

# Single Crystal Diffraction

William Ratcliff

NCNR



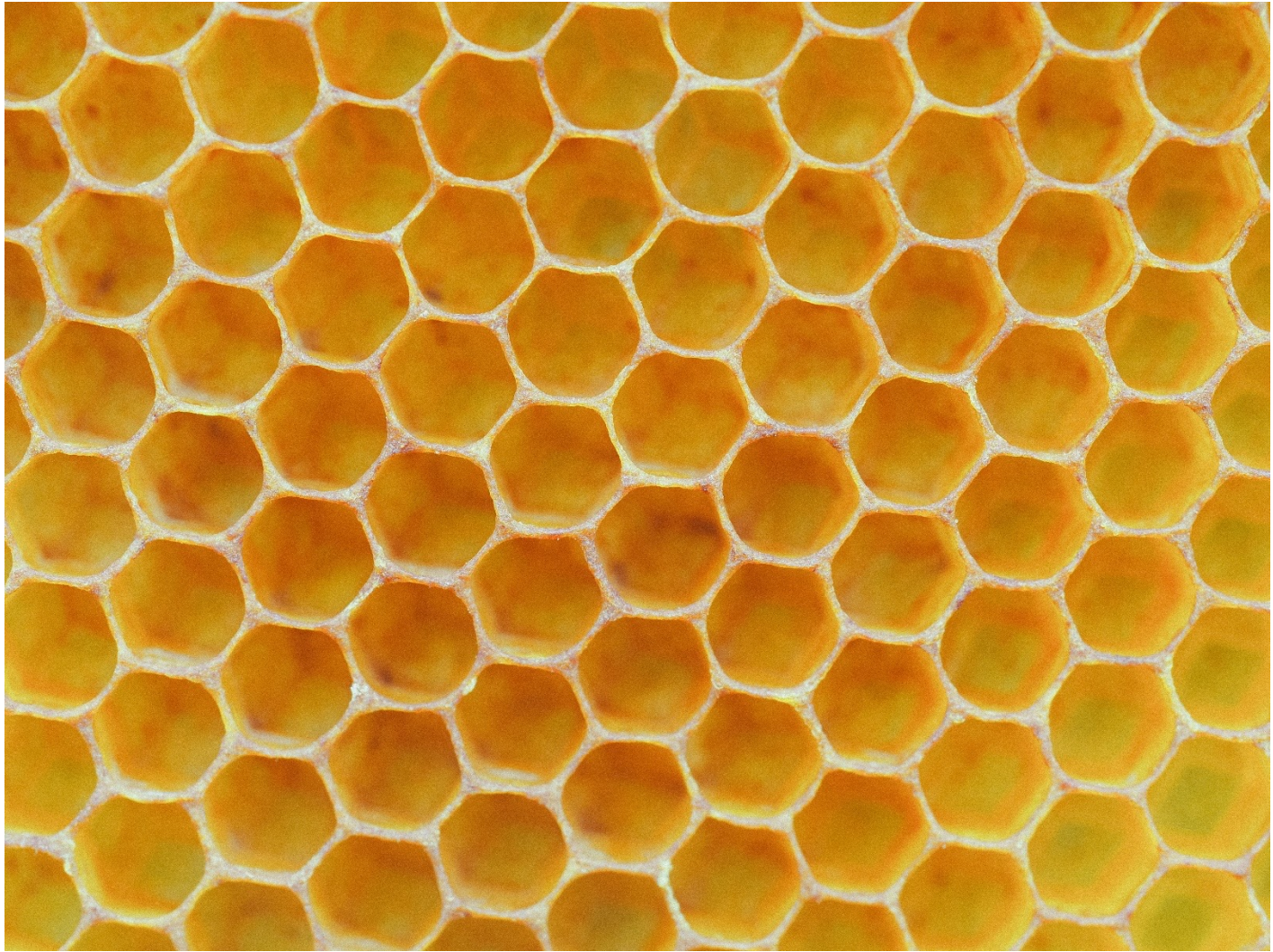
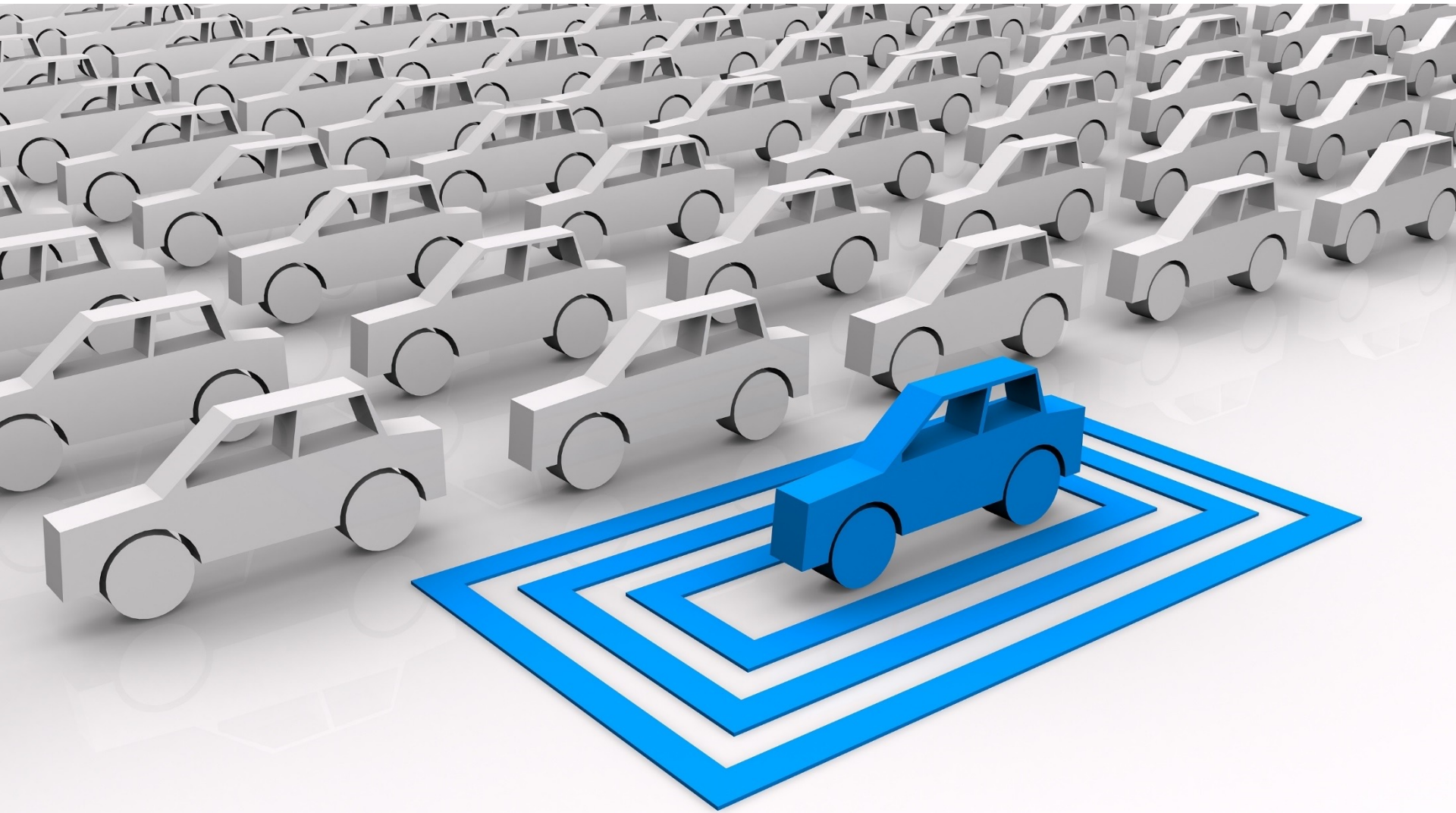


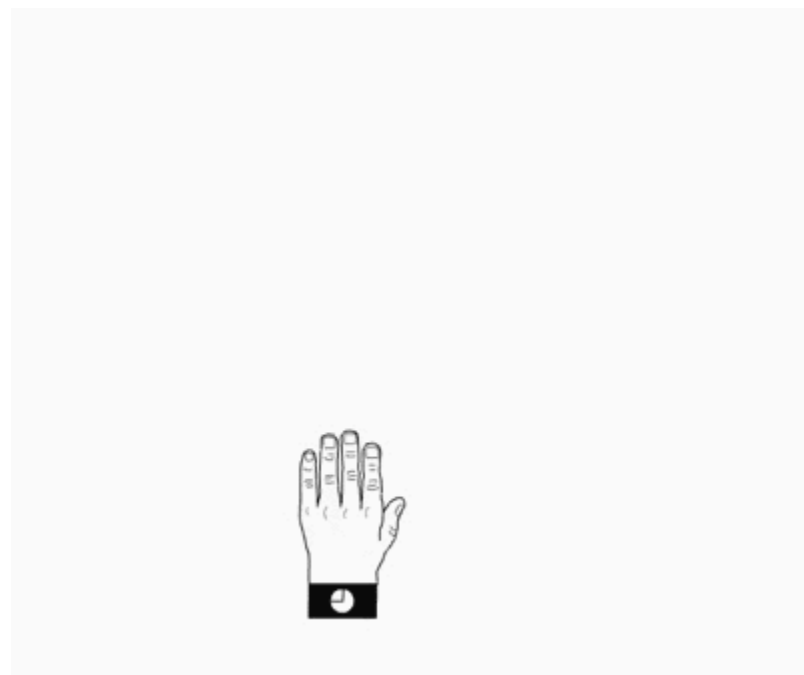
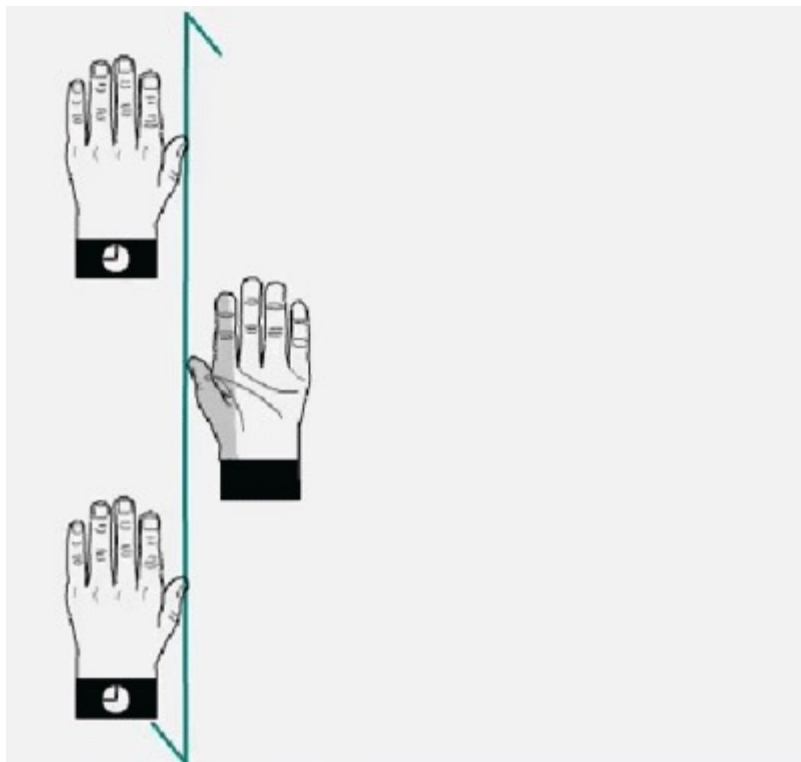
Photo by [Ante Hamersmit](#) on [Unsplash](#)





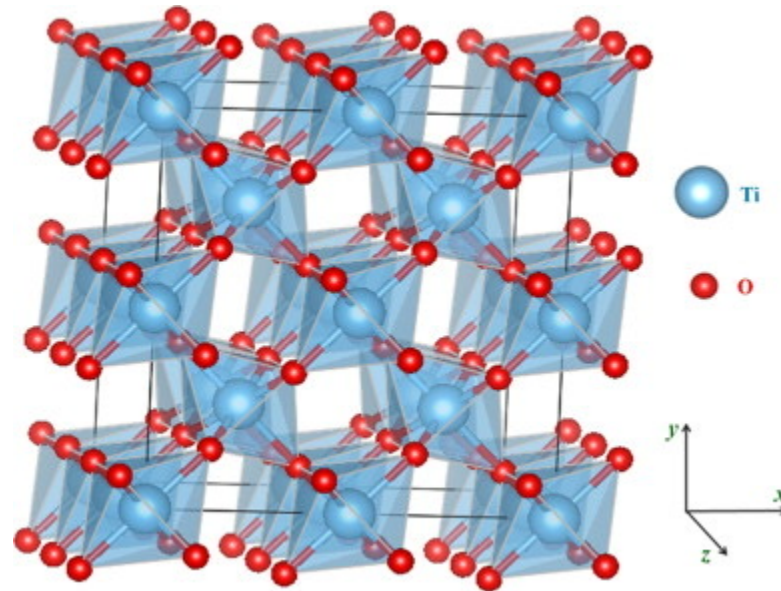






[https://www.xtal.iqfr.csic.es/Cristalografia/parte\\_03-en.html](https://www.xtal.iqfr.csic.es/Cristalografia/parte_03-en.html)





