SYNCHROTRON RADIATION: PRODUCTION & PROPERTIES

DENNIS M. MILLS
Deputy Associate Laboratory Director
Advanced Photon Source

National School for Neutron and X-ray Scattering
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AGENDA

1. A little history lesson and anniversaries
2. Properties of radiation from relativistic electrons
3. Properties of electron beams in storage rings
4. Next generation light sources
X-RAYS WERE A PUZZLE

- Röntgen discovery of X-rays was made by in 1895 and he first thought they might be some sort of **longitudinal vibrations of the ether**.

- These "rays" were not deflected by magnetic fields, so carried no charge, and, at the time, refraction by a prism could not be measured (we’ll see why in the talk about x-ray optics).

- Others, including Charles Barkla who studied the polarization of x-rays (Nobel prize winner in 1917 for this and other work on x-ray scattering), thought they were an **extension of the visible light spectrum**.

- William Bragg was a strong proponent of x-rays being "**particles**" or corpuscular in nature.

- In 1896 Lord Kelvin sent G. G. Stokes a letter ending thus: “In respect of the Rontgen X-rays, are you a longitudinalist, or an ultravioletist, or a tertium quidist? “

- The discussions must have been lively in the early 1900’s, as in his 1927 Nobel acceptance speech, Compton calls out that: “**Many will recall also the heated debate between Barkla and Bragg, as late as 1910, one defending the idea that X-rays are waves like light, the other that they consist of streams of little bullets...**”. 
100TH ANNIVERSARY OF THE DISCOVERY OF THE COMPTON EFFECT

1927 Nobel Prize citation for Compton: According to Einstein’s photoelectric effect theory, light consists of quanta, “packages” with definite energies corresponding to certain frequencies. A light quantum is called a photon. When Arthur Compton directed X-ray photons onto a metal surface in 1922, electrons were emancipated and the X-rays’ wavelength increased because some of the incident photon energy was transferred to the electrons. The experiment confirmed that electromagnetic radiation could also be described as photon particles following the laws of mechanics.

Arthur Compton
Born: September 10, 1892
Died: March 15, 1962

- Moved to the University of Chicago from Washington University in 1923.
- Awarded Nobel prize in 1927.
- In April 1941, the National Defense Research Committee created a special project headed by Compton to look into a uranium-based weapon.
- Compton decided to build Chicago Pile-1, the first nuclear reactor, under the stands at Stagg Field. Under Fermi’s direction, it went critical on December 2, 1942.
- Compton was placed in charge of the Manhattan Project’s plutonium production at Hanford, Washington.
- In 1946 he returned to Washington University as the Chancellor.
- The APS Compton Award, given to recognize an important scientific or technical accomplishment at the APS, was named Compton.
Synchrotron Radiation (SR) - radiation from charged particles traveling in circular orbits - was first observed from a 70 MeV synchrotron at GE in Schenectady in 1947.

On April 24, 1947, Langmuir and I [Herbert Pollack] were running the machine… Some intermittent sparking had occurred and we asked the technician to observe with a mirror around the protective concrete wall. He immediately signaled to turn off the synchrotron as "he saw an arc in the tube." The vacuum was still excellent, so Langmuir and I came to the end of the wall and observed. At first we thought it might be due to Cerenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk [i.e., synchrotron] radiation.

FROM SYNCHROTRONS TO STORAGE RINGS

- Synchrotrons were first used as sources of SR. However, the particles’ constantly changing energy was not attractive and the advent of storage rings provided a far more attractive source.

- We now use the name synchrotron radiation to describe radiation that is emitted from charged particles traveling at relativistic speeds, regardless of the accelerating mechanism and shape of the trajectory.

For more info on the early users of synchrotron radiation see:
PROPERTIES OF THE X-RAYS
RADIATION PATTERNS FROM ACCELERATING CHARGES

Definitions:

\[ \beta = \frac{v}{c} \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{E}{m_0c^2} \]

- When \( v << c \), (\( \beta \approx 0 \)), the shape of the radiation pattern is a classical dipole pattern.

- But as \( \beta \) increases, the shape of the radiation pattern changes; it is more forward directed.

Lorentz transformation

\[ \tan(\psi)_{lab} = \frac{\sin(\psi')}{\gamma(\cos(\psi') + \beta)} = \frac{1}{\gamma \beta} \approx \frac{1}{\gamma} \]

When \( \psi \) is small, \( \tan(\psi) \approx \psi \) so \( \psi \approx 1/\gamma \).

