Argonne NATIONAL LABORATORY

Synchrotron Radiation: Production & Properties

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National School for Neutron

and X-ray Scattering

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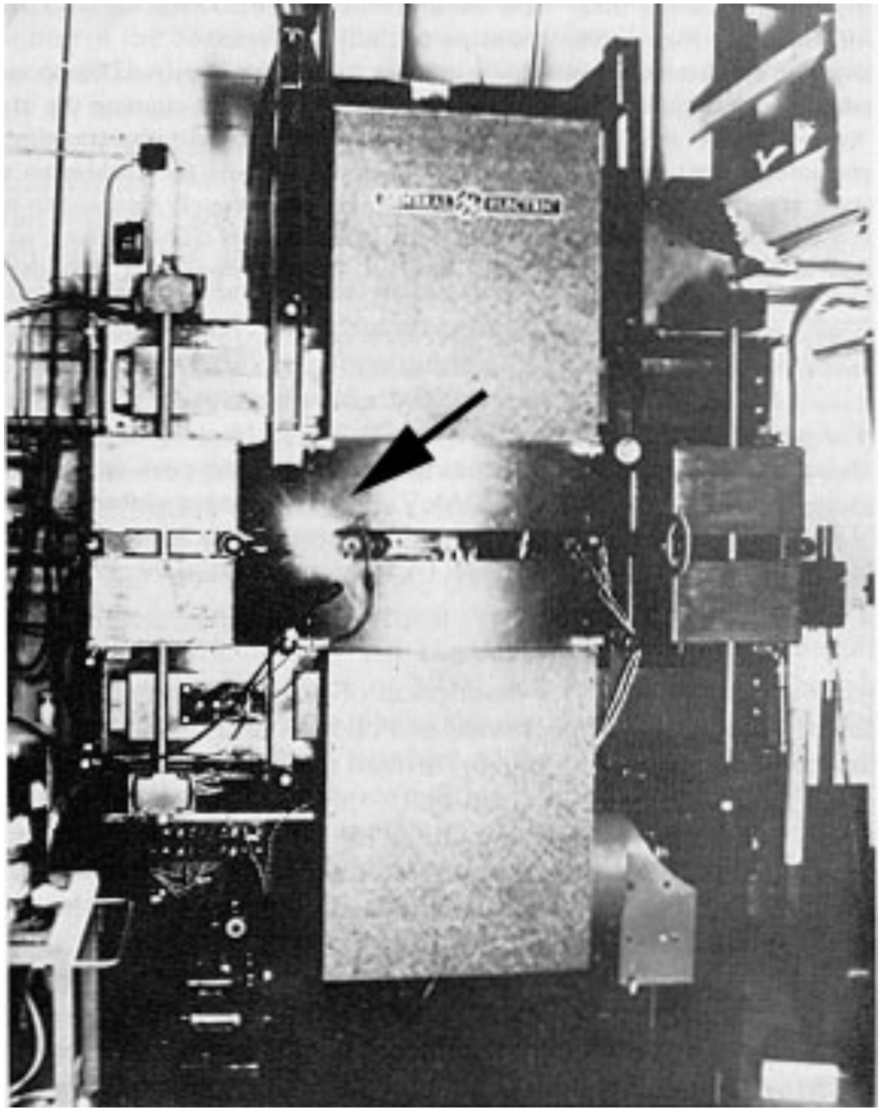
Synchrotron Radiation - Some Background

Synchrotron Radiation (SR) - radiation from charged particles traveling in circular orbits - was first observed from a 70 MeV synchrotron at GE in Schenectady in 1947.

On April 24,[1947] Langmuir and I [Herbert Pollack] were running the machine… Some intermittent sparking had occurred and we asked the technician to observe with a mirror around the protective concrete wall. He immediately signaled to turn off the synchrotron as "he saw an arc in the tube." The vacuum was still excellent, so Langmuir and I came to the end of the wall and observed. At first we thought it might be due to Cerenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk [i.e., synchrotron] radiation.

Excerpted from Handbook on Synchrotron Radiation, Volume 1a, Ernst-Eckhard Koch, Ed., North Holland, 1983.



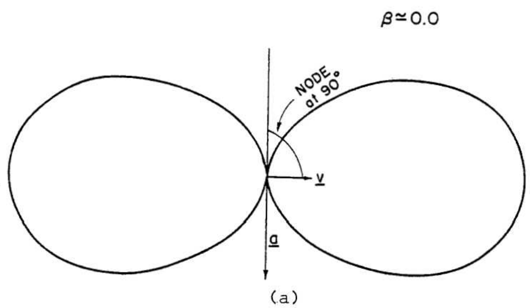
Radiation Patterns From Accelerating Charges

Definitions:

$\beta = v/c$

$\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$

When v << c, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.

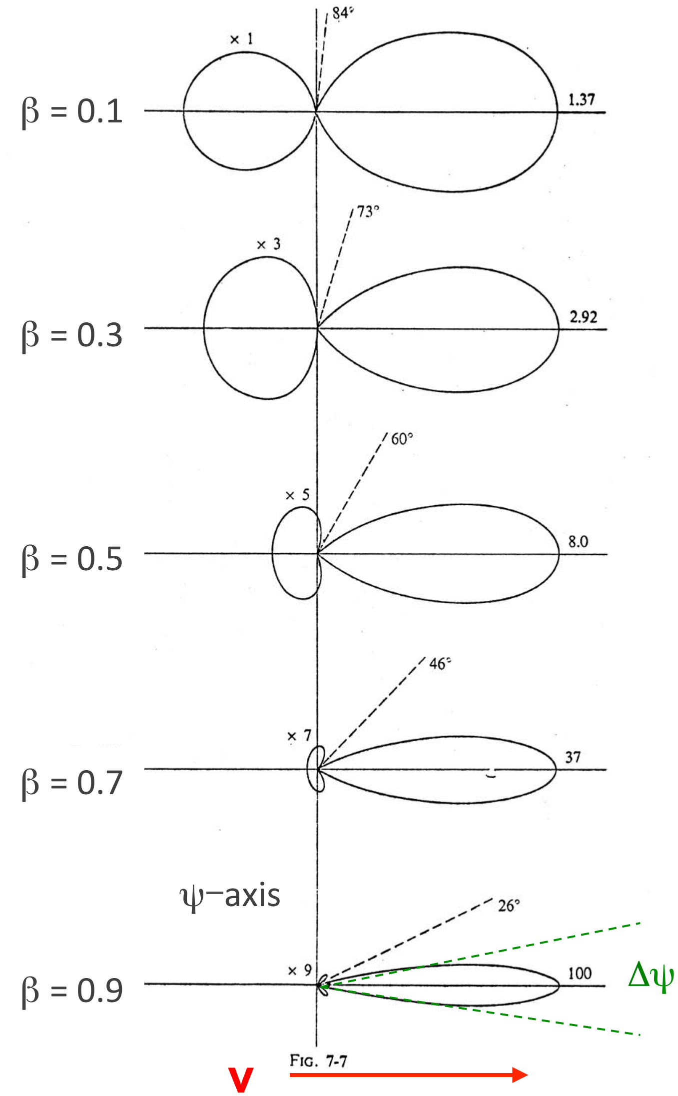


$\beta = 0$

But as β approaches 1:

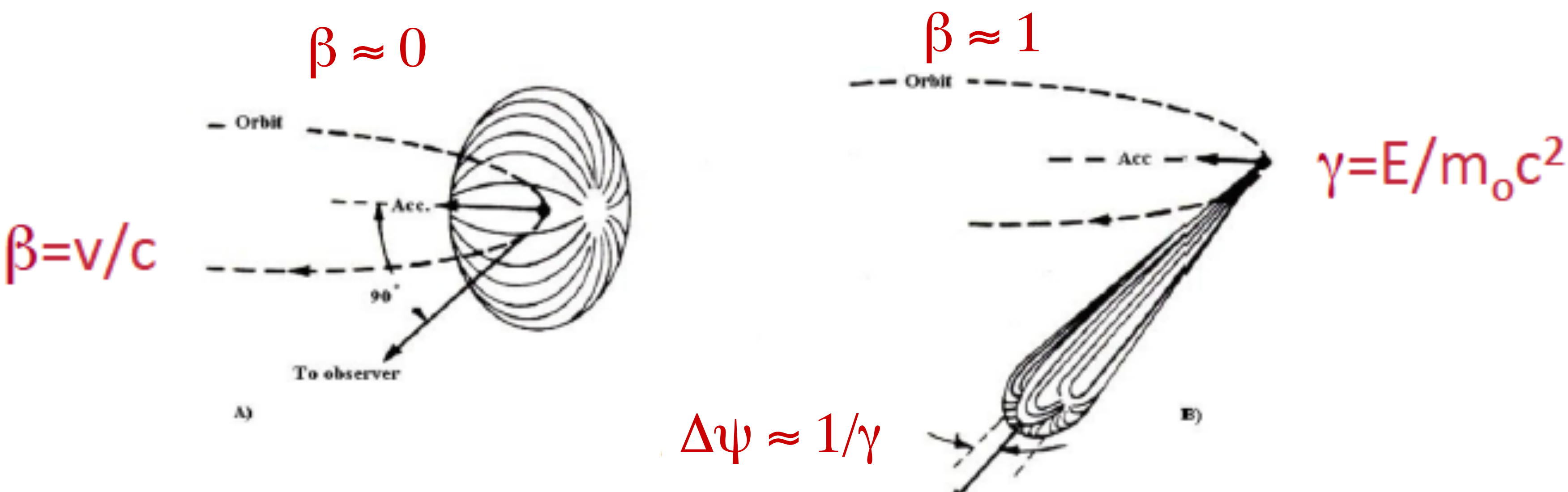
- The shape of the radiation pattern changes; it is more forward directed

$$\tan(\psi)_{lab} = \frac{\sin\psi'}{\gamma\left(\cos\psi' + \beta\right)} = \frac{1}{\gamma\beta} \approx \frac{1}{\gamma}$$



The angular divergence of the radiation (sometimes called the opening angle), $\Delta\psi$, is approximately $1/\gamma$.

Radiation from Highly Relativistic Particles



- The opening angle in both the horizontal and vertical directions is given approximately by:

$$\Delta\psi_{vert} = \Delta\psi_{hor} \approx 1/\gamma,$$

when $\beta \approx 1$. (@ APS $\beta \approx 1 - 2 \times 10^{-9}$)

- Relativistic velocities are good!!
  - radiation forward directed
  - radiated power $\propto E^4$

At the APS with E = 7 GeV,

$\gamma = E/m_o c^2 = 7\ GeV/0.511\ MeV$

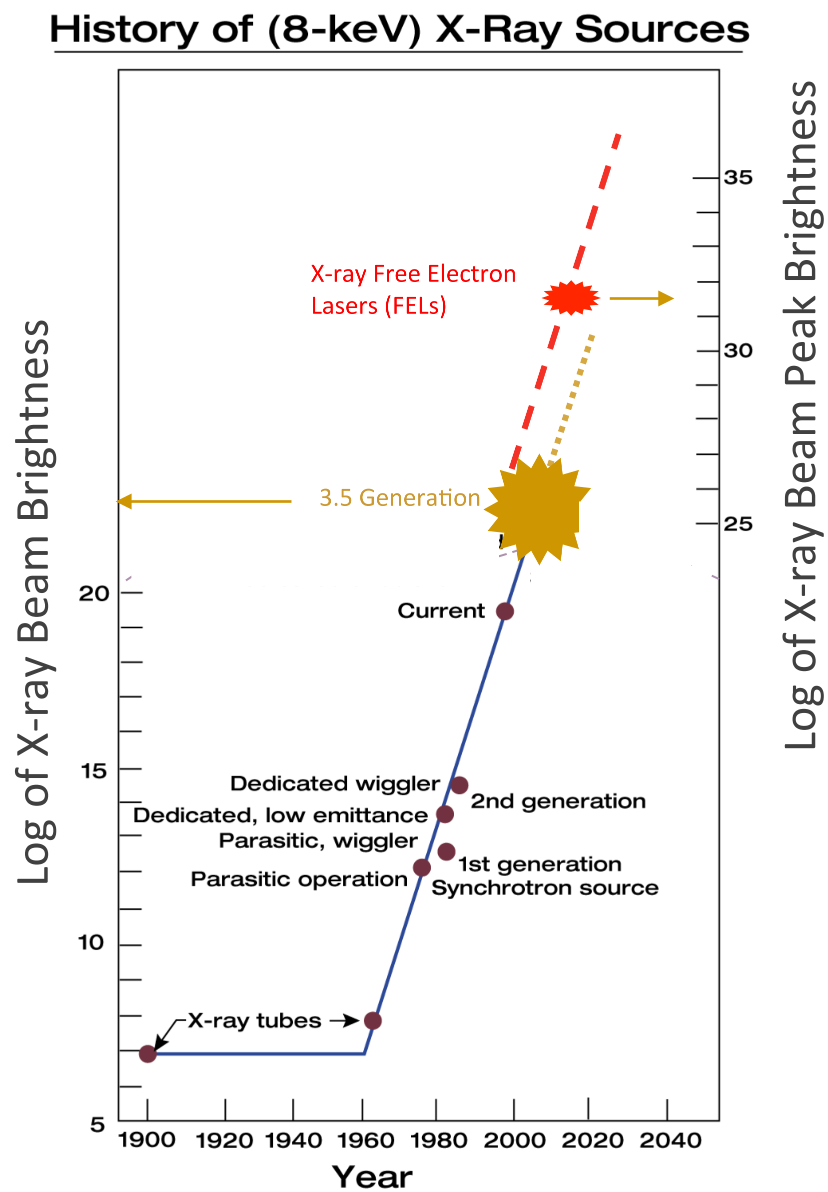
$\gamma = 1.4 \times 10^4$

$1/\gamma = 73 \times 10^{-6}$

See Appendix 1 for more details

The Evolution of Brightness from SR Sources

- 1st Generation Sources
  - *Ran parasitically on accelerations for high-energy physics (CHESS and SSRL originally)*
- 2nd Generation Sources
  - *Built to optimize synchrotron radiation from the bending magnets (NSLS)*
- 3rd Generation Sources
  - *Built to optimize synchrotron radiation from insertion devices (ALS, APS, NSLS II)*
- 3.5 (?) Generation Sources
  - *Based on multibend achromat (MBA) storage ring lattices to go towards a diffraction-limited source (ALS-U, APS-U) (more later)*
- 4th Generation Sources
  - *fully coherent sources*
    - *X-ray Free Electron Lasers - X-FELs (LCLS)*
    - *Diffraction Limited Storage Rings (DLSRs)*



A Vital National Resource for Science and Technology

The **Advanced Photon Source (APS)** is a fully optimized, insertion-device-based, third-generation x-ray source for the production of high intensity (brightness) hard x-ray beams.

ALS, APS, NSLS & SSRL are funded by the Department of Energy, Office of Science, Basic Energy Science (DOE/BES).

The Cornell High Energy Synchrotron Source – CHESS - is funded by NSF.

DOE/BES also operates several neutron user facilities as well as the nanoscience research centers.

Over **5500 unique researchers** from around the world came to Argonne last year to perform experiments at the APS.