= Symmetry + Translation

INTERNATIONAL TABLES  
for CRYSTALLOGRAPHY

Volume

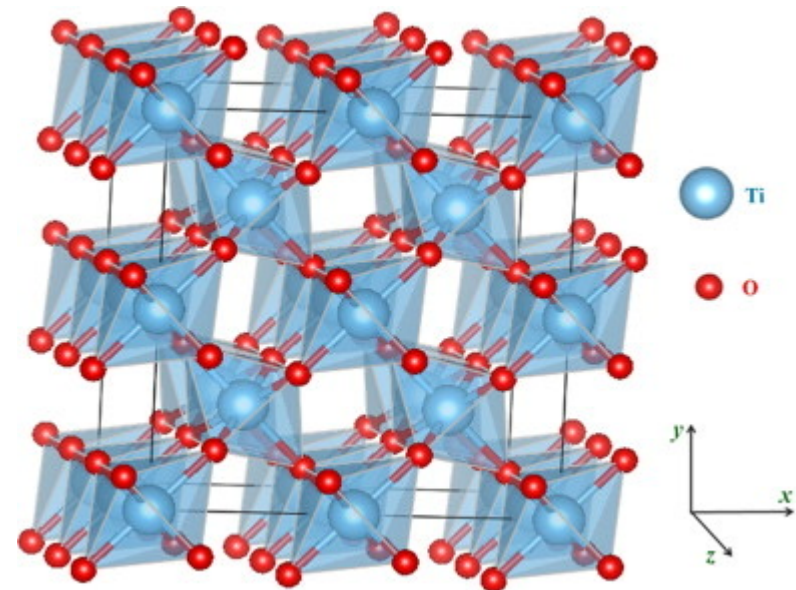
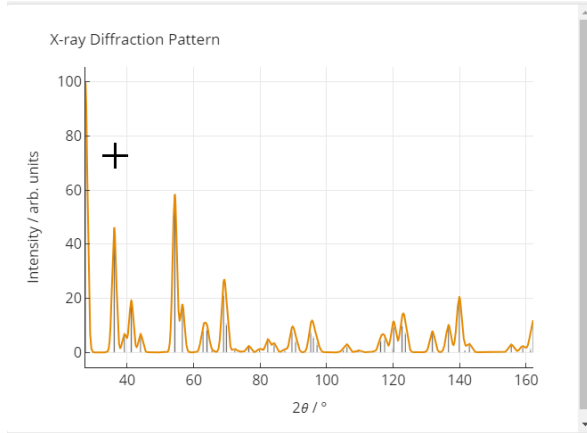
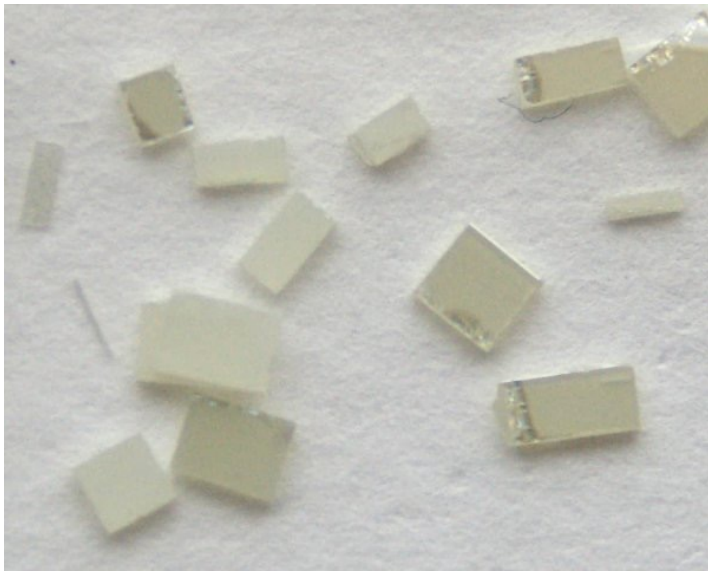
**A**

Space-group symmetry

Edited by Th. Hahn

Fifth edition





[https://en.wikipedia.org/wiki/Titanium\\_dioxide](https://en.wikipedia.org/wiki/Titanium_dioxide)

<https://next-gen.materialsproject.org/materials/mp-2657/#:~:text=TiO%E2%82%82%20is%20Rutile%20structured%20and,%20Ti%E2%80%93O%20bond%20lengths.>

[https://www.google.com/url?sa=i&url=https%3A%2Fhongtortai.com%2Fcollection%2Frutile-structure-of-tio2&psig=AOvVawZmp\\_ydJWopYk-xDw1u6F1&ust=1691324082267000&source=images&cd=vfe&opi=89978449&ved=0CBAQjRxqFwoTCLibqay\\_xYADFQAAAAAAdAAAAABAY](https://www.google.com/url?sa=i&url=https%3A%2Fhongtortai.com%2Fcollection%2Frutile-structure-of-tio2&psig=AOvVawZmp_ydJWopYk-xDw1u6F1&ust=1691324082267000&source=images&cd=vfe&opi=89978449&ved=0CBAQjRxqFwoTCLibqay_xYADFQAAAAAAdAAAAABAY)

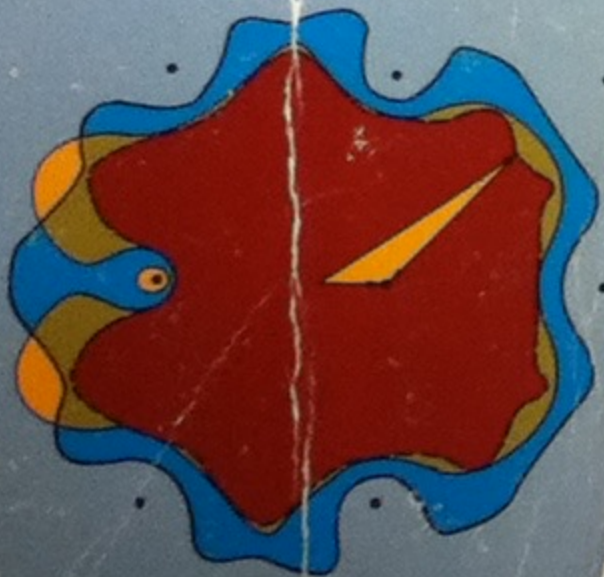
# Intermission





how**stuff**works  
It's good to know

INTRODUCTION TO  
THE THEORY OF  
THERMAL  
NEUTRON  
SCATTERING



G.L. Squires

Harald Ibach Hans Lüth

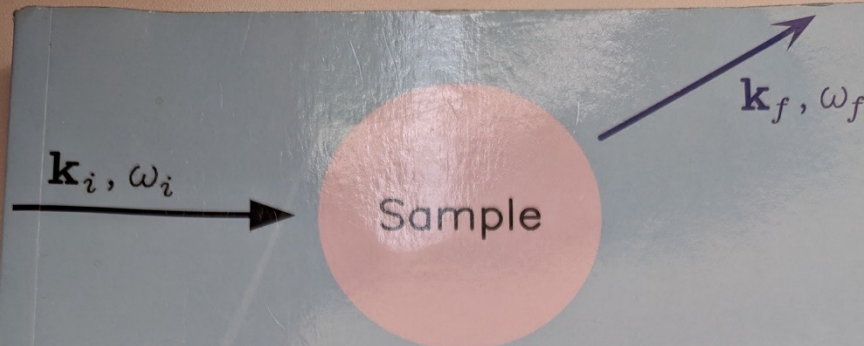
# Solid-State Physics

An Introduction to Principles of Materials Science

Second Edition



Springer

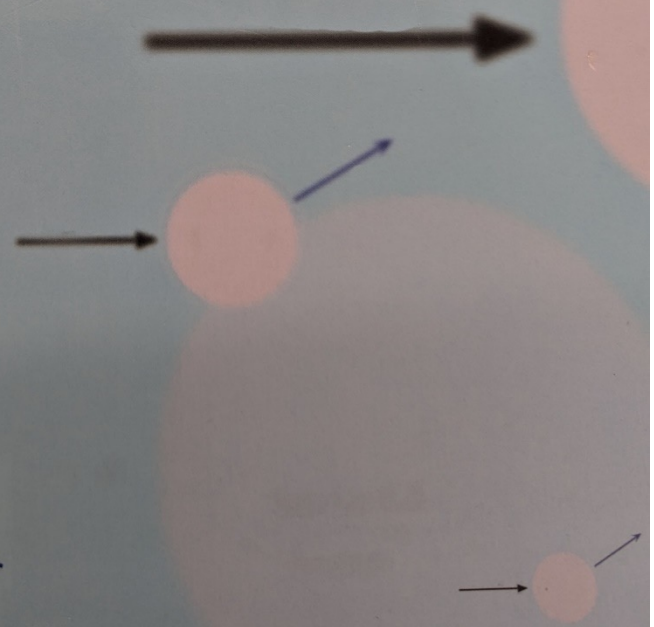


# Elementary Scattering Theory

For X-ray and Neutron Users

**D.S. SIVIA**

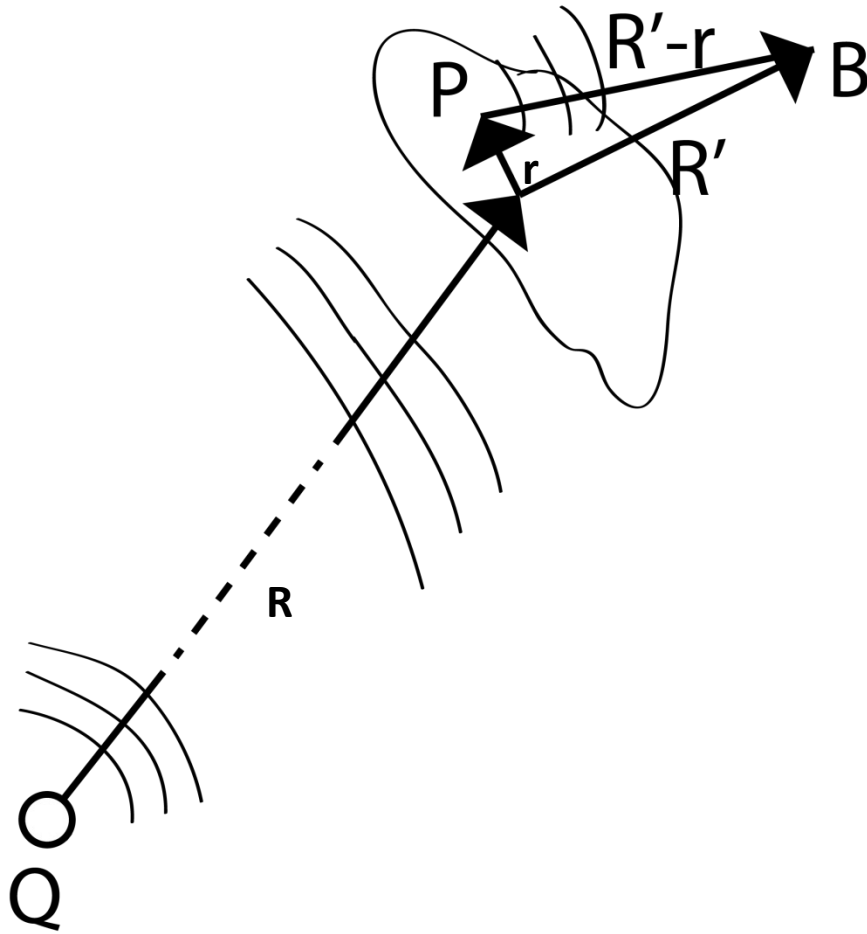
OXFORD





# Crystallography and the reciprocal space

<http://toutestquantique.fr/en/>



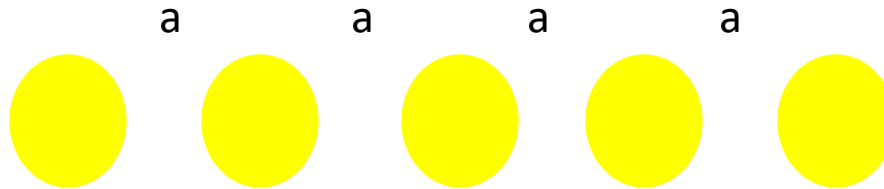
$$A_P = A_0 e^{i\vec{k}_0 \cdot (\vec{R} + \vec{r}) - i\omega_0 t}$$

$$A_B = A_P(r, t) \rho(r) \frac{e^{i\vec{k} \cdot (\vec{R}' - \vec{r})}}{|\vec{R}' - \vec{r}|}$$

$$A_B = A_P(R' \gg r, t) \rho(r) \frac{e^{i\vec{k} \cdot (\vec{R}' - \vec{r})}}{|\vec{R}'|}$$

$$I(K) \propto |A_B|^2 \propto \left| \int \rho(r) e^{i\vec{K} \cdot \vec{r}} dr \right|^2$$

# Reciprocal Space



$$\rho(x) = \rho(x + na)$$

$$\rho(x) = \sum_n \rho_n e^{i(n2\pi/a)x}$$

$$\rho(\vec{r}) = \sum_n \rho_{\vec{G}} e^{i\vec{G} \cdot \vec{r}} \quad \vec{r}_n = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3$$

$$\vec{G} \cdot \vec{r} = 2\pi m \quad g_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$I(K) \propto |A_B|^2 \propto \left| \int \rho(r) e^{-i\vec{K} \cdot \vec{r}} dr \right|^2$$



$$I(K) \propto \left| \sum_{\vec{G}} \rho_{\vec{G}}(r) \int e^{i(\vec{G}-\vec{K}) \cdot \vec{r}} dr \right|^2$$

$$\int e^{i(\vec{G}-\vec{K}) \cdot \vec{r}} dr = \begin{cases} V & \text{for } \vec{G} = \vec{K} \\ \sim 0 & \text{otherwise} \end{cases}$$