The angular spread of the forward emitted radiation (sometimes called the opening angle), is approximately \( 1/\gamma \).

\( V \) is instantaneous velocity vector.
RADIATION FROM HIGHLY RELATIVISTIC ($\gamma \gg 1$) PARTICLES IN A CIRCULAR ORBIT

\[ \beta \approx 0 \quad \beta \approx 1 \]

$\Delta x' \approx 1/\gamma$

$\gamma = E/m_0c^2$

At the APS with $E = 7$ GeV,

$\gamma = 1.4 \times 10^4$

$1/\gamma = 73 \times 10^{-6}$

See Appendix 1 for more details

Relativistic velocities are good!!

- radiation forward directed
- radiated power $\propto E^4$

Primes denote angles

$x'$ ($y'$) is the angle from the velocity vector, $V$, in the $x$- ($y$) plane

The opening angle in both the horizontal and vertical directions is given approximately by:

$\Delta x'_{\text{rad}} = \Delta y'_{\text{rad}} \approx 1/\gamma$, when $\beta \approx 1$.

@ APS $\beta \approx 1 - (2 \times 10^{-9})$
ANATOMY OF SYNCHROTRON RADIATION FACILITY

ADVANCED PHOTON SOURCE
BEAM ACCELERATION & STORAGE SYSTEM

(A) ELECTRON GUN

(B) ELECTRON LINEAR ACCELERATOR
   Output energy: 375 MeV

(C) PARTICLE ACCUMULATOR RING

(D) BOOSTER SYNCHROTRON
   Nominal extraction energy: 7.0 GeV

(E) STORAGE RING
   Nominal energy: 7.0 GeV

(E1) INSERTION DEVICE
(E2) BENDING MAGNET

NOTE: Diagrams not to scale
BEND MAGNET (BM) X-RAY RADIATION PROPERTIES

Bend Magnet Radiation

Spectrum characterized by the critical energy:

\[ E_c[\text{keV}] = 0.066 \cdot B[\text{kG}] \cdot E^2[\text{GeV}] \quad (E_c \approx 20 \text{ keV} @ \text{APS}) \]

Recall: \( \lambda[\text{Å}] = 12.4/E[\text{keV}] \) so 20 keV is 0.62Å

Vertical opening angle (\( \Delta y' \)) is \( 1/\gamma \). At the APS:

\[ \gamma = 1.4 \times 10^4 \]

so \( 1/\gamma = 73 \times 10^{-6} \text{ radians} \)

You get this for “free” since you need the bending magnets to keep the electrons orbiting in the storage ring.

All the BMs have the same B-field to maintain a closed orbit, so spectrum from each BM is the same.

See Appendix 2 for more details
Planar Insertion Devices (IDs)

Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.

“Planar” refers to the magnetic field being in one direction (in this case vertical or y-direction).

IDs can have fields in both the vertical and horizontal directions to produce circularly polarized x-rays and for other applications.

The strength of the magnetic field, $B$, is varied by changing the gap between the permanent magnets in the upper and lower halves of the insertion device.

Beam of electrons continues around the ring being bent by the bending magnets (not shown for clarity).

The magnetic period of the ID, $\lambda_{ID}$, is defined as the distance between the centers of adjacent magnets.

Magnetic field is in the y-direction for a “planar” ID (i.e., no field in the x-direction).

Steel or permandur poles with permanent magnets between them supplying the magnetic field.
CHARACTERIZING INSERTION DEVICES

- IDs are characterized by the so-called field index or deflection parameter, $K$

$$K = \frac{eB_0\lambda_{ID}}{2\pi m_0 c} = 0.0934 \lambda_{ID}[\text{cm}] B_0[\text{kG}]$$

where $\lambda_{ID}$ is the magnetic period of the insertion device and $B_0$ the peak magnetic field. The length of the insertion device, $L$, is equal to the number of periods, $N$, times the length of the period, i.e.,

$$L = N\lambda_{ID} .$$

- The maximum deflection angle of the particle beam, $x'_{ID,\text{max}}$, is given by:

$$x'_{ID,\text{max}} = \pm \left( \frac{K}{\gamma} \right)$$

$\gamma$

**Important thing here is $K$ is proportional to mag field, $B$.**

APS Undulator A has a period of 3.3 cm and operates with $K \approx 1$, therefore:

$$x'_{ID,\text{max}} \approx \frac{1}{\gamma}$$

See Appendix 3 for more details on $K$
**WIGGLER (K>> 1)**

Horizontal deflection angle of electrons given by $x'_{\text{ID, max}} = \left(\frac{K}{\gamma}\right)$