Synchrotron Radiation Sources Around the World

The Anatomy of Synchrotron Radiation Facility

ADVANCED PHOTON SOURCE

BEAM ACCELERATION & STORAGE SYSTEM

(A) ELECTRON GUN

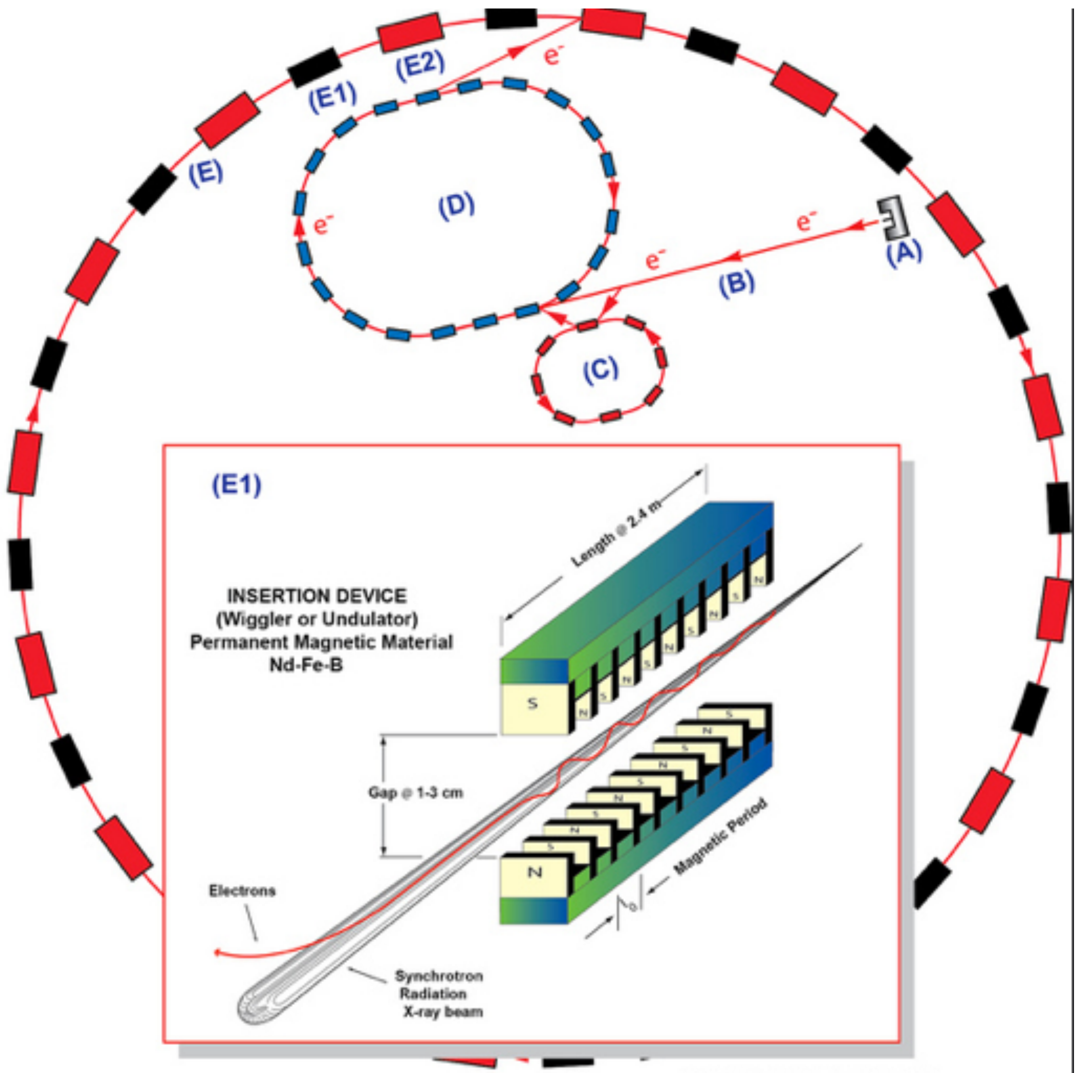
(B) ELECTRON LINEAR ACCELERATOR
Output energy: 375 MeV

(C) PARTICLE ACCUMULATOR RING

(D) BOOSTER SYNCHROTRON
Nominal extraction energy: 7.0 GeV

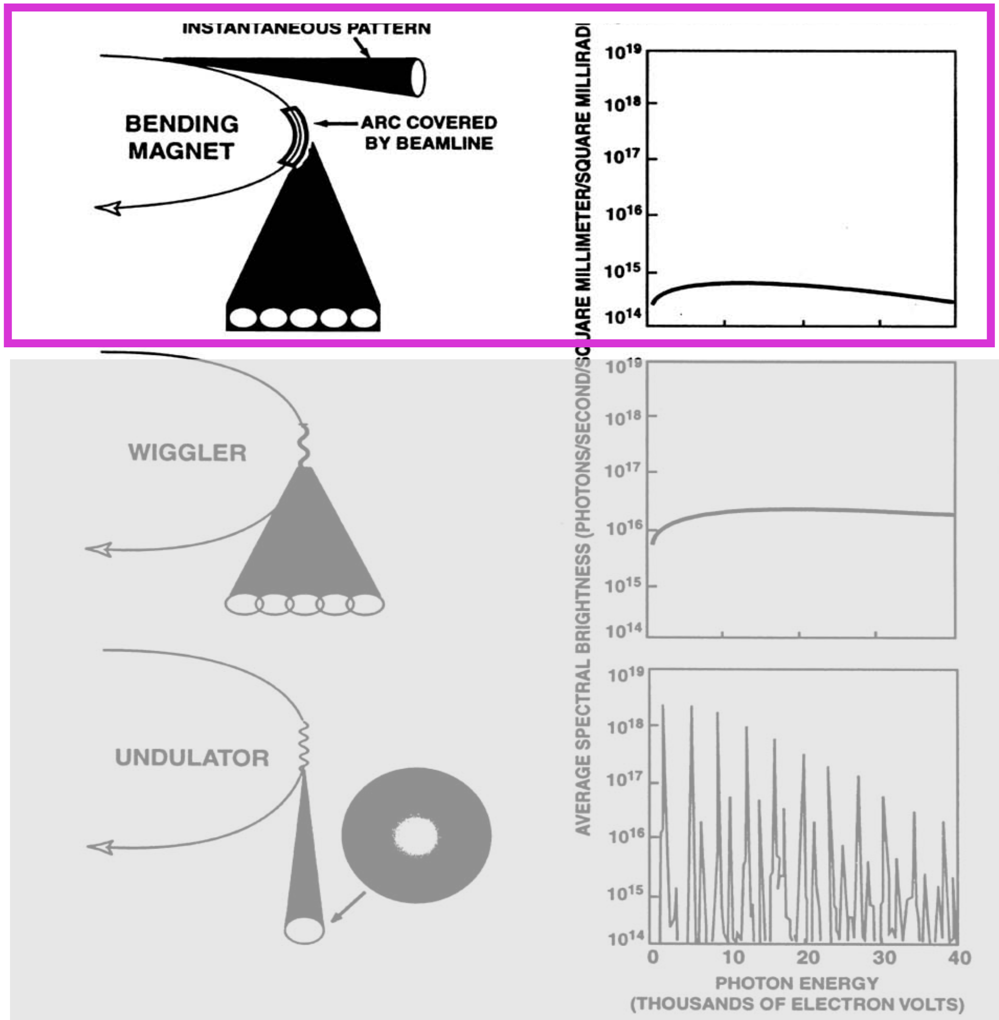
(E) STORAGE RING
Nominal energy: 7.0 GeV

(E1) INSERTION DEVICE
(E2) BENDING MAGNET



NOTE: Diagrams not to scale

BM Spectral Properties



Bend Magnet Radiation

- Spectrum characterized by the critical energy:

  $E_c = 3hc\gamma^3/4\pi r$ ($E_c \approx 20$ keV @ APS)

  E_c[keV] = 0.066 B[kG] E^2 [GeV]

Recall: λ[Å] = 12.4/E[keV] so 20 keV is 0.62Å

- Vertical opening angle ($\Delta\psi_v$) is $1/\gamma$.

  At the APS:

  $\gamma = E/m_oc^2 = 7$ *GeV/0.511 MeV*

  $\gamma = 1.4 \times 10^4$

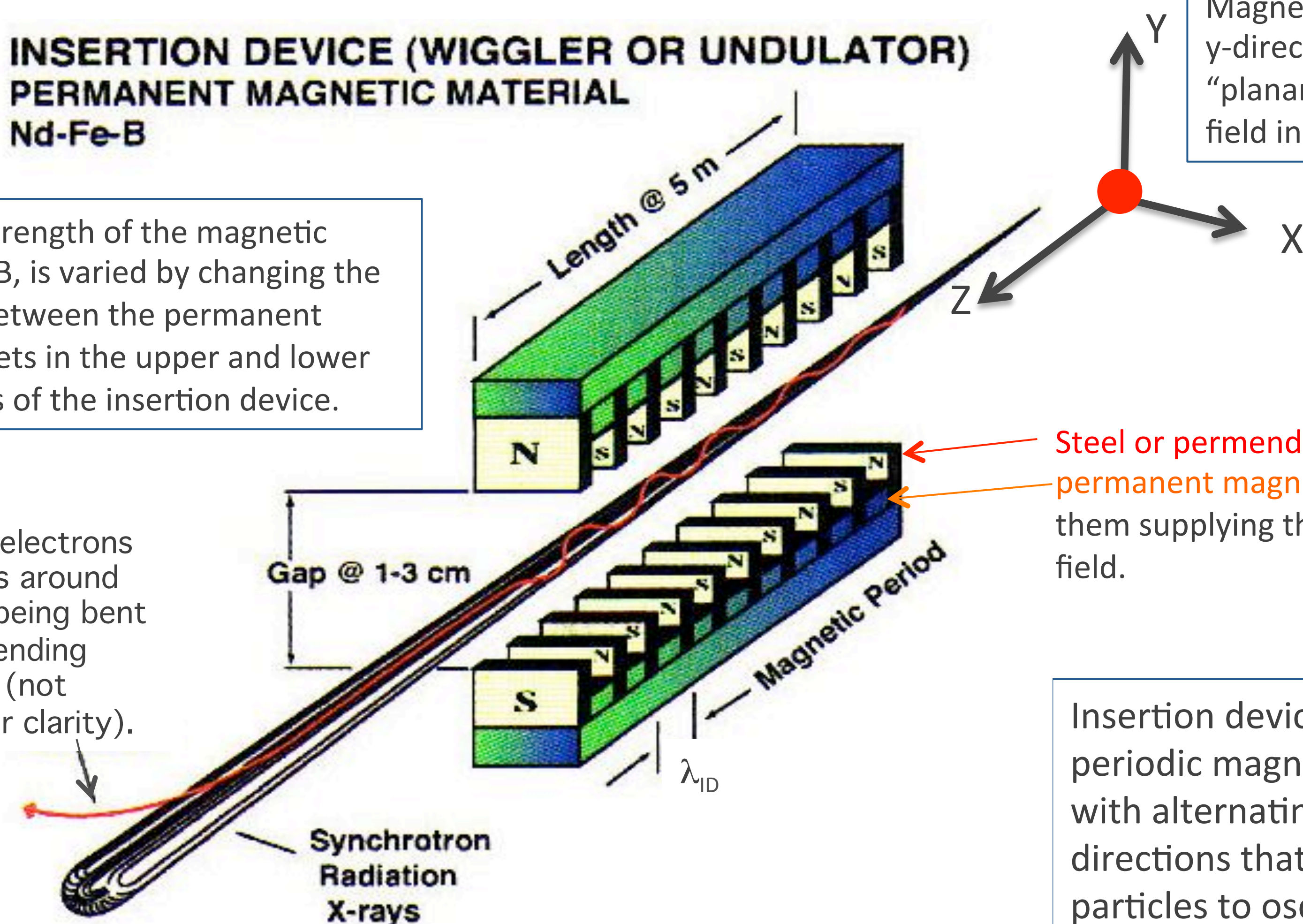
  so $1/\gamma = 73 \times 10^{-6}$ radians

- Horizontal opening angle determined by apertures

- In the plane of the orbit, the polarization is linear and parallel to the orbital plane.

See Appendix 2 for more details

Planar Insertion Devices



Magnetic field is in the y-direction for a "planar" ID (i.e., no field in the x-direction).

The strength of the magnetic field, B, is varied by changing the gap between the permanent magnets in the upper and lower halves of the insertion device.

Steel or permendur poles with permanent magnets between them supplying the magnetic field.

Beam of electrons continues around the ring being bent by the bending magnets (not shown for clarity).

Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.

Characterizing Insertion Devices

- IDs are characterized by the so-called field index or deflection parameter, K **(See Appendix 3)**:

$$K = eB_o\lambda_{ID}/2\pi m_o c = 0.0934\ \lambda_{ID}[cm]\ B_o[kG]$$

where λ_{ID} is the period of the insertion device and B_o the peak magnetic field. (The length of the ID, L, is equal to the number of periods, N, times the length of the period, i.e., $L = N\lambda_{ID}$.)

- The maximum deflection angle of the particle beam, θ_{max}, is given by:

$$\theta_{max} = \pm(K/\gamma)$$

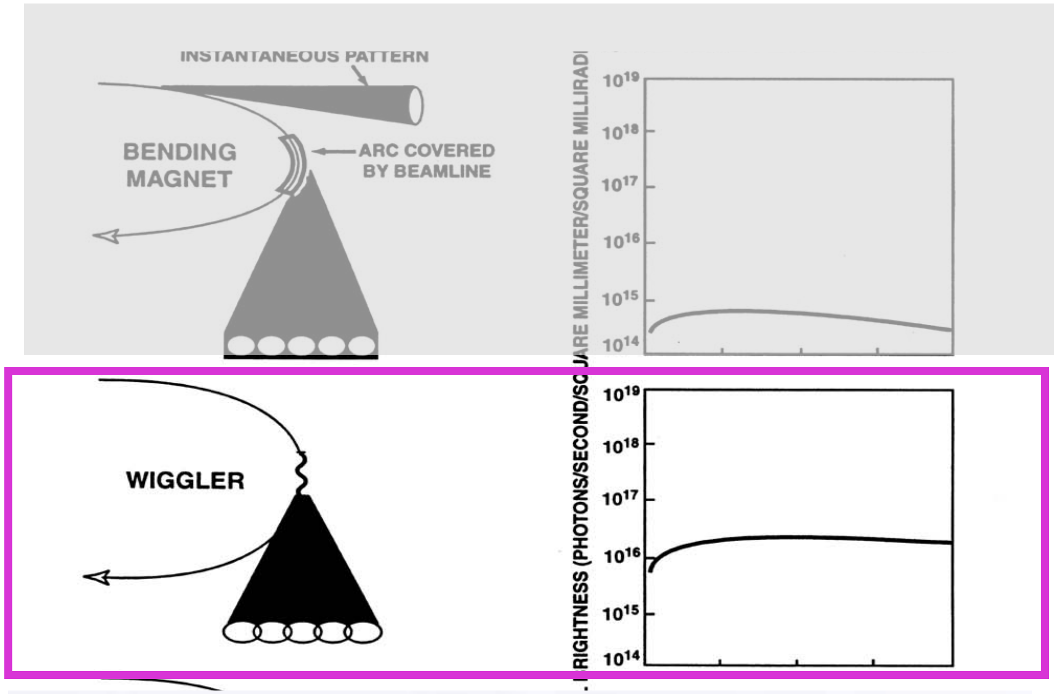
and the amplitude of the oscillation of the particles, x_{max}, by:

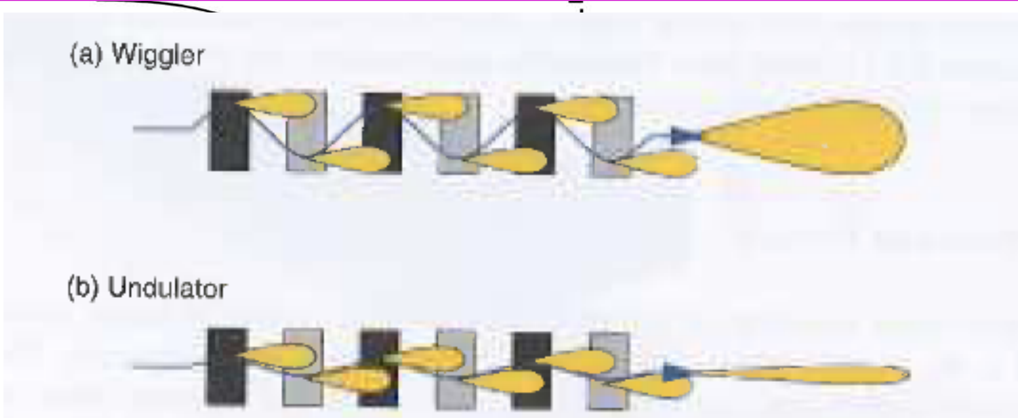
$$x_{max} = (K/\gamma)(\lambda_{ID}/2\pi)$$

APS Undulator A has a period of 3.3 cm and operates with K ≈ 1, therefore:

$\theta_{max} \approx 1/\gamma$ **and** $x_{max} \approx 0.38$ *microns.*

Wiggler Radiation Pattern and Spectrum





Wiggler Radiation (K>>1)

- Radiation spectrum looks like 2N dipole sources (N = number of periods)
- θ_{max} = (K/γ) >> 1/ γ, i.e. the angular deflection of the particle beam is much greater than the natural opening angle of the radiation (1/ γ).
- Spectrum characterized by the critical energy (which may be different than BM critical energy)
- Presently, there are NO planar wigglers installed at the APS.
- Wigglers with fields in both the horizontal and vertical directions produce elliptically polarized radiation. These are sometimes called elliptical multipole wigglers (EMWs).