Laue Condition

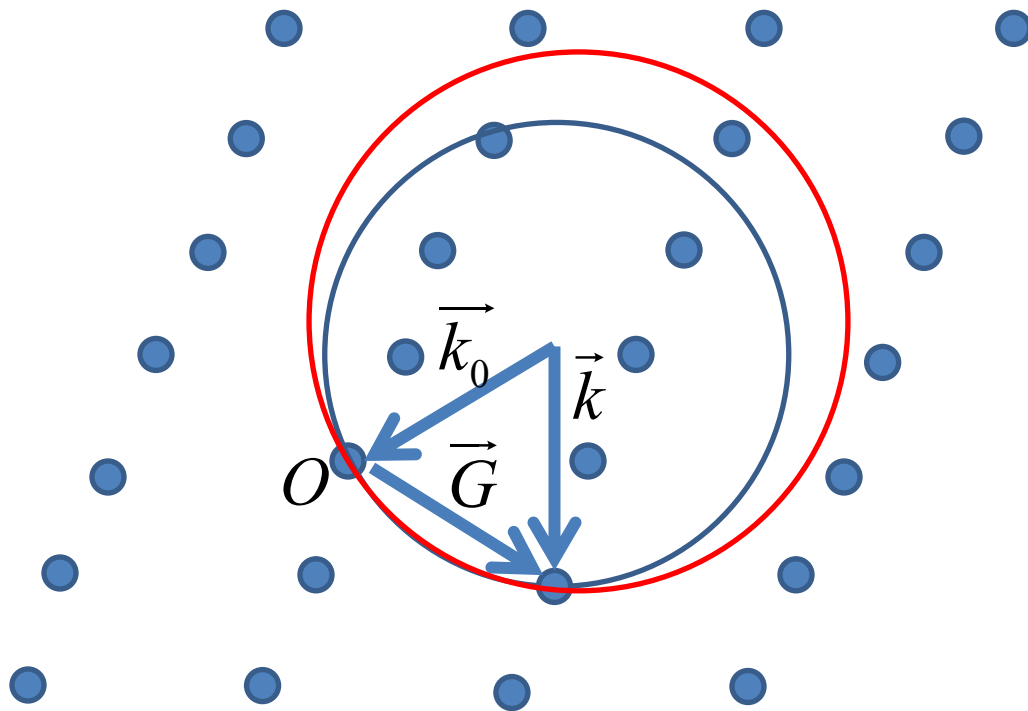
# Here there be dragons...

THE LENOX GLOBE



The Hunt-Lenox Globe, as transcribed by B.F. da Costa

# Ewald Sphere



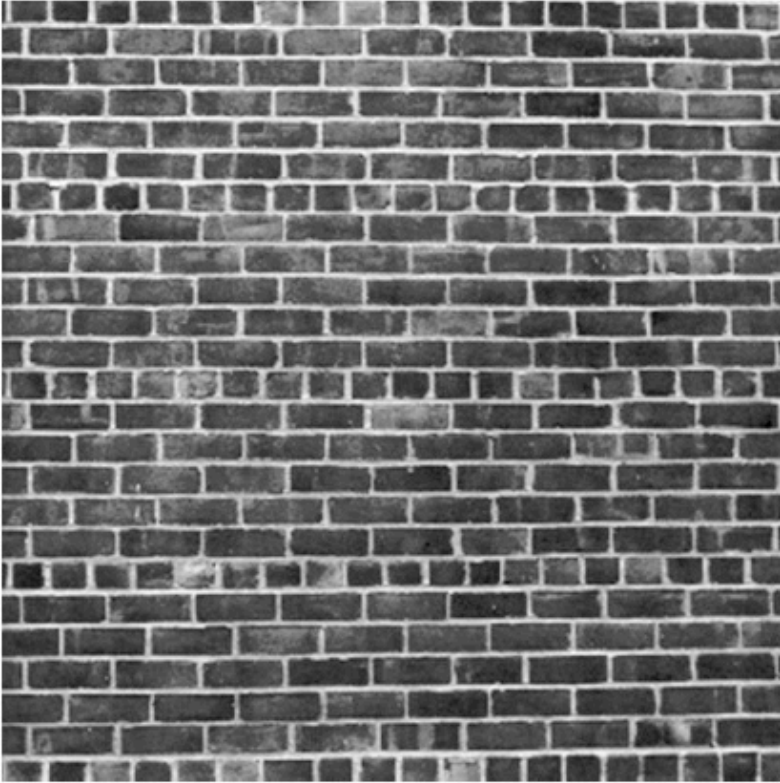
$$I(K) \propto |A_B|^2 \propto \left| \int \rho(r) e^{-i\vec{K}\cdot\vec{r}} dr \right|^2$$



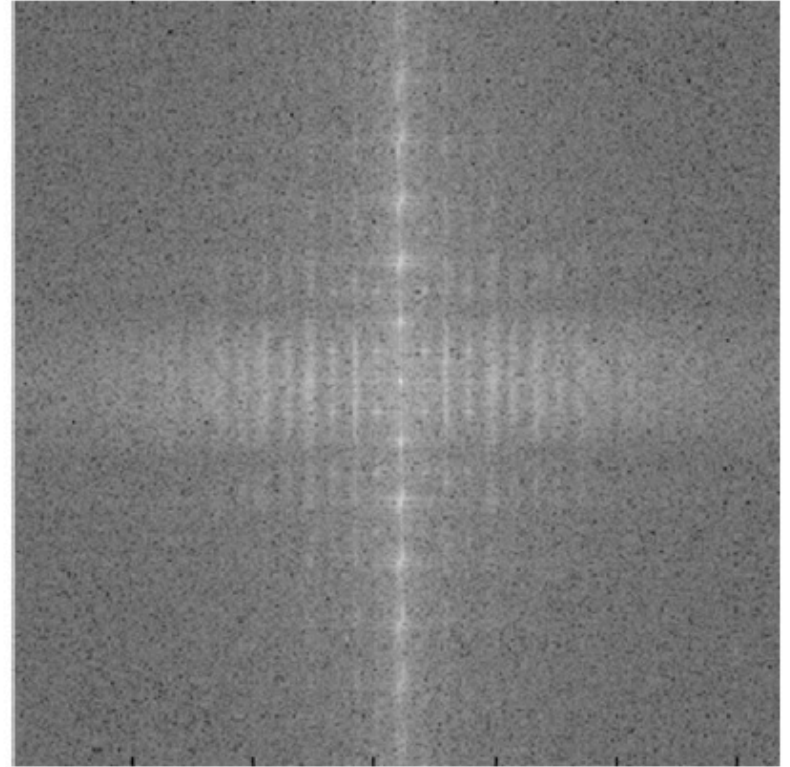
$$I(K) \propto \left| \sum_{\vec{G}} \rho_{\vec{G}}(r) \int e^{i(\vec{G}-\vec{K})\cdot\vec{r}} dr \right|^2$$

$$\int e^{i(\vec{G}-\vec{K})\cdot\vec{r}} dr = \begin{cases} V & \text{for } \vec{G} = \vec{K} \\ \sim 0 & \text{otherwise} \end{cases}$$

Laue Condition

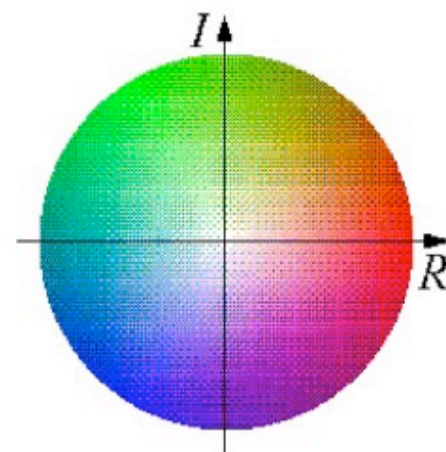
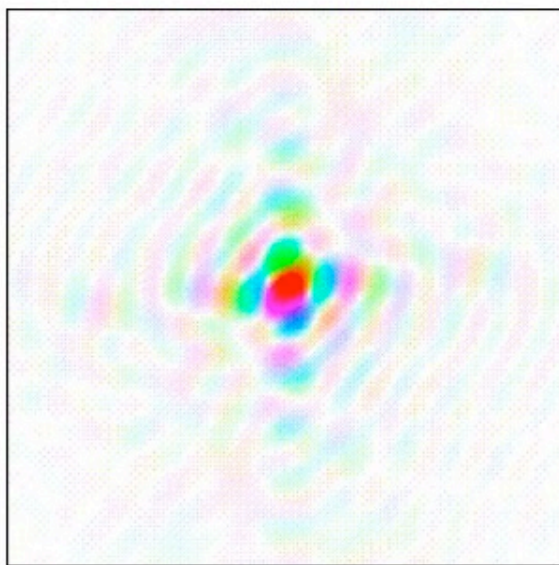
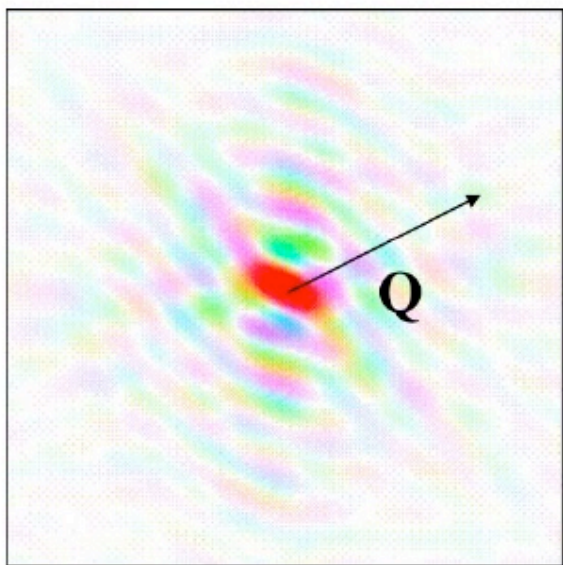
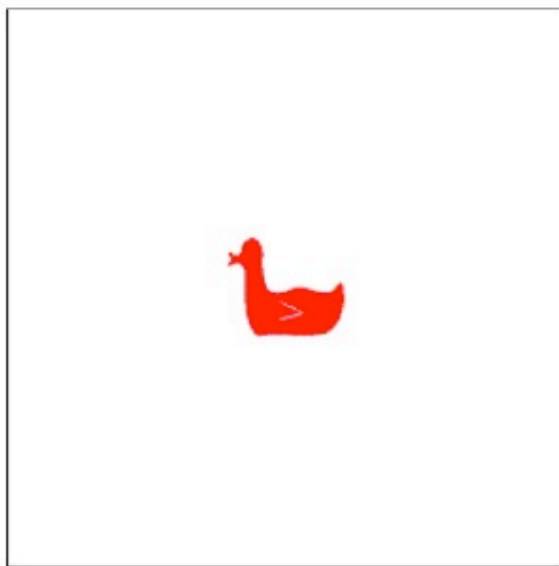
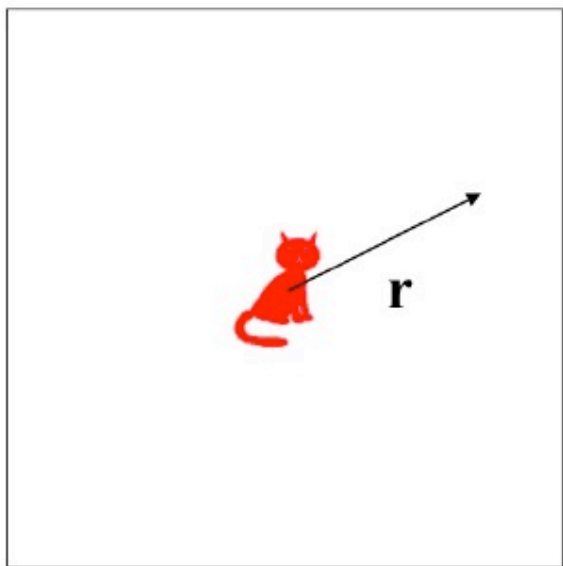