Radiation spectrum looks like 2N dipole sources ($N =$ number of periods)

$$x'_{\text{ID, max}} = \left(\frac{K}{\gamma}\right) \gg \frac{1}{\gamma}$$

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**UNDULATOR (K ≈ 1)**

When the maximum deflection of the electron beam is on the order of the natural opening angle of the emitted radiation, i.e.

$$x'_{\text{ID, max}} = \left(\frac{K}{\gamma}\right) \approx \frac{1}{\gamma},$$

radiation from each of the magnetic pole overlaps.
This overlap can cause interference effects in the spectrum.

On-axis (x' = y' = 0), the constructive interference occurs at a particular x-ray energy ($E_{n,x-ray}$) and its odd harmonics, i.e. $n = 1, 3, 5...$ when:

$$E_{n,x-ray} = \frac{[nhc \gamma^2]}{[\pi \lambda_{ID}(1 + K^2/2)]}$$

$h=\text{Planck's constant}$ and $c$ the velocity of light.

Recall that $K$ is proportional to the peak magnetic field generated by the undulator.
- In permanent magnet undulators vary the gap between the upper and lower poles to change magnetic field
- In a superconductor undulator, vary the current in the windings to change magnetic field

At the fundamental (1$^{\text{st}}$ harmonic or $n = 1$), the horizontal and vertical opening angles of the radiation is given by:

$$\Delta x'_{\text{rad}} = \Delta y'_{\text{rad}} \approx (1/\gamma) \left[1/N\right]^{1/2}$$

where $N = \text{number of periods}$ [typically 100 or so].
THE DIFFRACTION LIMIT FOR LIGHT

- Just as the Uncertainty Principle sets a lower limit for the product of the size, $\Delta x$, and momentum $\Delta p_x$, this relationship can be re-written in terms of size $\Delta x_{\text{rad}}$ ($\Delta y_{\text{rad}}$) and angular divergence $\Delta x'_{\text{rad}}$ ($\Delta y'_{\text{rad}}$) for radiation.

  $$\Delta x \Delta p_x, \geq \frac{h}{4\pi}$$

  $$\Delta x_{\text{rad}} \Delta x'_{\text{rad}} \geq \frac{\lambda}{4\pi} \quad \text{and} \quad \Delta y_{\text{rad}} \Delta y'_{\text{rad}} \geq \frac{\lambda}{4\pi}$$

- A light beam that satisfies $\Delta x_{\text{rad}} \Delta x'_{\text{rad}} = \frac{\lambda}{4\pi}$ is called **diffraction limited**. When a radiation beam has this property, it is a **fully coherent source**.

- Another way to define a **diffraction limited beam** is if its potential to be focused to small spots is as high as possible for its wavelength.

- The diffraction limit at 1 Å is:

  $$\frac{\lambda}{4\pi} = \frac{1\text{Å}}{4\pi} = 10^{-10} \text{ meters}/4\pi \approx 8 \times 10^{-12} \text{ m}$$

  **8 picometers-radian** (radians are dimensionless)

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**APS Undulator A** has a length of 2.4 meters. For 1 Å radiation the natural opening angle is:

$$\Delta x'_{\text{rad}} = \frac{1}{\gamma} (1/N)^{1/2} = 4.5 \times 10^{-6} \text{ rad}$$

where $N =$ number of magnetic periods in the undulator.

The corresponding size is:

$$\Delta x_{\text{rad}} = \left[\frac{\lambda \gamma}{4\pi}[N]^{1/2}\right] = 1.7 \times 10^{-6} \text{ m}$$

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I've labeled the x-ray properties with the subscript “rad” to distinguish it from the particle beam source size and divergence that will be discussed in the second half of the talk.
ELECTRON BEAM PROPERTIES ARE KEY

- So far, this discussion has not taken into account the properties of the source of the x-rays – namely the electron beam.

- The electron beam has both a size and divergence in the x- and y-directions (transverse to the velocity of the beam).

- The electron beam also has a finite length in the z-direction (along the direction of the velocity of the beam – sometimes called the longitudinal properties).