Undulators

Undulators:

- $K \approx 1$

- θ_{max} is comparable ($1/\gamma$) and so the radiation from each pole overlaps causing interference effects in the spectral distribution.

- On-axis ($\psi_v = \psi_H = 0$), the constructive interference occurs at a particular x-ray energy (wavelength) when:

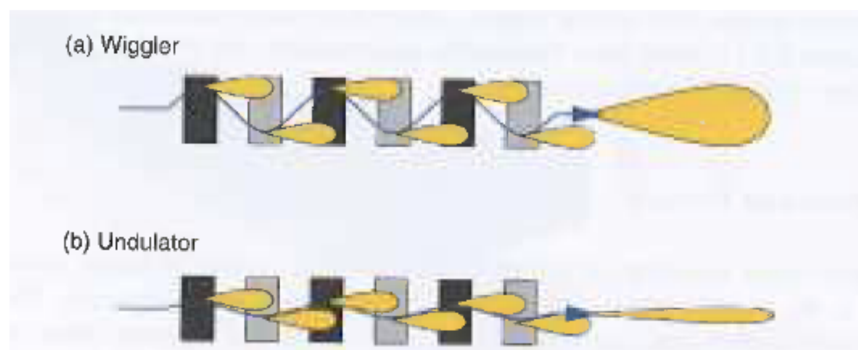
$$\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

- The wavelength, $\lambda_n^{x\text{-}ray}$, where constructive interference occurs, can be adjusted by varying the magnet field, B, in the undulator.

$$K = 0.093\ \lambda_{ID}[\text{cm}]\ B_o[\text{kG}]$$

- Since most undulators are made with permanent magnets, the B-field is changed by changing the gap between the magnets.

APS 2.4 m long Undulator A (λ_{ID} = 3.3 cm)
Permanent magnets

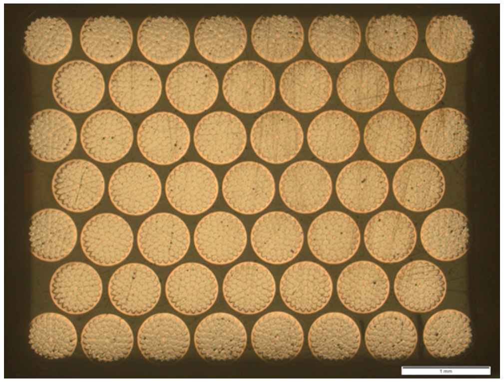


Superconducting Undulators

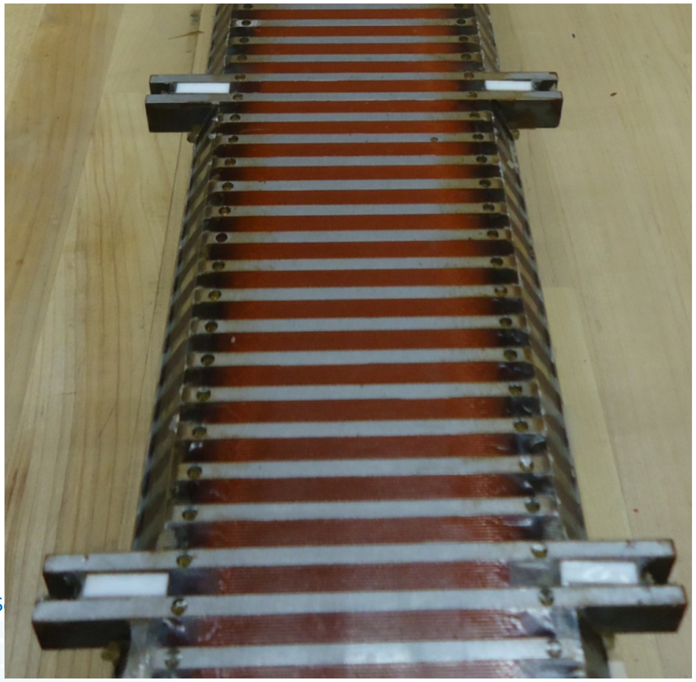
- There is a strong desire at the APS to increase x-ray energy of the 1st harmonic.

$$\lambda_n^{x\text{-ray}} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

- One way to do this is to go to shorter ID periods, i.e., reduce λ_{ID}.
- That is difficult to do with PMs as there is less room for the magnetic material.
- Room temp electromagnets cannot produce the necessary fields at short periods, but superconducting electromagnets can.



SCU1
- 18 mm period
- 1.1 m long
- 9.5 mm gap
- 0.6 mm NbTi conductor



Tuning the Peaks of Undulator Radiation

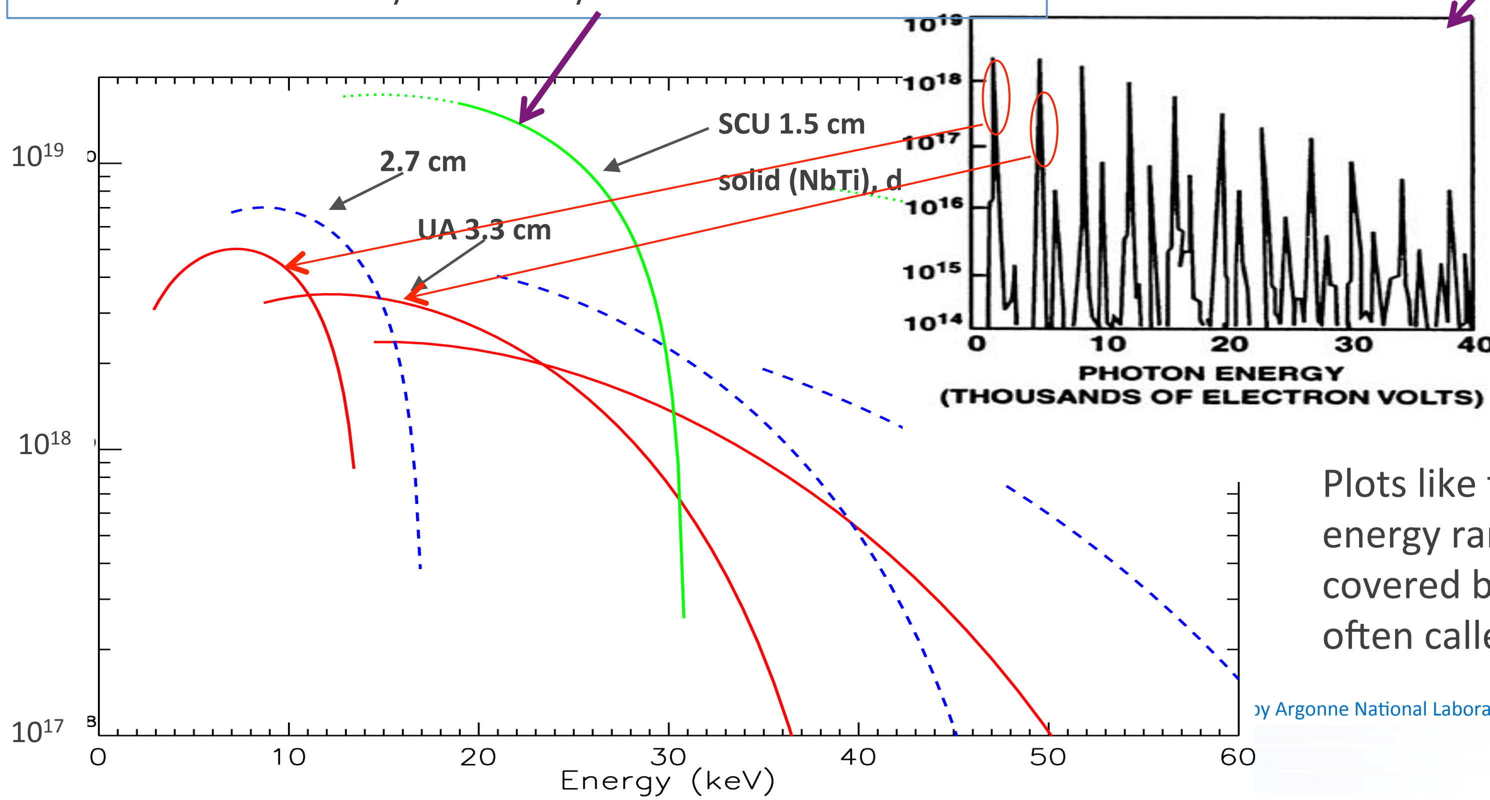
On-axis ($\psi_v = \psi_H = 0$), the constructive interference occurs when:

$$\lambda_n^{x\text{-ray}} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

where n is the harmonic number (n=1 is the fundamental). $\lambda_n^{x\text{-ray}}$, can be adjusted by changing the gap, which changes K since $K = 0.093\ \lambda_{ID}[cm]\ B_o[kG]$.