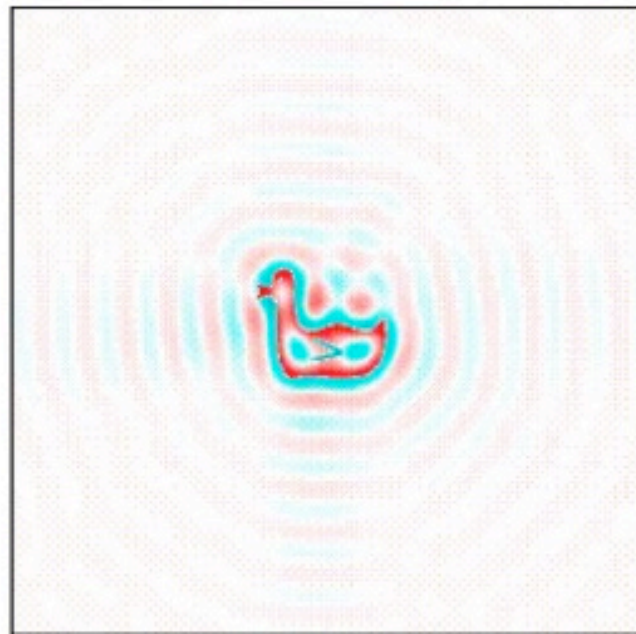
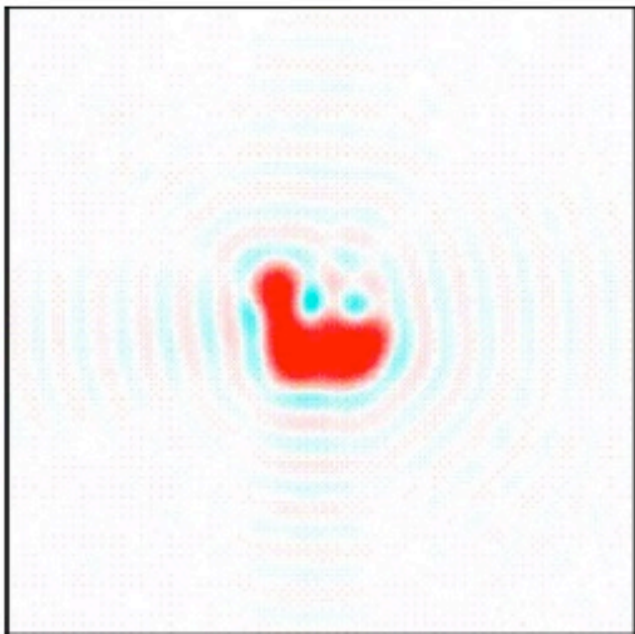
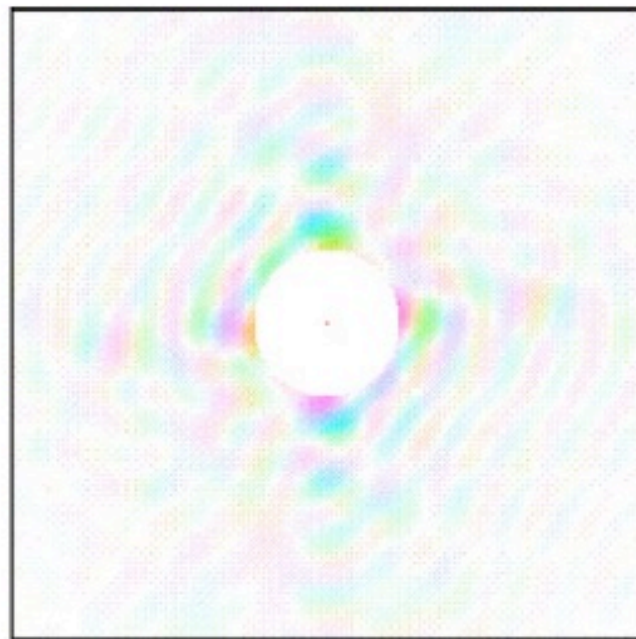
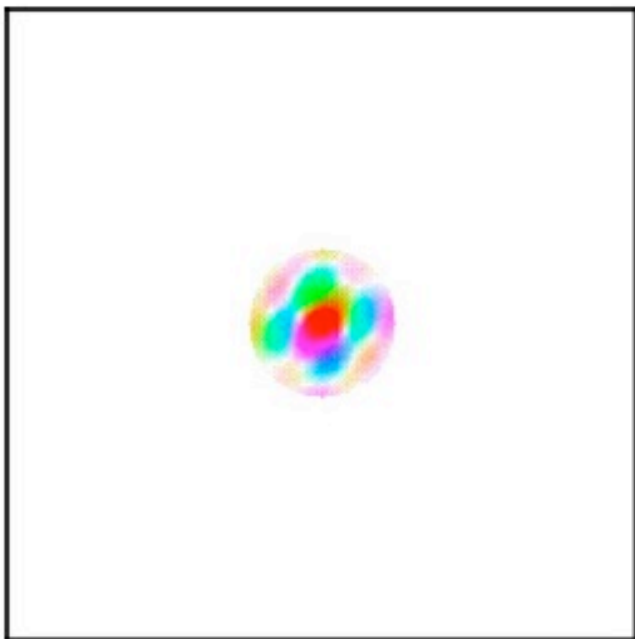
$f(x, y)$

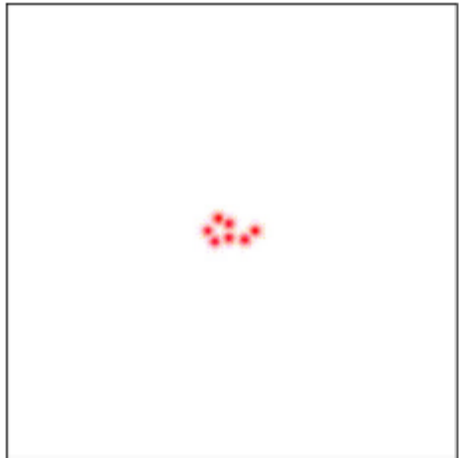


$|F(u, v)|$

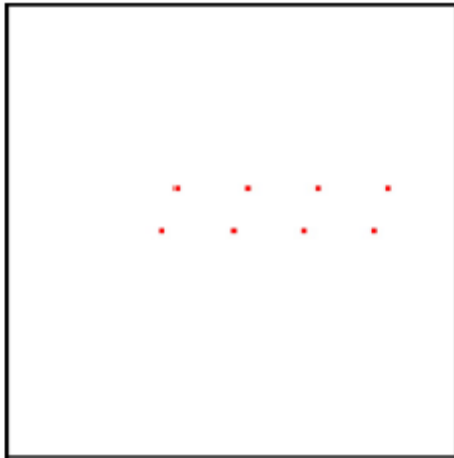




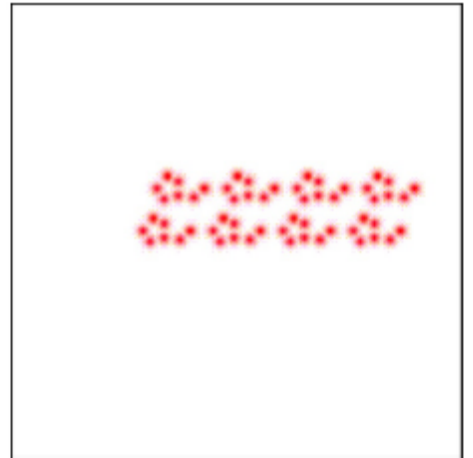


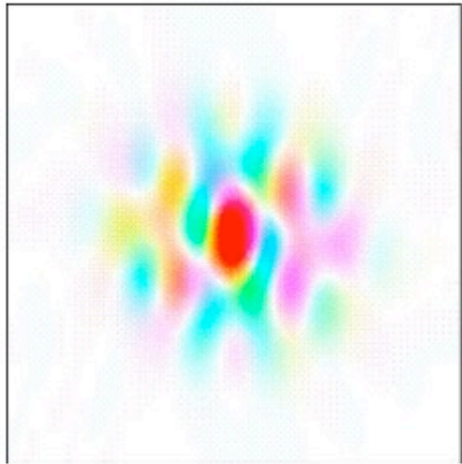


$\otimes$

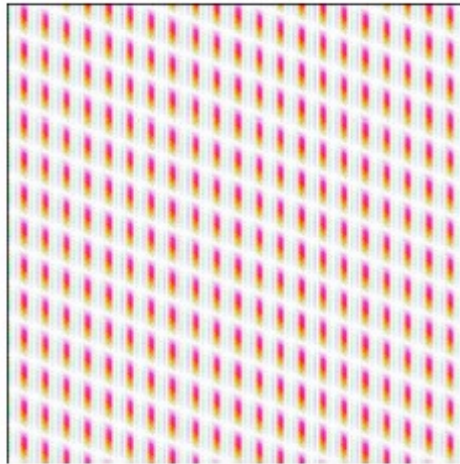


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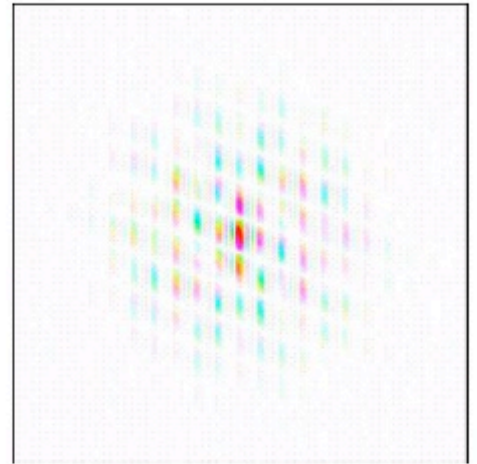


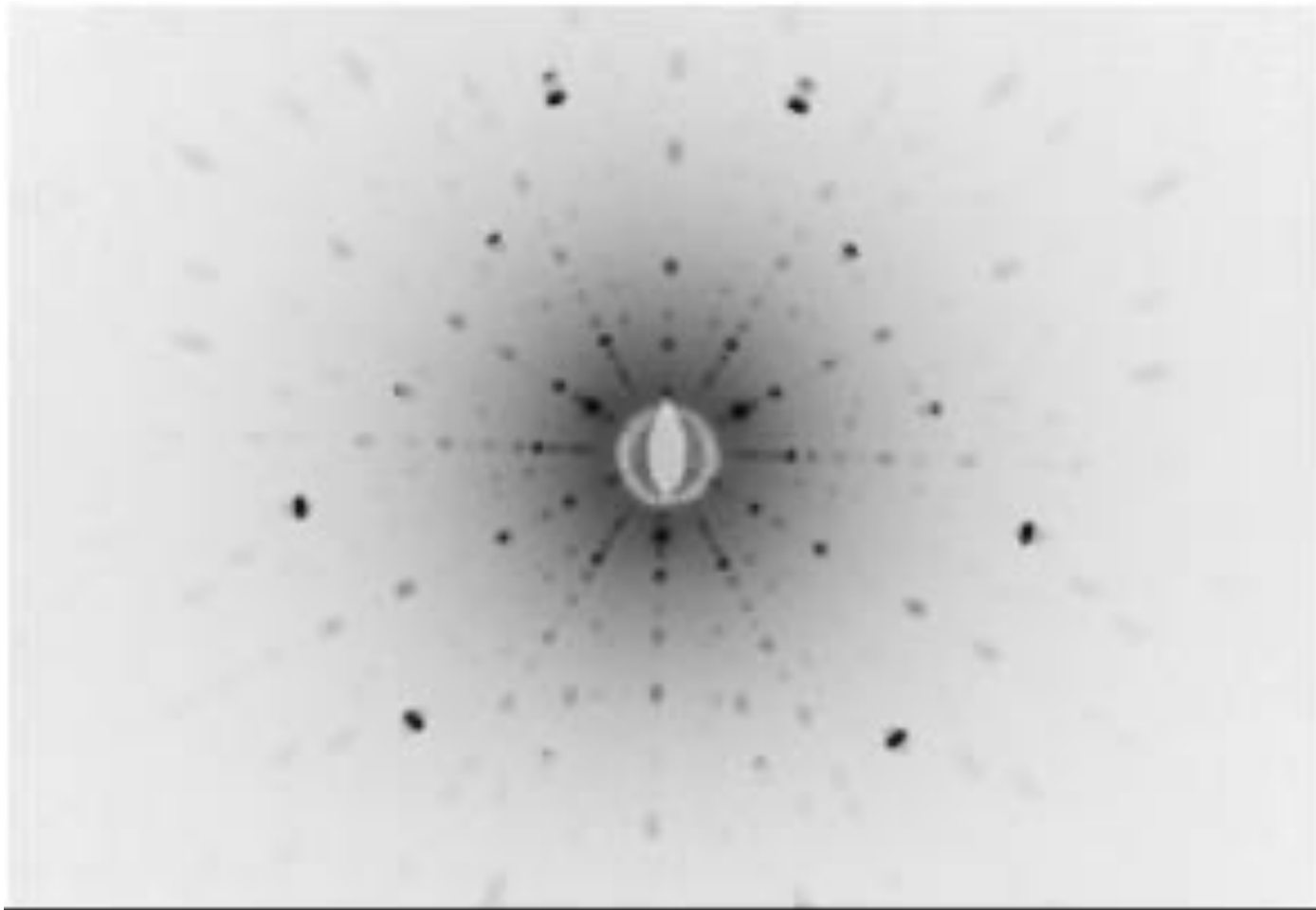


$\times$



$=$





$$I(K) \propto |A_B|^2 \propto \left| \int \rho(r) e^{-i\vec{K} \cdot \vec{r}} dr \right|^2$$