- To fully understand the properties of the emitted x-ray beams, we need to look a little more closely at the (transverse) properties of the electron beams.

- But first, let’s look at the longitudinal properties.
PROPERTIES OF THE ELECTRON BEAM: TRANSVERSE AND LONGITUDINAL
LONGITUDINAL PROPERTIES - PULSE DURATION

- Because the electrons are radiating x-rays, they are constantly losing energy, and so to restore the energy loss on each revolution, radio-frequency (RF) resonant cavities are installed in the storage ring to replenish the radiative energy losses.

- Particles are grouped together by the action of the radio-frequency (RF) cavities into bunches. At APS:
  - 1104 m circumference (3.68 microsecond period)
  - there are 1296 evenly spaced “RF buckets” (stable orbit positions) around the ring
  - the bunch length of the electron packet in one of these “buckets” is about 3 cm in length, corresponding to a pulse duration of about 100 psec

Electrons arrive at the right time to feel the correct E-field to gain the lost energy.
The time structure is determined by which of the rf buckets are filled with electrons.

24 equally spaced bunches (about 4 mA/bunch)
- compromise between quasi-continuous source and pulsed source

1 + 8x7 (12-14 mA in one bunch and the rest of the 100 mA distributed in 7 trains of 8 closely spaced bunches on the other side of the ring)
- timing experiments

324 equally spaced bunches (about 0.3 mA/bunch)
- approximates a quasi-continuous source
Although the flux from undulators can be determined without knowledge of the source size and divergence, one very important characteristic of the x-ray beam, namely \textit{brightness}, requires a more detailed knowledge of the particle beam’s size and divergence.

\textbf{Brightness} has units of: \textit{photons/sec/0.1\% BW/source area} / \textit{source divergence}

\[
\text{Flux} / 4\pi^2 \Sigma_x \Sigma_y \Sigma_x' \Sigma_y'
\]

where $\Sigma_i$ ($\Sigma_i'$) is the \textbf{effective} one sigma value of the source size (divergence) in the $i$\textsuperscript{th} direction. The total source size and divergence is the quadrature sum of the electron beam and the radiation beam, namely:

\[
\Sigma_x = \left[ \Delta x^2_{\text{rad}} + \Delta x^2_{\text{electron}} \right]^{1/2}
\]

\[
\Sigma_x' = \left[ \Delta x'^2_{\text{rad}} + \Delta x'^2_{\text{electron}} \right]^{1/2}
\]

We previously determined $\Delta x_{\text{rad}}$ and $\Delta x'_{\text{rad}}$ for undulator radiation.

and a similar expression for $\Sigma_y$

and a similar expression for $\Sigma_y'$
TRANSVERSE PROPERTIES OF PARTICLE BEAMS

- When a bunch of electrons are injected into a storage ring, the ensemble of electrons rattles around until it reached its equilibrium values, in both the transverse (x and y directions) and longitudinal (z) directions.

- Here we are interested in the **transverse** (x and y) properties.

- Accelerator physicists describe the electron beam in terms of its horizontal (x-plane) and vertical (y-plane) position and divergence or it’s **phase space**.

- This definition of **phase space** has as the two conjugate variables
  
  \[ x \text{ and } x' (\text{or, } y \text{ and } y') \]

- The area bounded by an ellipse that captures some given fraction of electrons in this xx’ (yy’) phase space is proportional to what is called the **horizontal (or vertical) emittance** of the electron beam.

Remember: this plot is not the beam cross-section, but a phase space representation!!!
ELECTRON BEAM PHASE SPACE AND EMITTANCE

- There is a separate horizontal emittance ($\varepsilon_x$) and vertical emittance ($\varepsilon_y$). In today’s storage rings:

  \[ \varepsilon_y \text{ is typically 1\% to 10\% of } \varepsilon_x \text{ (the percentage is called the coupling)} \]

- The emittance is a constant of the storage ring, although one can trade off electron beam size for divergence as long as the area of the phase-space remains constant.

- To determine the x-ray beam brightness, you need to know the electron beam source size and divergence at the spot in the storage ring where the undulator is located and add those values - in quadrature - with source size and divergence of the radiation.

\[ \varepsilon_x \approx 3 \times 10^{-9} \text{ m-rad or 3000 pm-rad} \]
\[ \varepsilon_y \approx 0.03 \times 10^{-9} \text{ m-rad or 30 pm-rad} \]
DIFFRACTION LIMITED SOURCES AND COHERENCE

- If both the horizontal and vertical emittance of the particle beam were small compared to the diffraction limit of the light ($\lambda / 4\pi$), then the x-rays that are emitted would have full transverse coherence.