Here is the spectrum over the tuning range of the undulator. The low energy is fixed by the minimum magnet gap (determined by the vacuum chamber). As the gap is opened, the x-ray energy increases, but as you open it further, the magnetic field decreases and so does the intensity of the x-rays.

Here is the spectrum for a particular gap (i.e., a particular K value).



Plots like this that show the energy range that can be covered by an undulator are often called "tuning curves".

Undulator Radiation

Undulator Radiation Spectra

- Undulators defined as IDs with deflection angle ≈ $1/\gamma$, i.e., $K \approx 1$

- The spectrum from an undulator is spikey, but peaks are tunable by varying K since $K = 0.093\, B[kG]\, \lambda_{ID}[cm]$

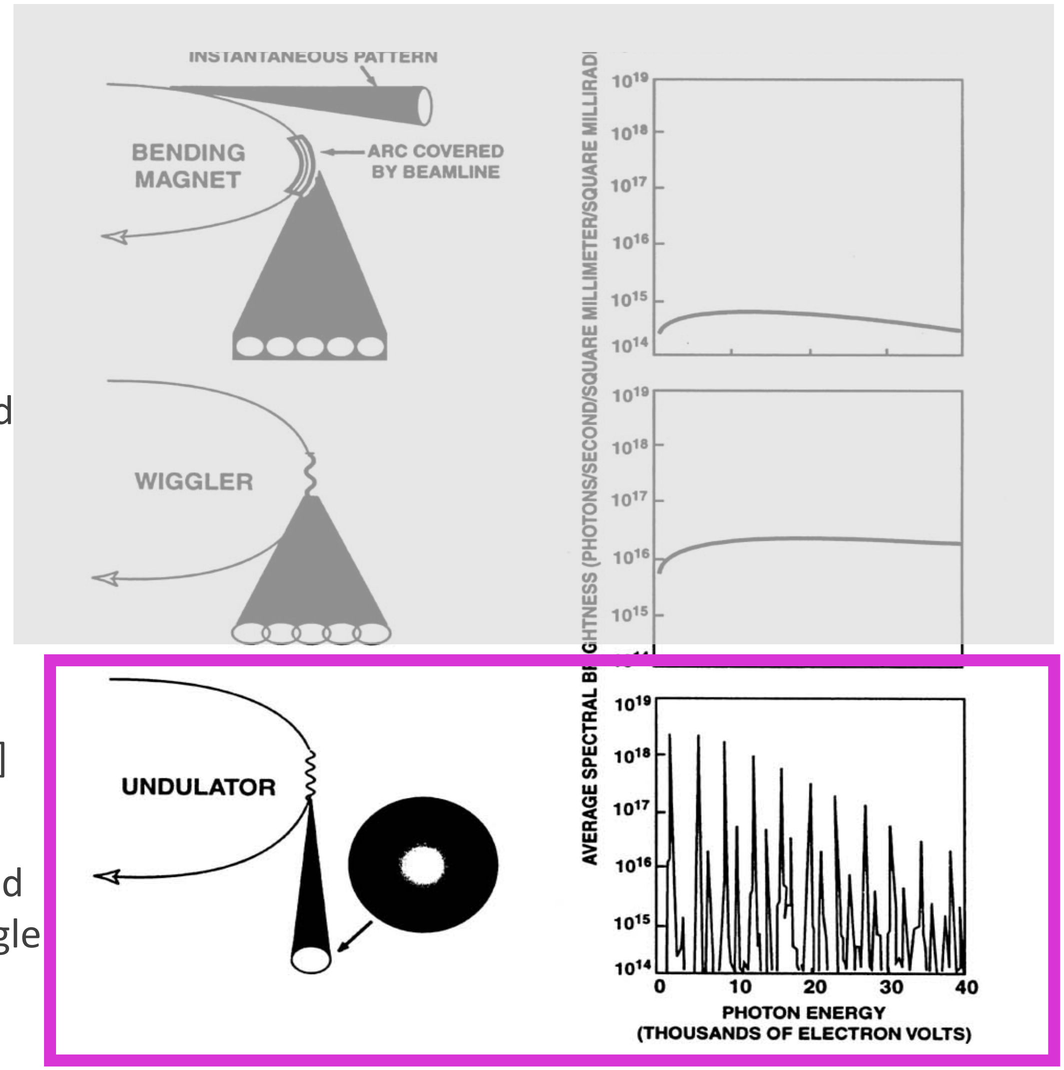
- At the peaks (harmonics) the horizontal and vertical opening angles of the radiation is given by:

$$\Delta\psi_V = \Delta\psi_H \approx (1/\gamma)\,[1/N]^{1/2}$$

where N = number of periods [typically 100]

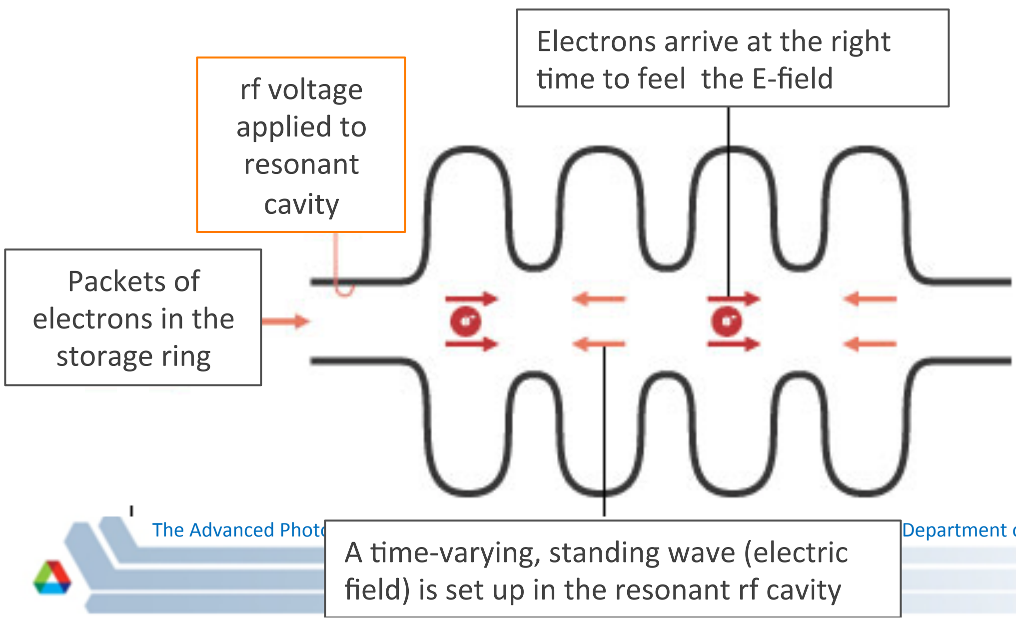
- To get the true opening angle of the emitted radiation, need to consider the opening angle of the emitting particles (more later on!)

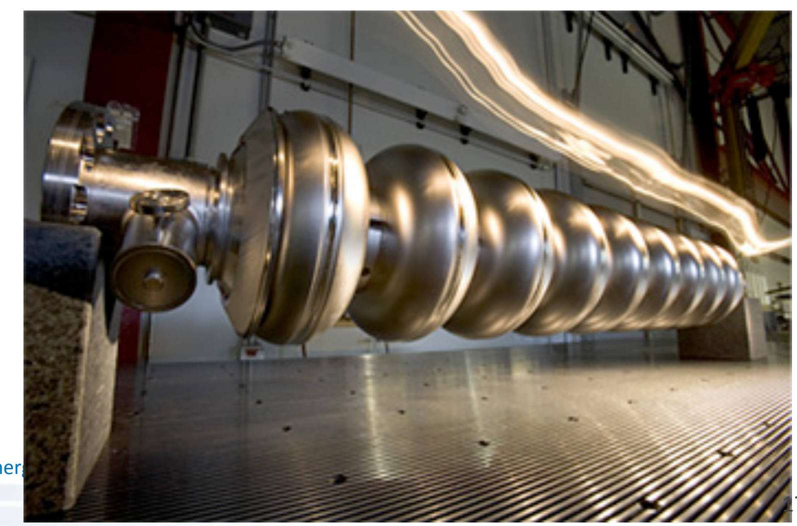
See Appendix 4 for more details



Time Structure of the Radiation

- Because the electrons are radiating x-rays, they are constantly losing energy.
- To restore the energy loss on each revolution, radio-frequency (RF) cavities are installed in the storage ring to replenish the radiative energy losses.
- Particles are grouped together by the action of the radio-frequency (RF) cavities into bunches. At APS:
  - typically about 100 psec FWHM (about 3 cm in length)
  - 1104 m circumference (3.68 microsecond period)
  - there are 1296 evenly spaced “RF buckets” (stable orbit positions”) around the ring
  - minimum spacing is 2.8 nsec between bunches (determined by the RF frequency- 352MHz)
- Details of the time structure depends on the fill pattern, i.e. which RF buckets have electrons in them.