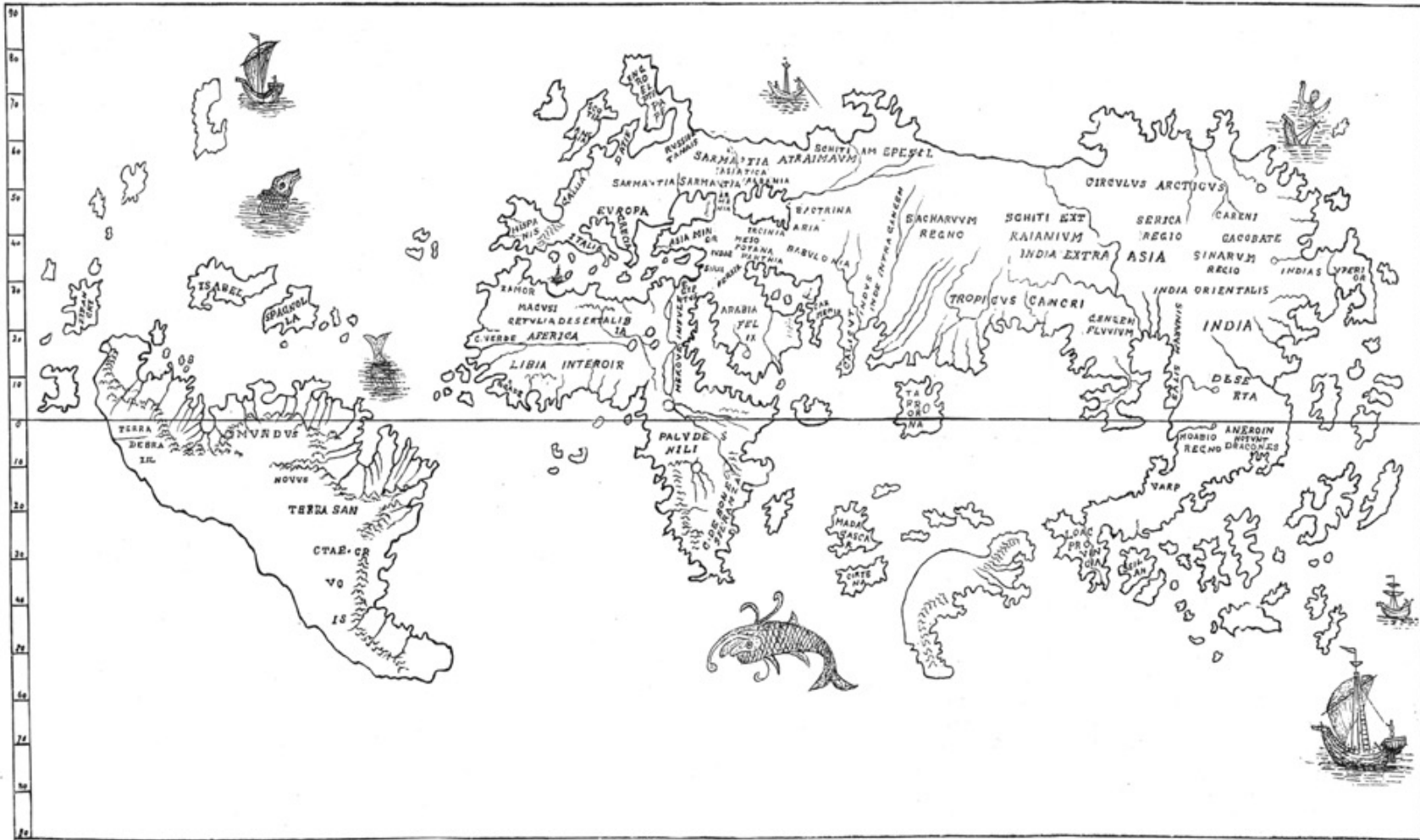
$$I(K) \propto \left| \sum_{\vec{G}} \rho_{\vec{G}}(r) \int e^{i(\vec{G}-\vec{K}) \cdot \vec{r}} dr \right|^2$$

$$\int e^{i(\vec{G}-\vec{K}) \cdot \vec{r}} dr = \begin{cases} V & \text{for } \vec{G} = \vec{K} \\ \sim 0 & \text{otherwise} \end{cases}$$

Laue Condition

# Here there be dragons...

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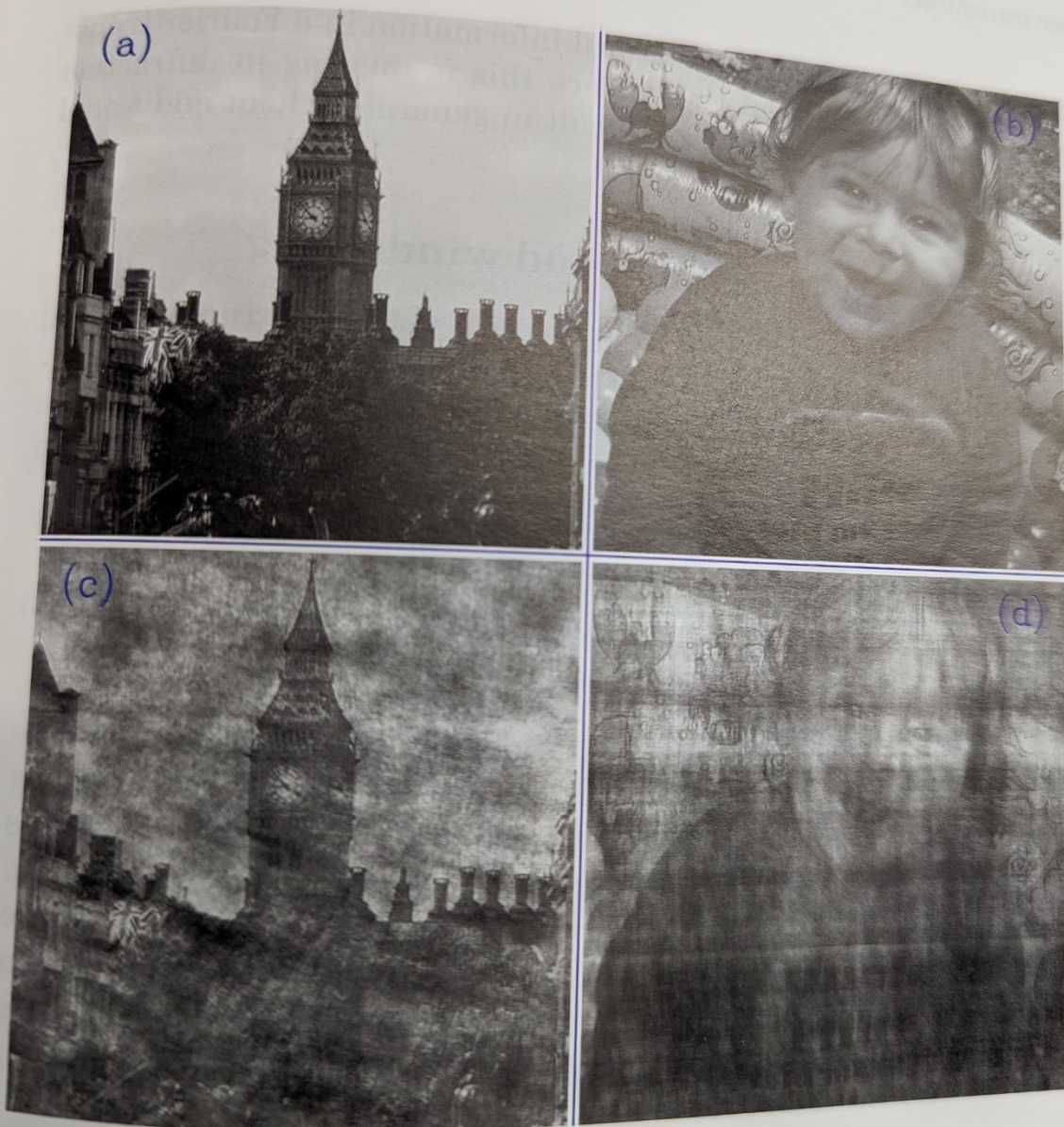


Fig. 2.20 The phase problem: (c) has the Fourier phases of (a) and the Fourier amplitudes of (b), while (d) has the phases of (b) and the amplitudes of (a).



# Intermission



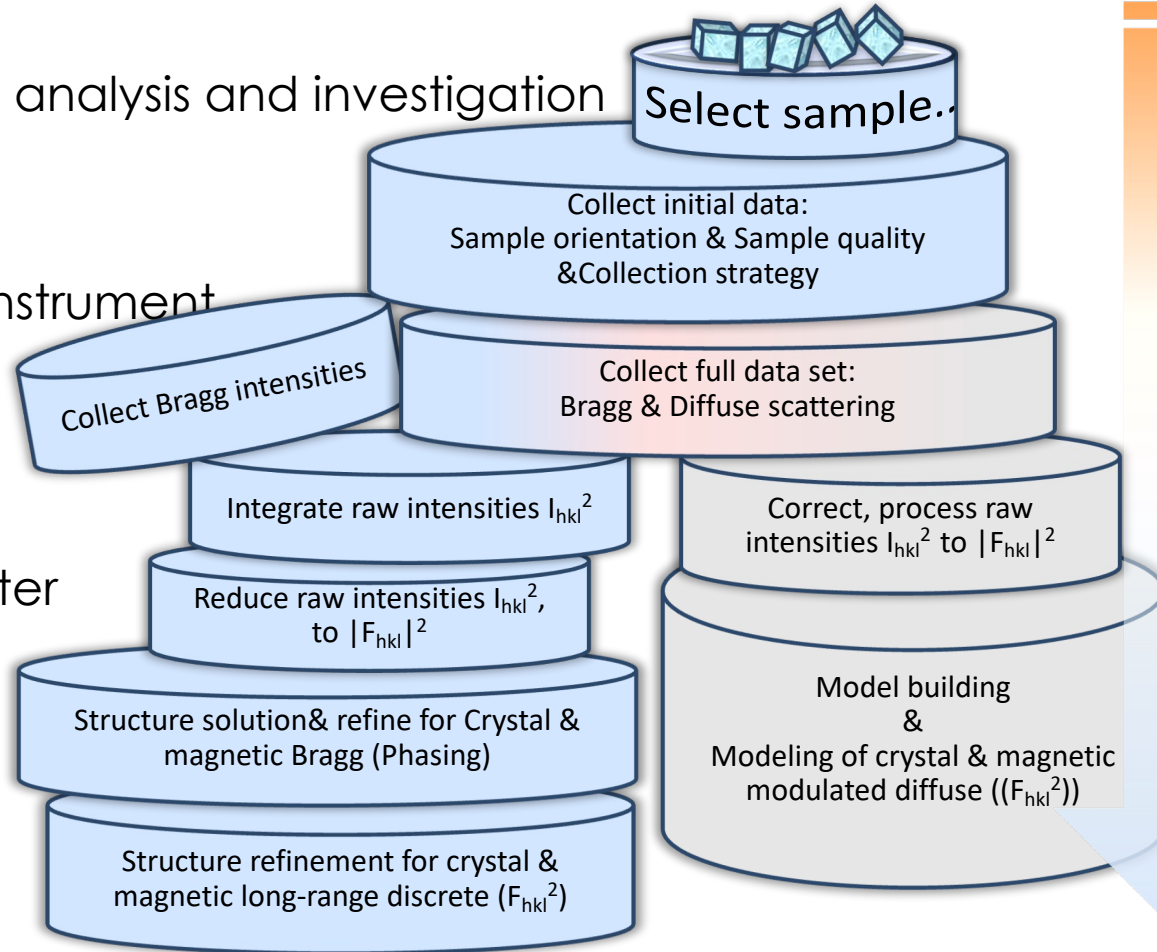


# Single Crystal Diffraction (SCD) @ TOPAZ

single crystal structure analysis and investigation

- A single crystal
- A diffraction experiment

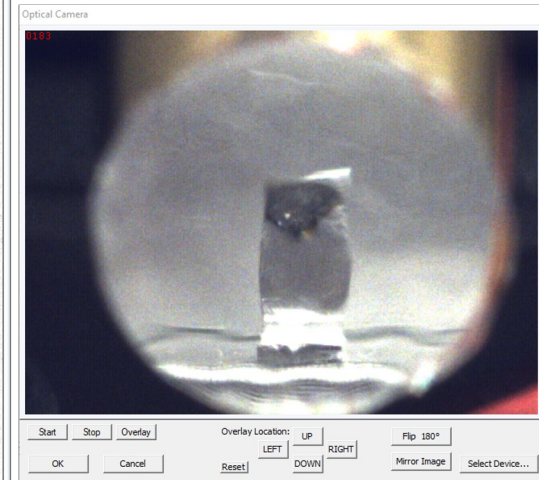
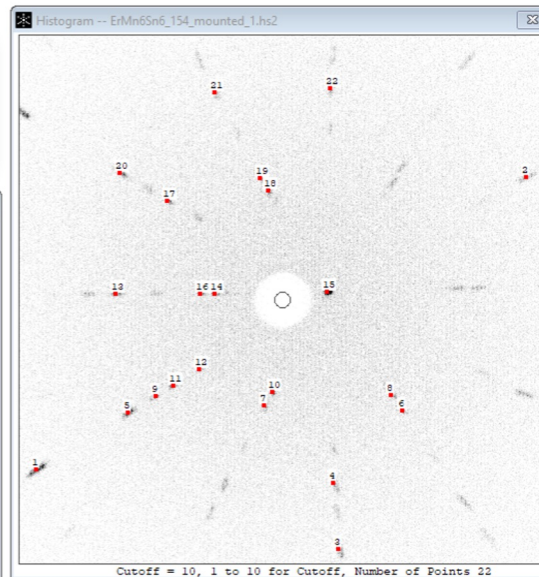
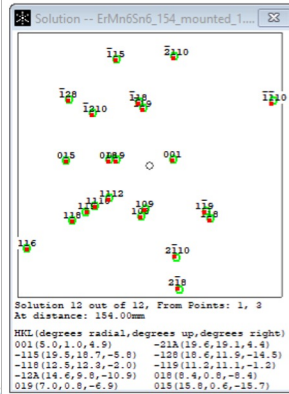
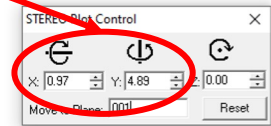
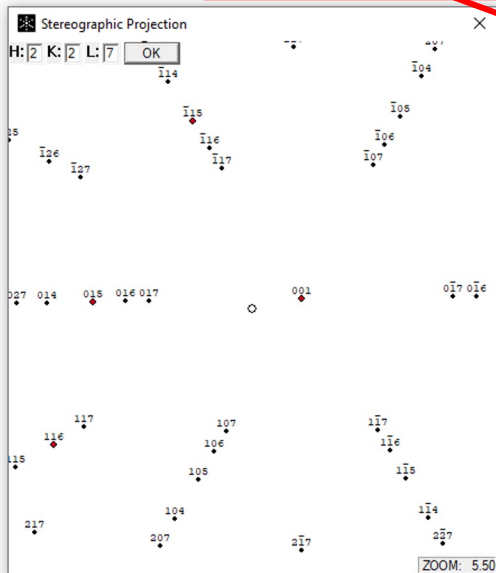
- Data collection at instrument
- Data reduction &
- Data processing  
at powerful computer
- Data analysis