- But at the APS, as with most 3rd generation synchrotron radiation sources, the electron beam emittance dominates - see figure below.

- Hence the radiation is partially coherent

- Partially coherent sources are characterized by the coherent fraction, the fraction of the x-ray flux that is coherent.

- Coherent fraction = ratio of diffraction-limited emittance to actual beam emittance.

- For the APS at 1Å, the coherent fraction is $\approx 10^{-3}$ – useful for experiments that rely on coherence but at the expense of throwing away a lot of (incoherent) flux.
NEXT GENERATION LIGHT SOURCES
THE EVOLUTION OF X-RAY BRIGHTNESS FROM STORAGE RING SOURCES

- **1st Generation Sources**
  - Ran parasitically on accelerations for high-energy physics (CHESS and SSRL originally)

- **2nd Generation Sources**
  - Built to optimize synchrotron radiation from the bending magnets (NSLS)

- **3rd Generation Sources**
  - Built to optimize synchrotron radiation from insertion devices (ALS, APS, NSLS II)

- **3.5 (?) Generation Sources**
  - Based on multi-bend achromat (MBA) storage ring lattices to go towards a diffraction-limited source (ALS-U, APS-U) (more later)

- **4th Generation Sources**
  - Fully coherent sources
    - X-ray Free Electron Lasers - X-FELs (LCLS)
    - Diffraction Limited Storage Rings (DLSRs)

See presentation by P. Fouss for details on FELs
THE DRIVE FOR MORE COHERENT SOURCES

- How do we get from where we are today ($\varepsilon_x = 3000$ picometer-radian) to something closer to the $8$ picometers-radian emittance for a fully coherent beam at $1\text{Å}$?

- Presently there are two approaches to obtain a more coherent x-ray source:
  - Storage rings with so-called Multi-bend Achromat (MBA) magnet structures
  - Using LINACs that satisfy the conditions for a free electron laser (FEL) - – See Appendix 6

The accelerator physics for both these approaches has been known and understood for sometime – WHY NOW??

In my humble opinion, is has been advances in modeling/simulations along with the availability of high performance computing that gave designers and builders (and funders) the confidence that these machines could be successfully built.


THE APS-U PROJECT: A MULTI-BEND ACHROMAT (MBA) LATTICE

\[ \varepsilon_x = C_L \frac{E^2}{N_d^3} \]

- \( C_L = \text{constant} \)
- \( E = \text{beam energy} \)
- \( N_d = \text{dipoles per sector} \)

**APS MBA Upgrade:**

*Beam energy:* 7 GeV => 6 GeV

*Dipoles per sector:* 2 => 7

Red magnets are dipoles

~50-fold reduction in horizontal emittance

10^2 to 10^3 X increase in brightness
APS-U Project Scope

**Total Project Cost**
- $815M

**Feature beamlines**
- Suite of beamlines, including long beamlines, designed for best-in-class performance

**New storage ring**
- 6 GeV with 200 mA, 42 pm-rad emittance
- Hybrid 7BA lattice with reverse bends
- Improved electron and photon stability

**New insertion devices**
- Including superconducting undulators

**New/upgraded front ends**

**Beamline enhancements**
- Improvements to make beamlines “Upgrade Ready”
- Existing beamlines are planned to come back on-line after the upgrade

**Injector improvements**
- Increase performance beyond present capability

**On-axis “swap-out” injection**

**42 pm-rad**
HIGHER MAGNETIC FIELDS FOR FOCUSING REQUIRES SMALL VACUUM CHAMBERS

The small vacuum chamber permits the focusing magnets pole pieces (blue in fig. on right) to get closer to the electron beam producing higher magnetic focusing fields.

Existing APS storage ring through the dipole (bending) magnets compared to the APS-U vacuum chamber.

Close-up of one of the 1,321 magnets that will make up the new APS electron storage ring.

More than 60 of the needed 200 magnet modules have been partially assembled and are in storage.

