklist: “HALO” Mnemonic

- **Harmonics** - get rid of them using mirrors, detuning, or other means, especially for thick transmission samples.

- **Alignment** - the beam should only see homogeneous sample and windows between the $I_0$ and $I$ (or $I_f$) detectors.

- **Linearity** - ionization chambers must be plateaued. Other detectors may need deadtime corrections.

- **Offsets** - dark currents must be measured and subtracted to compensate for drifts.
Standard EXAFS Detectors

- Integrating (non-energy resolving)
  - Ionization chambers
  - Fluorescence ionization chambers (Stern/Heald)
  - PIN diodes/PIPS detectors

- Pulse counting (energy resolving)
  - Solid State (Ge/Si) detectors
  - Silicon Drift Detectors (SDD)
  - Scintillator/Photomultiplier (PMT)
  - Proportional Counters (PC)
  - Avalanche Photodiodes (APD)
Fluorescence ion chamber

Often used with filter and soller slits to keep scattered background out of detector

Fluorescence line(s)

filter absorption edge

Sample

Filter Slits  Ion chamber

sample absorption edge
“Lytle Detector”

www.exafsco.com

Filter located against inside surface of slit assembly

Sample position

X-ray Beam

Slit assembly

Ion chamber

Gas flow in

He purge Aluminized Mylar

Ni mesh

Amplifier

Gain control
Performance for dilute systems depends critically on filter and slit quality, and correct choice of filter thickness. This approach cannot eliminate fluorescence at lower energies.

excellent filter and ideal slits

for more info see: http://gbxafs.iit.edu/training/tutorials.html
Limitations to common slit systems seriously degrade performance at high dilution.

Even with optimized filters, efficiency drops to a few percent for large (>100) background to signal ratios.

For more info see: [http://gbxafs.iit.edu/training/tutorials.html](http://gbxafs.iit.edu/training/tutorials.html)
Multielement Germanium Detector

detector->
preamp->
shaping amp->
multichannel analyzer
or SCA & scaler

Maximum count rates of several hundred KHz total (signal+background)/channel.

Can use together with Z-1 filters and Soller slits
SDD Arrays

higher count rates are under active development

4-element high count rate silicon drift detector

77 element prototype silicon drift detector

C. Fiorini et al

Total active area 6.7 cm$^2$
X-ray Analyzers

- Conventional solid state detectors can be easily saturated at high flux beamlines.

- They spend most of their time counting background photons you throw out anyway.

- Multilayer, bent crystal Laue, and other analyzers eliminate background before it gets to detector.

- Graphite log-spiral analyzer (Pease), Bragg log spiral analyzer (Attenkofer et al) are also good approaches.

- Effectively no count rate limits, and good collection efficiency, or better resolution.

- No count rate limit due to pulsed nature of source.
Multilayer Array Analyzer Detector

This device uses arrays of synthetic multilayer structures to diffract the signal and eliminate scattered background. It makes possible some experiments that are otherwise intractable.

Advanced versions of these analyzers are under development.

www.hdtechinc.com
Bent Crystal Laue Analyzers

Extremely bent silicon crystals have very high efficiency and wide angular acceptance.

Logarithmic spiral bent crystal

Diagram showing the setup of a bent crystal analyzer system with labels for Ag sample, transmitted beam blocked by slits, Spirally Bent Si (111) Crystal, Sn Soller slits, and diffracted beam.
Bent Laue Analyzer

(set in bend & angle to diffract desired emission line)

Soller Slits
(matches beam divergence)

Area Integrating Detector
(i.e. ionization detector)

Sample’s x-ray fluorescence
Bent Crystal Laue Analyzer

www.quercustech.com

www.fmb-oxford.com
Data Analysis

Modern codes for calculating theoretical XAFS spectra are accurate enough to use to fit experimental data directly. “FEFF9” (J.J. Rehr et al) is a leading program for calculating spectra. Others include GNXAS and EXCURV.