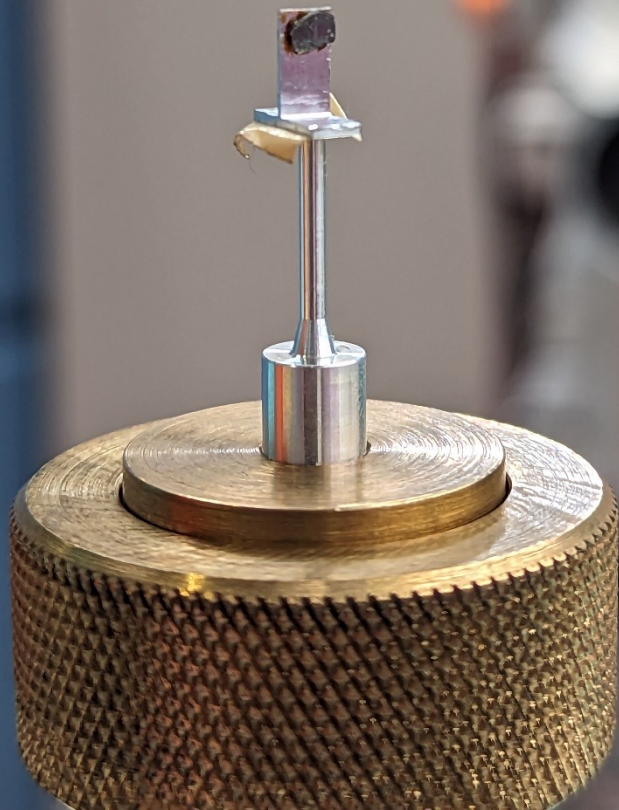




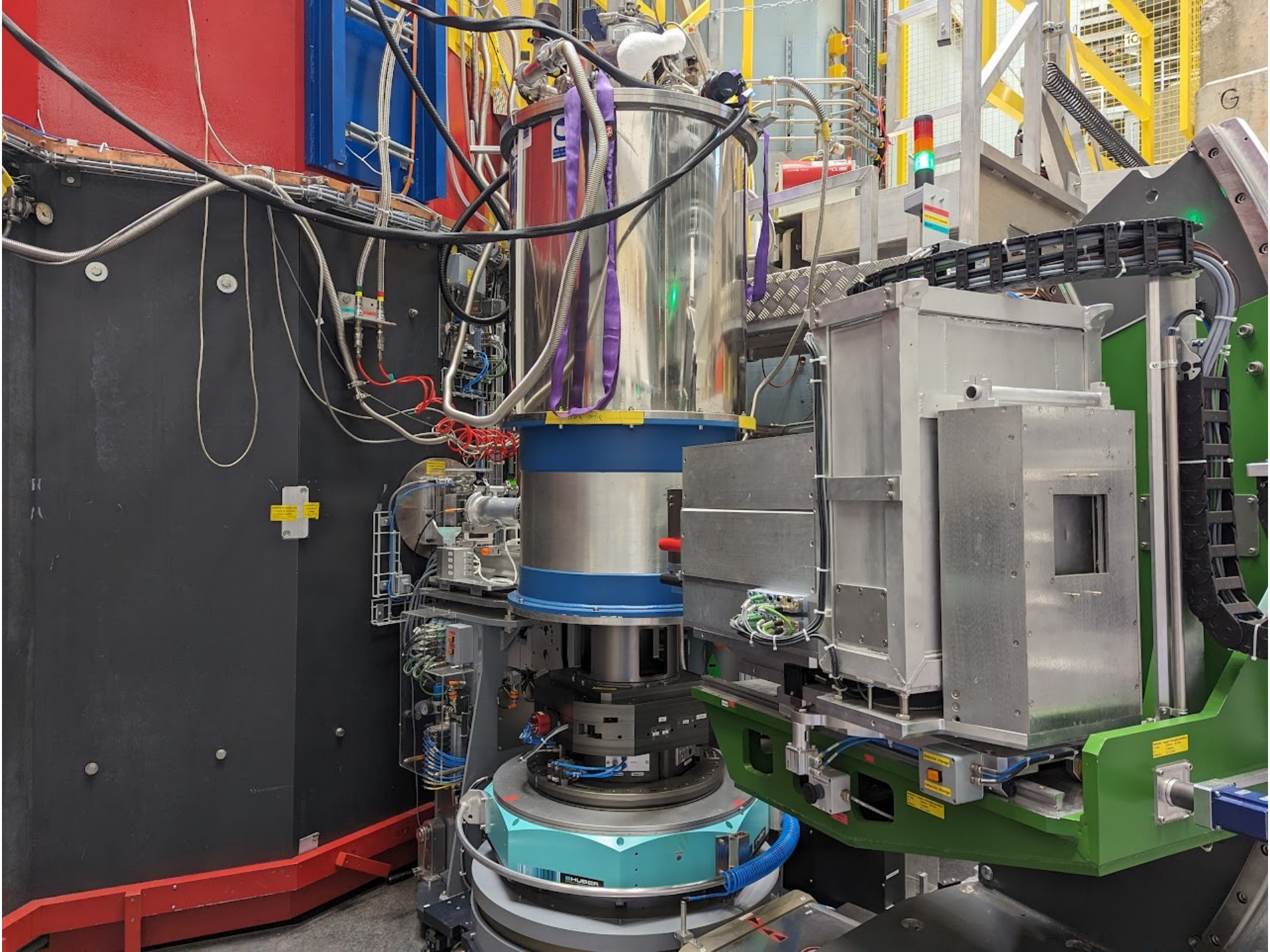
# X-ray Laue Alignment X-tal X

(001) reflection lies 0.97 degrees out of plane. Horizontal rotation is not relevant.









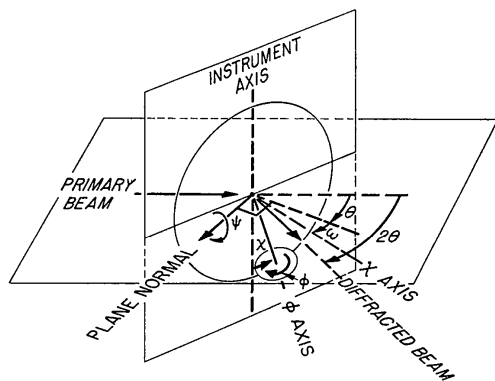


# Angle Calculations for 3- and 4- Circle X-ray and Neutron Diffractometers\*

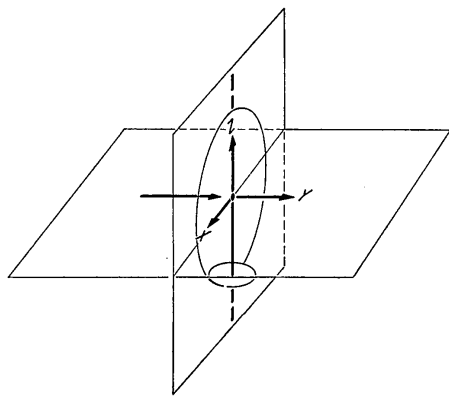
BY WILLIAM R. BUSING AND HENRI A. LEVY

*Chemistry Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830, U.S.A.*

(Received 13 June 1966)



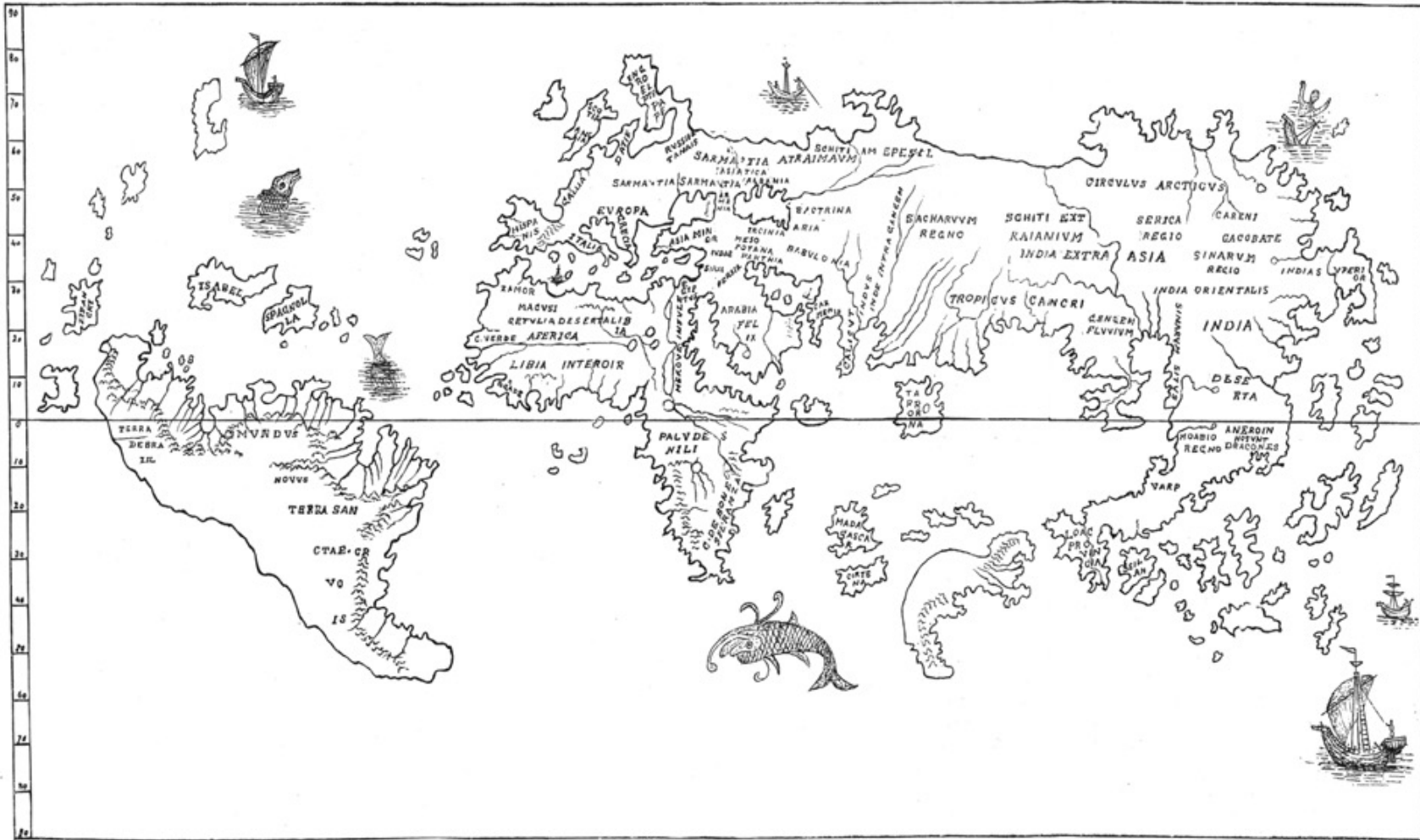
(a)



(b)

# Here there be dragons...

THE LENOX GLOBE



The Hunt-Lenox Globe, as transcribed by B.F. da Costa

# Integration and Corrections

- Raw/measured integrated intensities – background
- Volumetric data = diffuse scattering
- **Convert to structure factor amplitudes from intensities:**

– Sample dependent corrections:

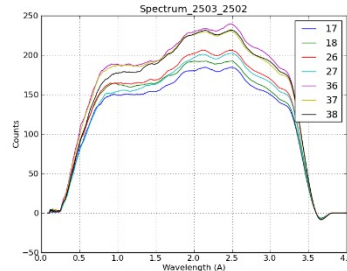
- Absorption correction
  - Density, chemical composition, volume
  - Absorption coefficient
- Path length correction  $\mu = \left[ \mu_s + \frac{\mu_a}{1.798} \times \lambda \right] \text{ cm}^{-1}$

- Lorentz correction
  - Geometric or Lambda contribution

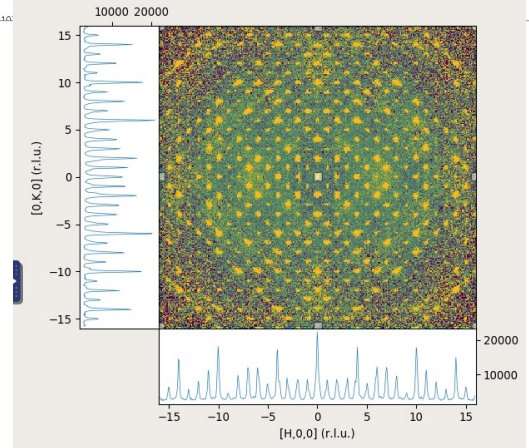
– Instrument specific corrections:

- Incident spectrum correction
- Detector efficiency correction
- Normalization
- Scaling