Typical APS Filling Patterns

- The APS has a 1104 m circumference (3.68 microsecond period).

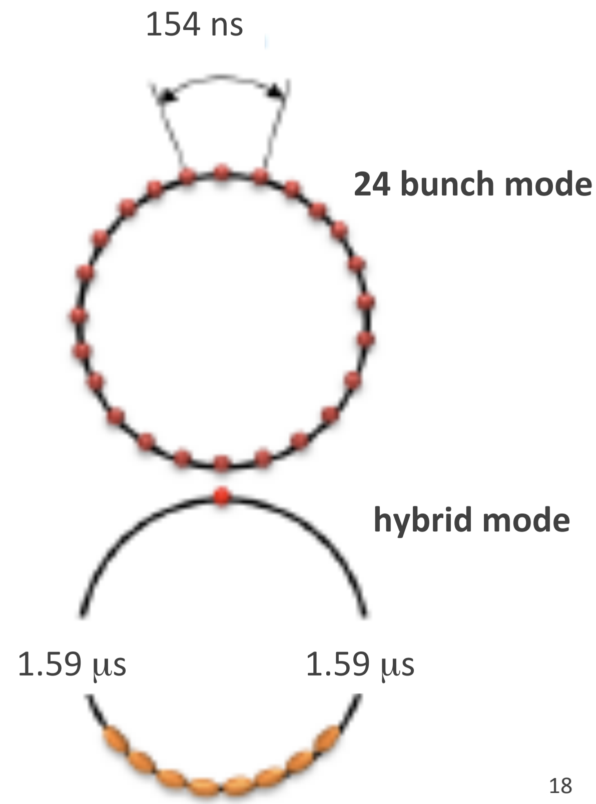
- There are 1296 evenly spaced "RF buckets"(stable orbit positions) around the ring that electrons could be stored in.

- This would correspond to a spacing of 2.8 nsec between bunches

- 324 equally spaced bunches
  - approximately 11 nsec between bunches
  - approximates a continuous source

- 24 equally spaced bunches
  - approximately 154 nsec between bunches
  - compromise between quasi-continuous source and pulsed source

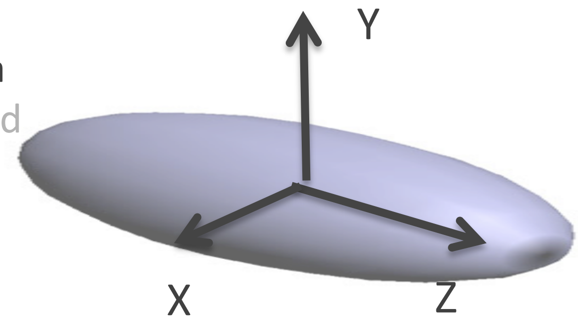
- 1 + 7x8 (hybrid mode)
  - a single bunch followed by 8 groups of 7 bunches
  - timing experiments



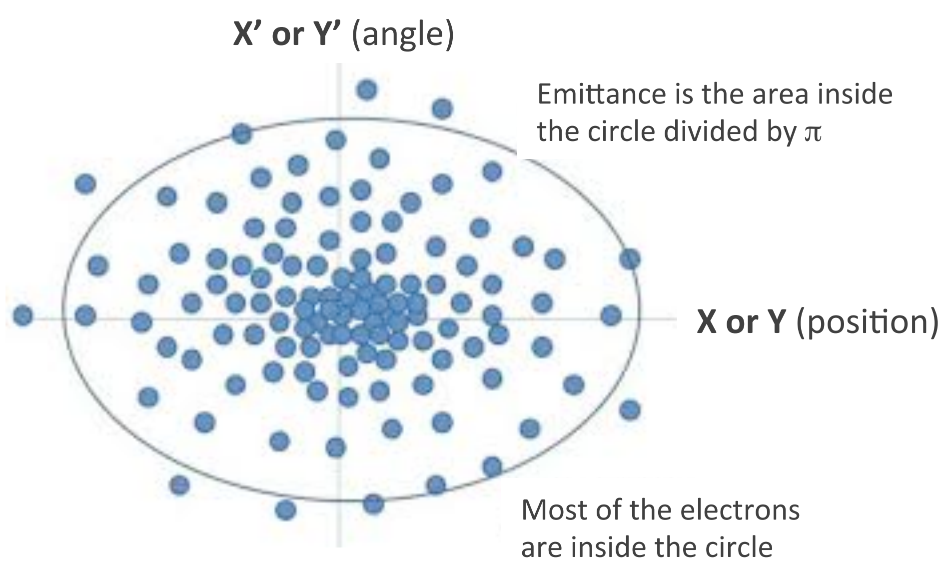
Transverse Properties of Particle Beams

- Up until now, we have calculated the radiation properties from a single electron, however in a storage ring, the radiation is emitted from an ensemble of electrons with some finite size and divergence distribution.

- Both the transverse and longitudinal properties of the particle beam in a storage ring are the equilibrium properties of the particle beam, but here we are interested in the ***transverse*** properties.



X, Y are transverse directions
Z is the longitudinal direction



- In particular, we need to look at the X (horizontal) and Y (vertical) the phase space of the particle beam.

- Typical coordinates for a phase space plot are P_x and X, for instance, but P_x is proportional to X' .

- This is not the beam cross-section!

Phase Space and Emittance

- The product of the particle beam size and divergence is proportional to a parameter of the beam called the emittance (units are length x angle).

- The formal definition is that the emittance is the (elliptical) phase space area occupied by the system of particles divided by π.

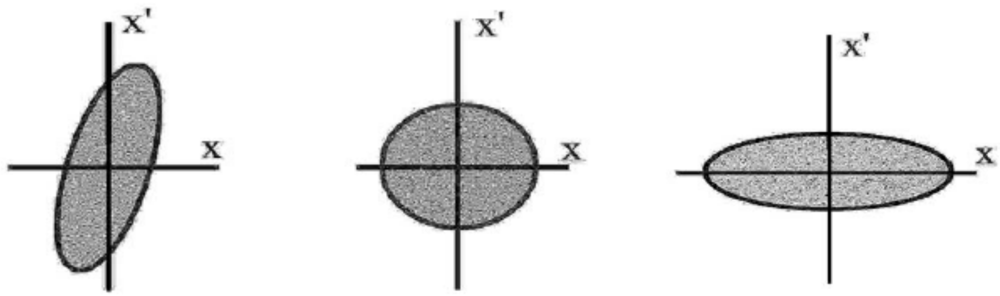
- There is a separate horizontal emittance (ε_x) and vertical emittance (ε_y). Typically in today's storage rings:

  ε_y is 1% to 10% of ε_x (called the coupling)

$\varepsilon_H = 3 \times 10^{-9}$ m-rad @ APS and

$\varepsilon_V = 0.03 \times 10^{-9}$ m-rad.

- The emittance is a constant of the storage ring, although one can trade off beam size for divergence as long as the area of the phase-space remains constant.



This is a consequence of Liouville's Theorem – the conditions for which are satisfied in most accelerators.

Why Do We Need to Know about the Transverse Particle Beam Properties?

- Although the flux from undulators can be determined without detailed knowledge of the source size and divergence, one very important characteristic of the x-ray beam, namely **brightness,** requires a more detailed knowledge of the particle beam's size and divergence.