FEFF does not analyze the data for you, however. Add-on programs of various kinds (e.g. Artemis/Athena/Horae/Demeter, Larch, Sixpack, EXAFSPAK…) use (or can use) FEFF-calculated spectra to fit the data by perturbing from an initial guess structure. Parameterizing the fitting process can be simple or quite involved.

Another approach (Dimakis & Bunker) basically uses FEFF as a subroutine and combines it with other info (e.g. DFT calculations) to estimate DWFs.
Data Reduction

- Apply instrumental Corrections (e.g. detector dead-time)
- Normalize data to unit edge step (compensates for sample concentration/thickness)
- Convert from E -> k space (makes oscillations more uniform spatial frequency, for BKG and Fourier transform)
- Subtract background using cubic splines or other methods
- Weight data with $k^n$, $1 \leq n \leq 3$; (compensates for amplitude decay)
- Fourier transform to distinguish shells at different distances
- Fourier Filter to isolate shells (optional)
Data Modeling

- Fit data in k-space, r-space, or E-space using single or multiple scattering theory, and theoretical calculations (e.g. Feff9, GNXAS, EXCURV)
- Fitting is done by describing an approximate hypothetical structure in terms of a limited number of parameters, which are adjusted to give an acceptable fit.
- Good open-source software is available e.g. feff6 (Rehr), ifeffit/Artemis/Athena (Ravel/Newville), SixPack (Webb) GNXAS (Di Cicco/Filliponi), RoundMidnight(Michalowicz), EXAFSPAK (George)...
- FEFF9 must be licensed, but it’s at reasonable cost.
- Other programs e.g. Mathematica, R can be useful.
Example: Raw XAFS data

$\mu(E)$

Energy (KeV)

$\begin{align*}
9.4 & \quad 9.6 & \quad 9.8 & \quad 10 & \quad 10.2 & \quad 10.4 & \quad 10.6 \\
\end{align*}$

Figure 2 – Transmission XAFS spectrum of ZnS

$\rightarrow$ normalize, convert to k space, subtract spline background
$K^3$ weighted EXAFS

$k^3 \chi$, 300K vs 30K
Average EXAFS signal decreases at higher temperatures because of increased thermal DWFs.
Fourier Filtered First Shell

determine single shell’s amplitude and phase from real and imaginary parts of inverse FT
Log-Ratio Amplitude

Slope gives difference in $\sigma^2$, intercept gives $\ln[CN \text{ ratio}]$ vs reference spectrum
Single Scattering fitting

- If SS is a good approximation, and shells are well isolated, you can fit shell by shell.
- Complications still occur because of large disorder, accidental cancellations, and high correlation between fitting parameters.
- Multishell fits in SS approximation.
Multiple scattering fitting

- MS often cannot be neglected (e.g. focussing effect)
- MS fitting introduces a host of complications but also potential advantages
  - SS contains no information about bond angles
  - MS does contain bond angle information (3-body and higher correlations)
- Parameter explosion -> how to handle DWFs?
  - Dangers of garbage-in, garbage-out
- (more on this later in the talk)
Theory

* Improved Theory and Practical Implementations

- Fast sophisticated electron multiple scattering codes
- Still limitations in near-edge (XANES) region
- Solves the forward problem (structure->spectrum), but not the inverse problem (spectrum -> structure),
- More work on better fitting direct methods is needed
- Sophisticated quantum chemistry codes have been made easier to use; they can be leveraged to combine DFT and XAFS
- Correlate electronic and vibrational structure
Computing Multiple Scattering with FEFF9

- Rewrite golden rule squared matrix element in terms of real-space Green's function and scattering operators; expand GF in terms of multiple scattering from distinct atoms.
- Initial atomic potentials generated by integration of Dirac equation (relativistic analog of Schrödinger); modified atomic potentials generated by overlapping (optional self-consistent field; use for XANES).
- Complex exchange correlation potential computed → mean free path.
- Scattering from atomic potentials described through k-dependent partial wave phase shifts for different angular momentum l.
- Radial wave function vs E obtained by integration to calculate mu zero.
- Unimportant scattering paths are filtered out (except FMS).
- Feffs for each path calculated (e.g. Rehr Albers formalism).
- Final spectrum generated by summing finite number of paths, or, over restricted energy range, FMS (use for XANES).