Isotropic scatterer  
= vanadium correction



| Version: | 2.0       | Facility: | SNS   | Instrument: | SNAP    |        |       |          |          |          |          |          |          |          |         |          |
|----------|-----------|-----------|-------|-------------|---------|--------|-------|----------|----------|----------|----------|----------|----------|----------|---------|----------|
| 6        | L1        | TO_SHIFT  |       |             |         |        |       |          |          |          |          |          |          |          |         |          |
| 7        | 1505.5382 | 4.168     |       |             |         |        |       |          |          |          |          |          |          |          |         |          |
| 4        | DETNUM    | NROWS     | NCOLS | WIDTH       | HEIGHT  | DEPTH  | DETD  | CenterX  | CenterY  | CenterZ  | BaseX    | BaseY    | BaseZ    | UpX      | UpY     | UpZ      |
| 5        | 10        | 256       | 256   | 16.1725     | 16.2027 | 0.2000 | 55.26 | -50.0977 | 16.6157  | -16.3538 | -0.00594 | -0.00142 | -0.99998 | -0.00179 | 1.00000 | -0.00137 |
| 5        | 11        | 256       | 256   | 16.1141     | 16.0675 | 0.2000 | 52.69 | -49.9941 | 16.6391  | 0.4007   | -0.00623 | -0.00138 | -0.99998 | -0.00194 | 1.00000 | -0.00137 |
| 5        | 12        | 256       | 256   | 16.0775     | 16.1339 | 0.2000 | 55.33 | -49.8906 | 16.6626  | 17.1552  | -0.00637 | -0.00140 | -0.99998 | -0.00189 | 1.00000 | -0.00139 |
| 5        | 13        | 256       | 256   | 16.1362     | 16.1297 | 0.2000 | 52.66 | -50.0661 | -0.1391  | -16.3305 | -0.00604 | -0.00142 | -0.99998 | -0.00194 | 1.00000 | -0.00141 |
| 5        | 14        | 256       | 256   | 16.0790     | 16.1810 | 0.2000 | 49.96 | -49.9625 | -0.1156  | 0.4239   | -0.00623 | -0.00140 | -0.99998 | -0.00189 | 1.00000 | -0.00139 |
| 5        | 15        | 256       | 256   | 16.1808     | 16.0856 | 0.2000 | 52.74 | -49.8589 | -0.0922  | 17.1784  | -0.00613 | -0.00138 | -0.99998 | -0.00203 | 1.00000 | -0.00137 |
| 5        | 16        | 256       | 256   | 16.1341     | 16.2021 | 0.2000 | 55.27 | -50.0244 | -16.8588 | -16.3073 | -0.00618 | -0.00133 | -0.99998 | -0.00199 | 1.00000 | -0.00132 |
| 5        | 17        | 256       | 256   | 16.1124     | 16.1506 | 0.2000 | 52.71 | -49.9308 | -16.8704 | 0.4472   | -0.00618 | -0.00142 | -0.99998 | -0.00194 | 1.00000 | -0.00141 |
| 5        | 18        | 256       | 256   | 16.0624     | 16.1940 | 0.2000 | 55.34 | -49.8272 | -16.8469 | 17.2016  | -0.00604 | -0.00142 | -0.99998 | -0.00184 | 1.00000 | -0.00141 |



# Data reduction – single crystal

Convert raw integrated intensities,  $I_{hkl}$ , into relative structure factor amplitudes,  $|F_{hkl}|^2$ .

$$I_{hkl} \sim |F_{hkl}|^2$$

TOF Laue:

Lorentz factor

$$I_{hkl} = k \phi(\lambda) \varepsilon(\lambda) A(\lambda) y(\lambda) (V_s / V_c^2) |F_{hkl}|^2 \lambda^4 / \sin^2 \theta$$

Constant Wavelength:

$$I_{hkl} = k \phi(\lambda) \varepsilon(\lambda) A(\lambda) y(\lambda) (V_s / V_c^2) |F_{hkl}|^2 \lambda^3 / \sin 2\theta$$

$k$  = scale factor

$\phi(\lambda)$  = incident flux spectrum

$\varepsilon(\lambda)$  = detector efficiency as a function of wavelength  $\lambda$

$A(\lambda)$  = sample absorption

$y(\lambda)$  = secondary extinction correction

$V_s$  = sample volume

$V_c$  = unit cell volume

$F_{hkl}$  = structure factor

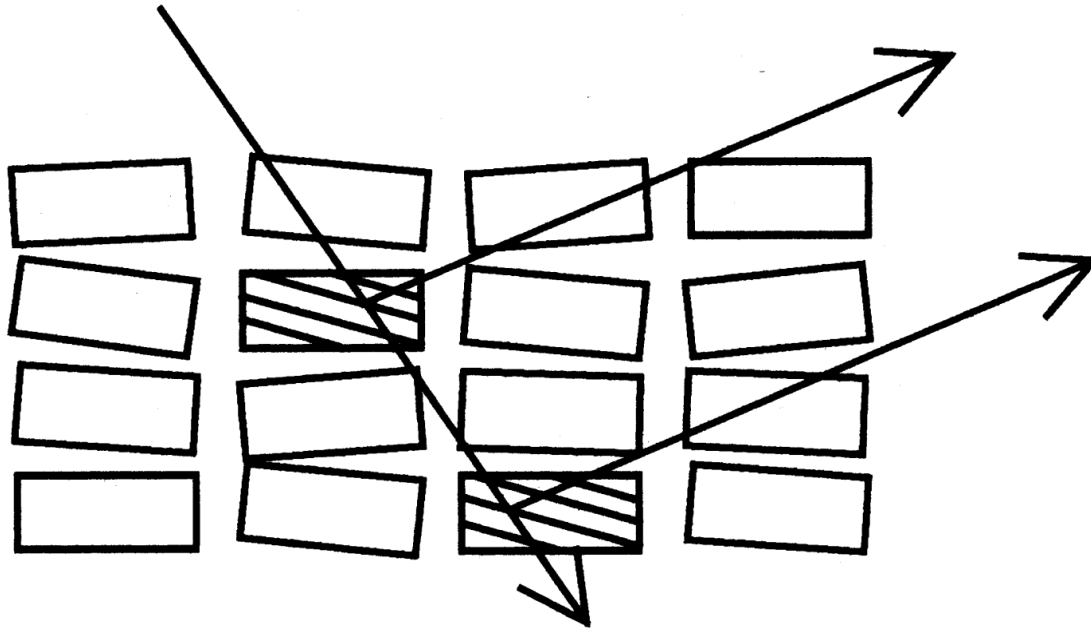
$\lambda$  = wavelength

| h  | k  | l   | $ F_{hkl} ^2$ | $\sigma_{( F_{hkl} ^2)}$ | batch | $\lambda$ | ...     |          |          |         |   |
|----|----|-----|---------------|--------------------------|-------|-----------|---------|----------|----------|---------|---|
| -2 | 0  | -4  | 4298.42       | 193.58                   | 1     | 2.50887   | 0.09279 | -0.73894 | -0.42849 | 0.58255 | 0 |
| -3 | 1  | -5  | 589.48        | 60.36                    | 1     | 1.96898   | 0.09435 | -0.73894 | -0.56656 | 0.58255 | 0 |
| -3 | -1 | -5  | 686.79        | 61.36                    | 1     | 1.79364   | 0.09198 | -0.73894 | -0.45863 | 0.58255 | 0 |
| -4 | 0  | -8  | 788.98        | 70.91                    | 1     | 1.25484   | 0.09281 | -0.73894 | -0.42849 | 0.58255 | 0 |
| -4 | 0  | -6  | 73.17         | 21.31                    | 1     | 1.50857   | 0.09348 | -0.73894 | -0.55873 | 0.58255 | 0 |
| -4 | 0  | -5  | 35.24         | 16.30                    | 1     | 1.65971   | 0.09375 | -0.73894 | -0.63859 | 0.58255 | 0 |
| -5 | 1  | -11 | 2530.43       | 144.21                   | 1     | 0.95726   | 0.09317 | -0.73894 | -0.48875 | 0.58255 | 0 |
| -5 | 1  | -10 | 522.83        | 63.81                    | 1     | 1.02775   | 0.09350 | -0.73894 | -0.45720 | 0.58255 | 0 |
| -5 | 1  | -9  | 2122.75       | 125.25                   | 1     | 1.10622   | 0.09384 | -0.73894 | -0.50912 | 0.58255 | 0 |
| -5 | -1 | -9  | 2180.89       | 127.34                   | 1     | 1.04810   | 0.09232 | -0.73894 | -0.44370 | 0.58255 | 0 |
| -5 | 1  | -8  | 679.26        | 65.36                    | 1     | 1.19332   | 0.09418 | -0.73894 | -0.56573 | 0.58255 | 0 |
| -5 | -1 | -8  | 555.22        | 58.64                    | 1     | 1.12724   | 0.09256 | -0.73894 | -0.49355 | 0.58255 | 0 |
| -5 | 1  | -7  | 2152.15       | 126.31                   | 1     | 1.28935   | 0.09446 | -0.73894 | -0.62574 | 0.58255 | 0 |
| -5 | -1 | -7  | 2153.33       | 126.42                   | 1     | 1.21387   | 0.09276 | -0.73894 | -0.54506 | 0.58255 | 0 |
| -6 | 0  | -12 | 245.58        | 68.94                    | 1     | 0.83602   | 0.09280 | -0.73894 | -0.42849 | 0.58255 | 0 |
| -6 | 2  | -11 | 3080.88       | 173.37                   | 1     | 0.92416   | 0.09430 | -0.73894 | -0.51903 | 0.58255 | 0 |
| -6 | -2 | -11 | 3360.52       | 193.75                   | 1     | 0.84529   | 0.09168 | -0.73894 | -0.41194 | 0.58255 | 0 |
| -6 | 2  | -10 | 2341.73       | 138.04                   | 1     | 0.98449   | 0.09458 | -0.73894 | -0.56656 | 0.58255 | 0 |
| -6 | 0  | -10 | 3770.93       | 187.90                   | 1     | 0.94440   | 0.09329 | -0.73894 | -0.51311 | 0.58255 | 0 |
| -6 | -2 | -10 | 2515.08       | 152.95                   | 1     | 0.89682   | 0.09186 | -0.73894 | -0.45863 | 0.58255 | 0 |
| -6 | 2  | -9  | 283.57        | 52.43                    | 1     | 1.05015   | 0.09486 | -0.73894 | -0.61690 | 0.58255 | 0 |
| -6 | -2 | -9  | 306.43        | 53.69                    | 1     | 0.95238   | 0.09204 | -0.73894 | -0.49076 | 0.58255 | 0 |
| -6 | 0  | -8  | 673.79        | 65.07                    | 1     | 1.07177   | 0.09374 | -0.73894 | -0.60763 | 0.58255 | 0 |
| -7 | 1  | -12 | 521.36        | 93.17                    | 1     | 0.81034   | 0.09380 | -0.73894 | -0.52385 | 0.58255 | 0 |
| -7 | -1 | -12 | 352.75        | 85.24                    | 1     | 0.77900   | 0.09265 | -0.73894 | -0.47567 | 0.58255 | 0 |
| -7 | 3  | -11 | 531.37        | 85.59                    | 1     | 0.88442   | 0.09511 | -0.73894 | -0.60876 | 0.58255 | 0 |
| -7 | 1  | -11 | 715.23        | 99.60                    | 1     | 0.85555   | 0.09404 | -0.73894 | -0.56432 | 0.58255 | 0 |
| -7 | -1 | -11 | 710.42        | 98.91                    | 1     | 0.82117   | 0.09282 | -0.73894 | -0.51235 | 0.58255 | 0 |
| -7 | -3 | -11 | 511.19        | 94.04                    | 1     | 0.78192   | 0.09148 | -0.73894 | -0.45254 | 0.58255 | 0 |
| -7 | 1  | -10 | 272.83        | 65.57                    | 1     | 0.90307   | 0.09427 | -0.73894 | -0.60506 | 0.58255 | 0 |

# Absorption

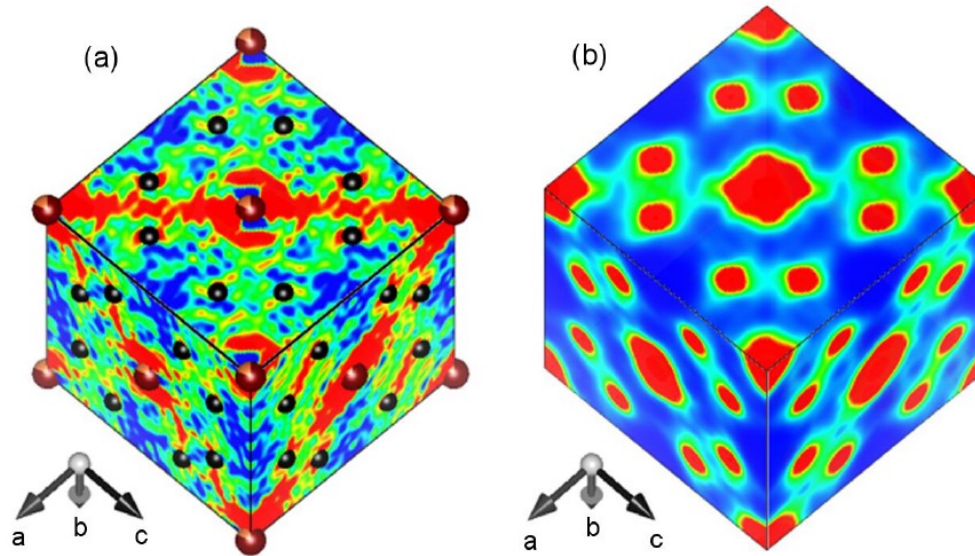
$$I = I_0 e^{-\mu t}$$

# Secondary Extinction



Single crystal time-of-flight neutron diffraction, J Peters and W. Jauch, Science Progress 85 (2002), pp. 297-317

# Structure Solution



**Figure 3.18** (a) Difference Fourier and (b) maximum-entropy-method maps of  $\text{Tm}_{0.19}\text{Yb}_{0.81}\text{B}_{12}$  are created in (100), (010), (001) faces of the unit cell. Electron density ( $g$ ) in the layer of any given thickness is automatically divided into several levels from  $g_{\min}$  to  $g_{\max}$ , each of them is assigned to a definite color from dark-blue over green to red. The values of  $g_{\text{MEM}}$  are cut at the level  $g_{\max} = 0.075\%$  of the maximal  $g_{\text{MEM}}$  value to show fine electron-density gradations in the thin layer. Difference electron-density values are cut at  $\pm 0.5 e/\text{\AA}^3$  [56].