Dipole vacuum chamber for APS-U
MBA UPGRADES – A COMPETITIVE LANDSCAPE

ESRF (France)
- Upgrade is complete and ESRF/Extremely Bright Source (EBS) is operational

MAX-IV (Sweden)
- A new (green field site) 3 GeV MBA lattice-based storage ring that is operational

SIRIUS (Brazil)
- A new (green field site) 3 GeV MBA lattice-based storage ring that is operational

SPRING-8 (Japan)
- Planning an MBA upgrading in 2020’s

ALS-U (US)
- Planning an MBA upgrading in 2020’s
SUMMARY

- Synchrotron radiation continues to be an important tool for researchers across all scientific and engineering disciplines.

- There is a strong science case for a new generation of sources such as:
  - Low-emittance storage rings
  - X-ray free electron lasers

- Storage rings can produce partially coherent x-ray beams with megahertz repetition rates but with lower peak intensity.

- FELs can provide very short pulses (femtoseconds) of high peak intensity, coherent x-rays but the repetition rate of many FELs limited to hundreds of hertz.
  - Superconducting radio frequency (SCRF) linacs will increase rep rates that to MHz (LCLS II and the European XFEL).

- The LCLS II, APS-U, and the upgrade for the Advanced Light Source (ALS-U at LBNL) will keep US x-ray facilities at the cutting edge to produce world class science in the years to come.
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QUESTIONS?
APPENDIX 1: RADIATED POWER FROM CHARGES AT RELATIVISTIC VELOCITIES

The classical formula for the radiated power from an accelerated electron is:

\[ P = \frac{2e^2}{3c^3} a^2 \]

Where \( P \) is the power and \( a \) the acceleration. For a circular orbit of radius \( r \), in the non-relativistic case, \( a \) is just the centripetal acceleration, \( v^2/r \). In the relativistic case:

\[ a = \frac{1}{m_o} \frac{dp}{d\tau} = \frac{1}{m_o} \gamma \frac{d\gamma m_o v}{dt} = \gamma^2 \frac{dv}{dt} = \gamma^2 \frac{v^2}{r} \]

Where \( \tau = t/\gamma = \) proper time, \( \gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2 \) and \( \beta = v/c \)

\[ P = \frac{2e^2 \gamma^4 v^4}{3c^3 r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8} \]
DEPENDENCE ON MASS AND ENERGY OF RADIATED POWER

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e− with the same total energy, E.)

2. Scales with the 4th power of the particle’s energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)
The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, $E_c$.

$$E_c = \frac{3hc\gamma^3}{4\pi r}.$$

Here $h$ is Planck’s constant and $r$ the radius of curvature of the trajectory. Note that the critical energy scales as $\gamma^3$. In practical units, the critical energy can be written as:

$$E_c[\text{keV}] = 2.218 E^3[\text{GeV}] / \rho[\text{m}] = 0.06651 B[\text{kG}] E^2[\text{GeV}]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at $E = 7 \text{ GeV}$. The critical energy of the radiation emitted from the BM is:

$$E_c[\text{keV}] = 0.06651 B[\text{kG}] E^2[\text{GeV}]$$

or

$$E_c = 0.06651(5.990)(7^2) = 19.5 \text{ keV} \text{ or } 0.64 \text{ Å}.$$
APPENDIX 3: WHERE DID “K” COME FROM?

\[ F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) \]

\[ \dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) \quad z = ct \]

\[ v_x = - \frac{ecB_0}{\gamma m_0} \frac{\lambda_{ID}}{2\pi c} \cos \left( \frac{2\pi ct}{\lambda_{ID}} \right) = - \frac{eB_0 \lambda_{ID}}{\gamma m_0} \cos \left( \frac{2\pi ct}{\lambda_{ID}} \right) \]

\[ x = \frac{eB_0}{\gamma m_0 c} \left[ \frac{\lambda_{ID}}{2\pi} \right]^2 \sin \left( \frac{2\pi ct}{\lambda_{ID}} \right) = \left[ \frac{eB_0}{m_0} \frac{\lambda_{ID}}{2\pi c} \right] \frac{1}{\gamma} \left[ \frac{\lambda_{ID}}{2\pi} \right] \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) = K \frac{1}{\gamma} \left[ \frac{\lambda_{ID}}{2\pi} \right] \sin \left( \frac{2\pi z}{\lambda_{ID}} \right) \]

\[ x_{max} = K \frac{1}{\gamma} \left[ \frac{\lambda_{ID}}{2\pi} \right] \quad \text{and} \quad \left[ \frac{dx}{dz} \right]_{max} = K \gamma \quad \text{where} \quad K = \left[ \frac{eB_0 \lambda_{ID}}{2\pi m_0 c} \right] \]