→ All of this is accomplished in a few seconds.

FEFFx: see papers of Rehr, Ankudinov, Zabinsky et al.
See also DLXANES, GNXAS, and EXCURV programs.
Example: Multiple Scattering within Histidine Imidazole Ring

- MS nlegs > 2
- Double Scattering
- First Shell SS
- All shells SS
- All shells SS + MS
Information content of XAFS spectra is limited

- Estimate from Nyquist criterion
- Can completely describe band limited function by finite set of Fourier coefficients
- $N$ degrees of freedom $= 2 \Delta k \Delta r / \pi$
- $2 \times 10 \times 3/\pi \sim 20$ for solution spectra
Parameter explosion in MS fitting

- Multiple scattering expansion
- May be tens or hundreds of important paths
- Each path has degeneracy, pathlength, debye waller factor, ...
- Geometry allows you to interrelate the pathlengths within certain limits
- Group fitting (Hodgson & Co)
- Determining all the MS Debye Waller parameters by fitting is a hopeless task
- What can you do?
Dealing with Parameter Explosion

- **Use a priori information; extend k-space range**
- Simultaneous fitting to multiple spectra e.g. different temperatures
- Suppress DWFs by measuring samples cryogenically --> zero point motion and static DWFs
- Minimize use of ad-hoc assumptions!
- Calculate DWFs on physical grounds (Dimakis & Bunker, Poiarkova & Rehr) using density functional theory or faster methods
- If you can orient your sample, do it - you can double or triple information for low symmetry sites with polarized XAFS; better yet, joint refinement with XRD
Parameters needed to describe structure

Neglecting inter-ligand MS, how many parameters needed to define structure for metal protein site?

- \( \rho, \alpha, \beta \) for 4 ligands \( \rightarrow \) 12 parameters
- \( \rho, \alpha, \beta \) for 6 ligands \( \rightarrow \) 18 parameters

Need more parameters to describe disorder

Neglects multiple scattering between ligands

Indeterminate or nearly so for 3D structure
Polarized XAFS helps

- Second rank tensor – 3 by 3 matrix – 9 components, each a function of energy
- Diagonalize to 3 independent functions
- Isotropic average in solution (and cubic symmetry) to one independent function – the usual XAFS
- Low symmetry structures – can get up to 3 times the information (~60 parameters) from polarized XAFS
- Can use crystals that are not perfect enough for atomic resolution diffraction
- In principle could solve for 3D active site structure in crystal
- Joint refinement: crystallography and XAFS
Ab initio XAFS: scattering + vibrations

By combining sophisticated electron multiple scattering codes with density functional based quantum calculations of molecular vibrations, one can accurately calculate spectra with no fudge factors.
His(3), Cys(1) Zn site: Automated fitting using a genetic algorithm, + FEFF7 + ab initio DWFs. (Dimakis & Bunker, Biophys. Lett. 2006)
Direct Methods

Direct methods for determining radial distribution functions from EXAFS using Projected Landweber–Friedman Regularization

Khelashvili & Bunker
Chemical Speciation

- Mobility and toxicity of metals in the environment strongly depends on their chemical state, which can be probed in situ with XAFS.

- Under appropriate conditions, the total absorption coefficient is linear combination of constituent spectra.

- Use singular value decomposition, principal component analysis, and linear programming (Tannazi) methods to determine species.

- These deliver direct methods for determining speciation.

- Nonlinearities arising from particle size effects theoretically and experimentally (Tannazi & Bunker).
XAFS is a powerful tool for studying the local structure in both disordered and ordered materials.

Recent advances have made the technique more powerful and flexible. Much more can be and is being done to build upon and exploit recent advances in theory, experiment, and data analysis.