# Intermission



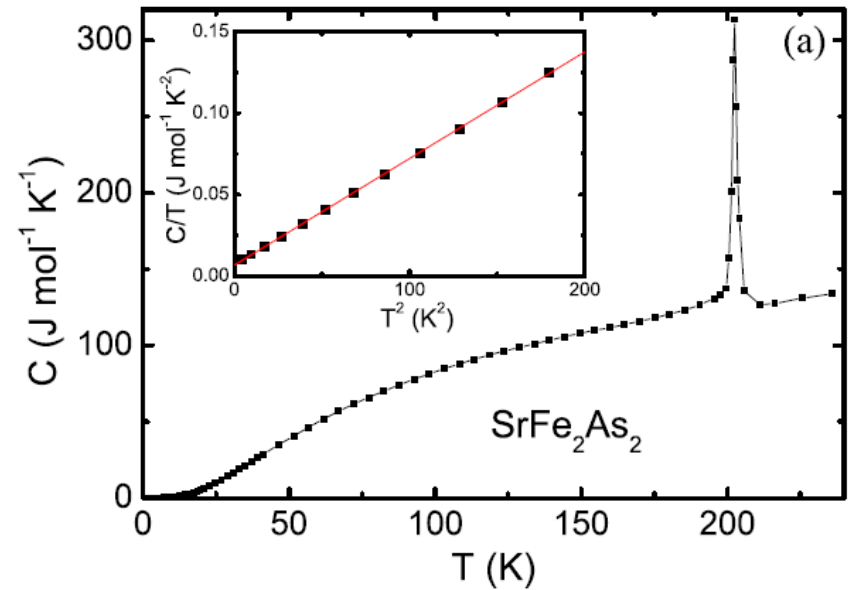
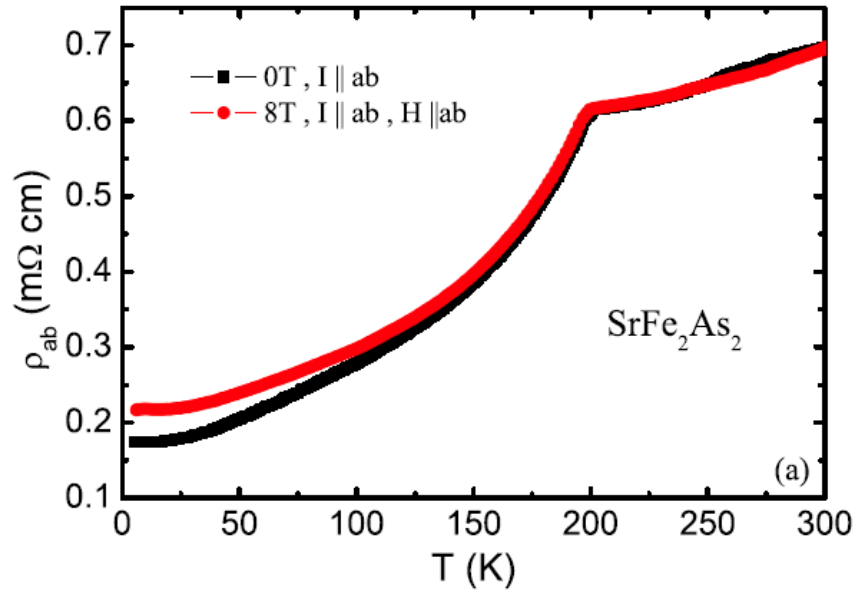




# How it starts?

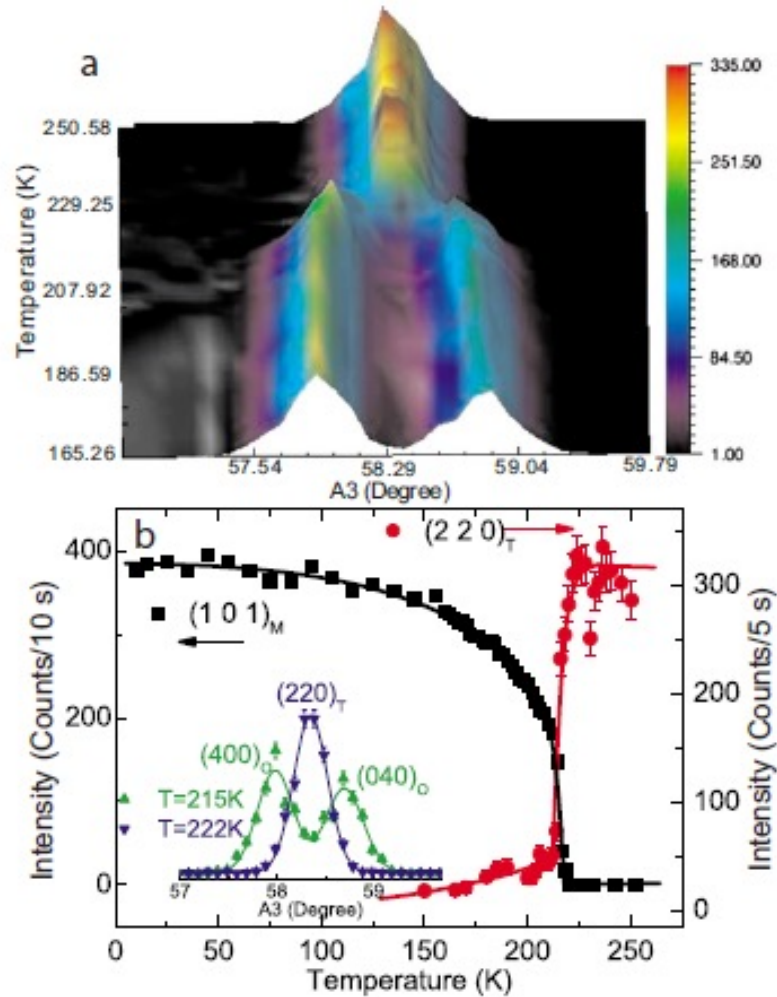


# Bulk Data Comes In



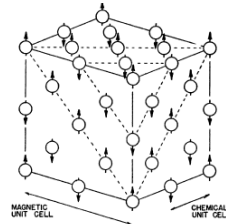


# Neutrons to the Rescue



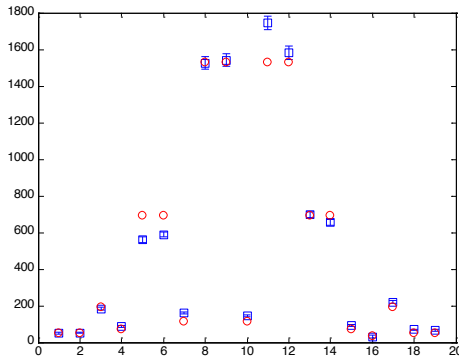
Physical Review B **78**, 140504 (2008)

# Guess and Check (Refinement)

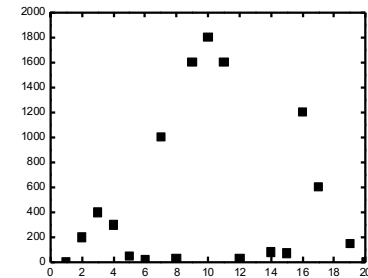


Update model

Predict Intensities from model



Compare predicted intensities to data



# Powder Diffraction

## Advantages

- You get the big picture
- Can get the propagation vector
- Avoids the muss and fuss of extinction
- It's often Good Enough™

## Disadvantages

- Can be hard to truly index  $k$ — is it  $[3\ 4\ 0]$  or  $[0\ 0\ 5]$ ?
- You average over all symmetry equivalent  $k$  at any particular Bragg angle
- You lose information in the powder averaging
- No domain info
- No multi- $k$  info
- Can be very hard to determine phase

# Single Crystal Diffraction

## Advantages

- Can fully determine  $\mathbf{k}$
- Can investigate domain populations
- Can apply probes (magnetic field, E-field, pressure, etc.) along a particular direction to see effect on magnetic ordering

## Disadvantages

- Extinction
- Absorption depends on shape
- Reciprocal space is large...
- Crystal growth is hard...



Questions?

NXS Lecture - Single Crystal  
Diffraction - William Ratcliff



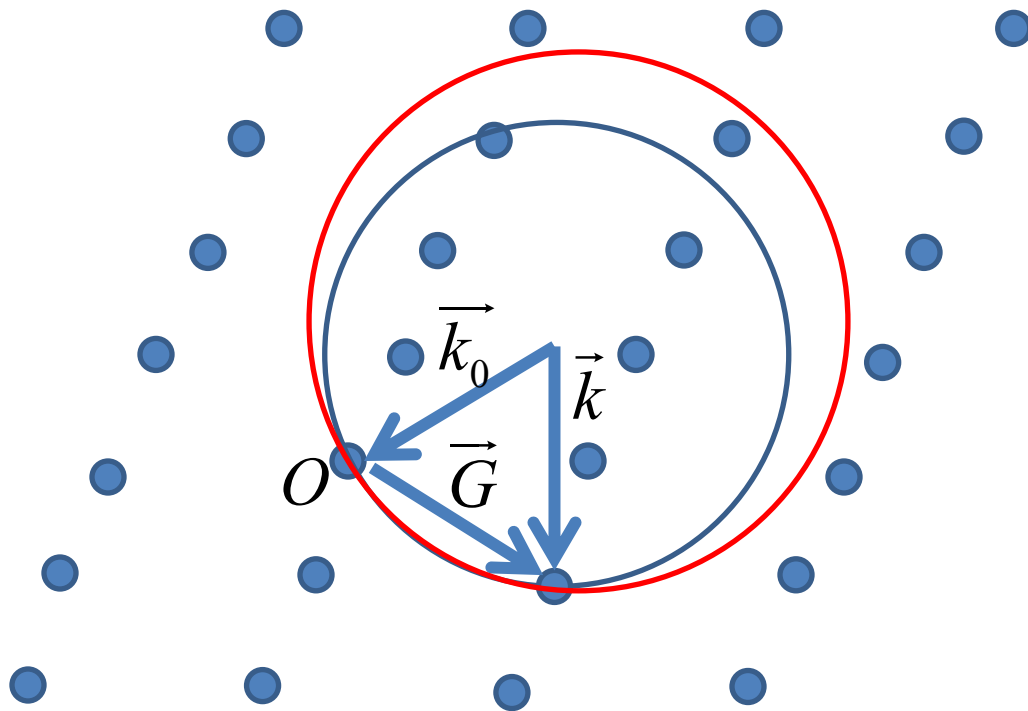
# Here there be dragons...

THE LENOX GLOBE



The Hunt-Lenox Globe, as transcribed by B.F. da Costa

# Ewald Sphere



# Intermission



# How I proceed

- Think about the problem
- Powder diffraction
- Think some more
- Try Representational Analysis (or Group theory)
- Single crystal diffraction
- Think a lot!!!
- Polarized diffraction
- Spherical polarimetry
- Think some more...

# YMn<sub>2</sub>O<sub>5</sub>

PRL 96, 097601 (2006)

PHYSICAL REVIEW LETTERS

week ending  
10 MARCH 2006

## Ferroelectricity Induced by Acentric Spin-Density Waves in YMn<sub>2</sub>O<sub>5</sub>

L. C. Chapon,<sup>1</sup> P. G. Radaelli,<sup>1,2</sup> G. R. Blake,<sup>1,3</sup> S. Park,<sup>4</sup> and S.-W. Cheong<sup>4</sup>

Powder

Journal of the Physical Society of Japan  
Vol. 76, No. 7, July, 2007, 074706  
©2007 The Physical Society of Japan

## Spiral Spin Structure in the Commensurate Magnetic Phase of Multiferroic RMn<sub>2</sub>O<sub>5</sub>

Hiroyuki KIMURA\*, Satoru KOBAYASHI<sup>1</sup>, Yoshikazu FUKUDA, Toshihiro OSAWA,  
Youichi KAMADA, Yukio NODA, Isao KAGOMIYA<sup>2</sup>, and Kay KOHN<sup>3</sup>

xtal

PHYSICAL REVIEW B 78, 245115 (2008)

## Spiral spin structures and origin of the magnetoelectric coupling in YMn<sub>2</sub>O<sub>5</sub>

J.-H. Kim,<sup>1</sup> S.-H. Lee,<sup>1,\*</sup> S. I. Park,<sup>2</sup> M. Kenzelmann,<sup>3</sup> A. B. Harris,<sup>4</sup> J. Schefer,<sup>3</sup> J.-H. Chung,<sup>5</sup> C. F. Majkrzak,<sup>6</sup>  
M. Takeda,<sup>7</sup> S. Wakimoto,<sup>7</sup> S. Y. Park,<sup>8</sup> S.-W. Cheong,<sup>8</sup> M. Matsuda,<sup>7</sup> H. Kimura,<sup>9</sup> Y. Noda,<sup>9</sup> and K. Kakurai<sup>7</sup>

Xtal+spherical polarimetry

PHYSICAL REVIEW B 79, 020404(R) (2009)

## Incommensurate magnetic structure of YMn<sub>2</sub>O<sub>5</sub>: A stringent test of the multiferroic mechanism

P. G. Radaelli,<sup>1,2</sup> C. Vecchini,<sup>1,3</sup> L. C. Chapon,<sup>1</sup> P. J. Brown,<sup>4</sup> S. Park,<sup>5</sup> and S.-W. Cheong<sup>5</sup>

Xtal+more representation analysis

# The Diffraction of Neutrons by Crystalline Powders

E. O. WOLLAN AND C. G. SHULL

*Oak Ridge National Laboratory, Oak Ridge, Tennessee*

(Received January 5, 1948)