Equation of motion for a relativistic charged particle in a magnetic field

Magnetic field in y-direction and electron traveling in z-direction
APPENDIX 4: THE DIFFRACTION LIMITED OF LIGHT

- The Heisenberg Uncertainty Principle sets a lower limit for the product of the size $\Delta x$ ($\Delta y$), and angular divergence $\Delta x'$ ($\Delta y'$), of radiation. Recall:

  \[ \Delta x \Delta p_x \geq \hbar / 2 \]

  \[ \frac{p_x}{p_z} = x' \text{ or } \frac{\Delta p_x}{p_z} = \Delta x' \text{ and } p_z = \hbar k = \frac{\hbar (2\pi)}{\lambda} \]

  \[ \text{so} : \Delta x \Delta x' = \Delta x \Delta x' p_z = \Delta x \Delta x' \left[ \frac{\hbar (2\pi)}{\lambda} \right] \geq \hbar / 2 \]

  \[ \Delta x \Delta x' \geq \lambda / 4\pi \]

- This says, for a given wavelength $\lambda$, the product of its size and divergence cannot be is less than $\lambda / 4\pi$. When the product is $\approx \lambda / 4\pi$ in both the x and y directions, the radiation is fully coherent and the source is said to be diffraction limited.
What is typically shown for the output of an undulator is not the spectrum at one gap setting but rather the energy range that can be covered over the available from the minimum gap.

The lowest x-ray energy (highest magnetic field) is set by the minimum magnet gap (determined by vertical size of the vacuum chamber).

As the gap is opened, the magnetic field gets smaller and the constructive interference occurs at higher x-ray energy. But when the magnetic field decreases so does the intensity so that limits the useful high energy range.

Here is the spectrum generated by an undulator for a particular magnetic gap or K value.

Plots like this that show the energy range that can be covered by an undulator are often called “tuning curves”.

ID Vacuum Chamber
Cross Section

Upper Magnet Poles

Lower Magnet Poles

Undulator A Tuning Curve

Brightness (arb. units)

Energy (keV)

Photon Energy
(Thousands of Electron Volts)

1st
3rd
5th
7th
9th

10^19
10^18
10^17
10^16
10^15
10^14
0
5
10
15
20
25
30
35
40

Argonne National Laboratory
• Another way to reduce the particle beam emittance is through linac-based x-ray free electron lasers (XFELs)
  - Full transverse (spatial) coherence and femtosecond pulses

• An x-ray FEL uses the high brightness of an electron gun coupled to an emittance-preserving linac. You don’t have to deal with the equilibrium transverse and longitudinal beam size (length of pulse) that occurs in a storage ring so can have a very low emittance and very short pulses (femtoseconds).

• The problem with a linac is you only use the electrons once, and so have to accelerate new electrons for each pulse. The acceleration process is what takes all the electrical power so a different paradigm is required. (In the storage ring you “recycle” the electrons after they have been accelerated up to energy and there is no further increase in particle beam energy as they go around.)
GETTING THE ELECTRONS TO RADIATE IN PHASE

- When the electrons in a storage ring go through the undulator, there is no correlation between their positions on the scale of the radiation wavelength. As a result the fields they generate superimpose at random and its intensity is proportional to the number of electrons, \( N_e \).

- If we could order the electrons so that they are all lined up in thin sheets, periodically spaced with spacing equal to the radiation wavelength, the radiation fields would superimpose in phase. The intensity is then proportional to \( N_e^2 \).

- Since \( N_e \) is a billion or more, one could obtain a huge gain.

See P. Fuoss’ talk for more details.
SUMMARY OF THE FEL PROCESS

- The electrons **slip** behind the EM wave by $\lambda_1$ per undulator period $\lambda_u$.

- Due to this sustained interaction, some electrons lose energy, while others gain energy -> **energy modulation** at $\lambda_1$

- In the undulator magnetic field, the trajectory of electrons with larger (smaller) energy is bent less (more) -> **micro-bunching** at $\lambda_1$

- Micro-bunched beam radiates coherently at $\lambda_1$, and in turn this interaction is enhanced -> **exponential growth of radiation power** (gain)

- This process is called Self-Amplified Spontaneous Emission (SASE)

\[ \lambda_1 = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right) \]