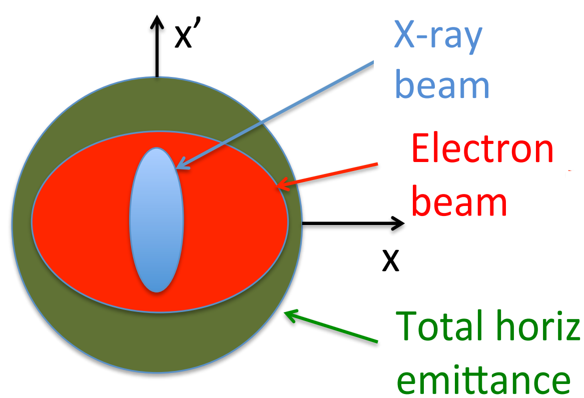
  Brightness has units of: photons/sec/0.1% BW/source area/source solid angle

$$\text{Flux}/4\pi^2 \Sigma_h \Sigma_v \Sigma_h' \Sigma_v'$$

where Σ_i (Σ_i') is the **effective** one sigma value of the source size (divergence) in the i[th] direction. If Gaussian distributions are assumed for both the particle beam and the radiation itself, the resultant source size and divergence is the quadrature sum of the two components, namely:

$\Sigma_i = [\sigma_r^2 + \sigma_i^2]^{1/2}$ and $\Sigma_i' = [\sigma_r'^2 + \sigma_i'^2]^{1/2}$.

- σ_r: radiation beam size
- σ_i: electron beam size (i= x or y)
- σ_r': radiation beam divergence
- σ_i': electron beam divergence (i= x or y)



- When the electron emittance ⬇ then brightness ⬆ (up to a point!)

Diffraction Limited Sources and Coherence

- The Heisenberg Uncertainty Principle sets a lower limit for the emittance of radiation. Recall:

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\frac{p_x}{p_z} = x' \text{ or } \frac{\Delta p_x}{p_z} = \Delta x' \text{ and } p_z = \hbar k = \frac{\hbar(2\pi)}{\lambda}$$

$$so: \Delta x \Delta p_x = \Delta x \Delta x' p_z = \Delta x \Delta x' [\frac{\hbar(2\pi)}{\lambda}] \geq \hbar / 2$$

$$\Delta x \Delta x' \geq \lambda / 4\pi$$

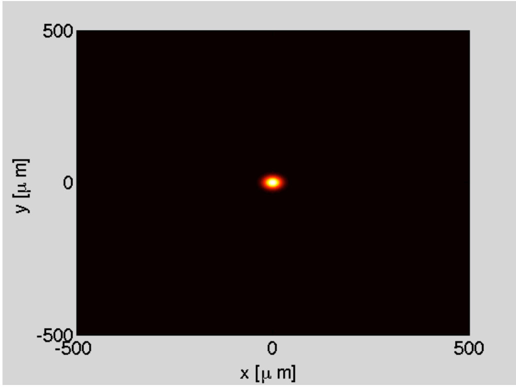
- If both the horizontal and vertical emittance of the beam is less than $\lambda/4\pi$, then the radiation emitted is fully coherent and the source is diffraction limited. For 1Å (12 keV) x-rays, the particle beam emittance would have to be less than:

1Å $/4\pi$ = 10^{-10} meters $/4\pi \approx 8 \times 10^{-12}$ m or 8 picometers – radian (radians are dimensionless)

- Coherence can be an important parameter in some experiments such as photon correlation spectroscopy, x-ray holography, imaging, etc.

Recall earlier lectures on XPCS & Coherence Based Imaging

Partial Coherent Sources



For 1Å (12 keV) x-rays → 8 picometers – radian for fully coherent beam.

- APS operates with:

  ε_H = 3 x 10^{-9} m-rad or 3000 picometer-radian

  ε_V = 0.025 x 10^{-9} m-rad or 25 picometer-radian

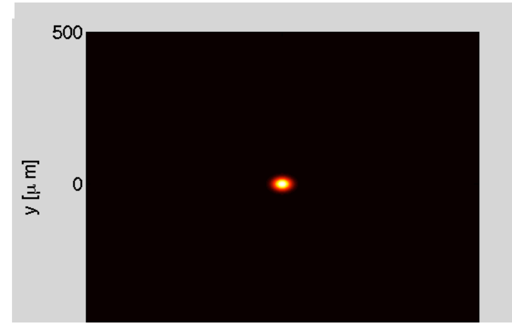
- Hence the APS is a partially coherent source at 1 Å.

- Partially coherent sources are sometimes characterized by the coherent fraction.
  - Coherent fraction = ratio of diffraction-limited emittance to actual beam emittance, or the fraction of the x-ray flux that is coherent.

- For the APS at 1Å, the coherent fraction is ≈ 10^{-3}. Marginally good enough for coherence-based experiments so the trend is to try to reduce the particle beam emittance to increase coherence.

- Why not just squeeze the electron beam tighter in the existing storage ring?
  - Intra-bunch scattering of the particles results in beam loss and shorter lifetimes *(increase the number of bunches and lengthen the bunch longitudinally to reduce particle density)*
  - Need stronger focusing magnetic fields than can be reached with iron magnets with present electron beam energy and size of the storage ring vacuum chamber *(reduce the size of the vacuum chamber)*
  - Present bend magnet configuration limits *emittance (change from 2 bend magnets per sector to multiple bend geometry, i.e., a larger number of shorter bending magnets)*

Partial Coherent Sources



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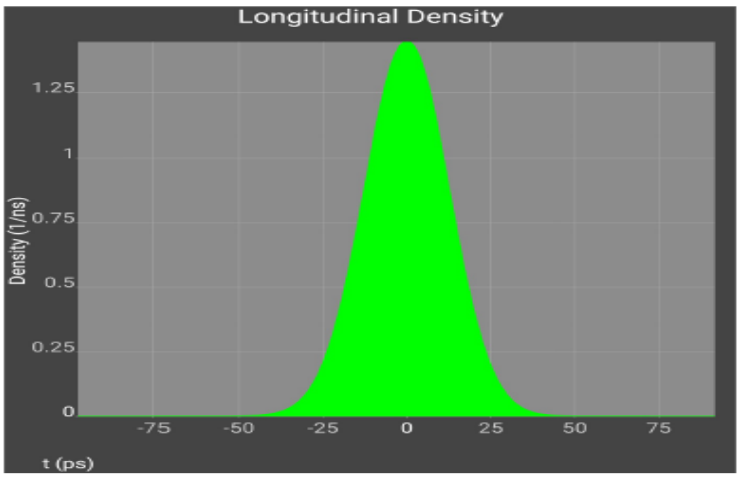
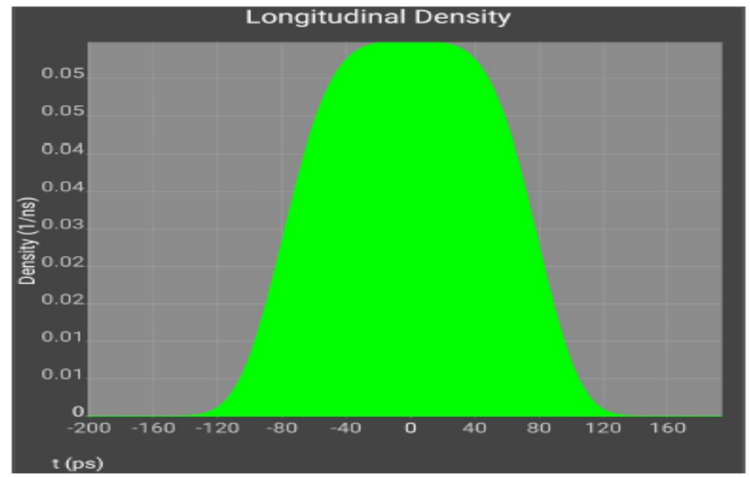


Stretch the bunch with rf cavities →



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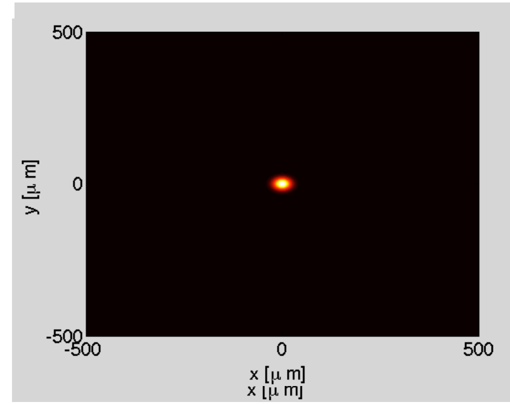

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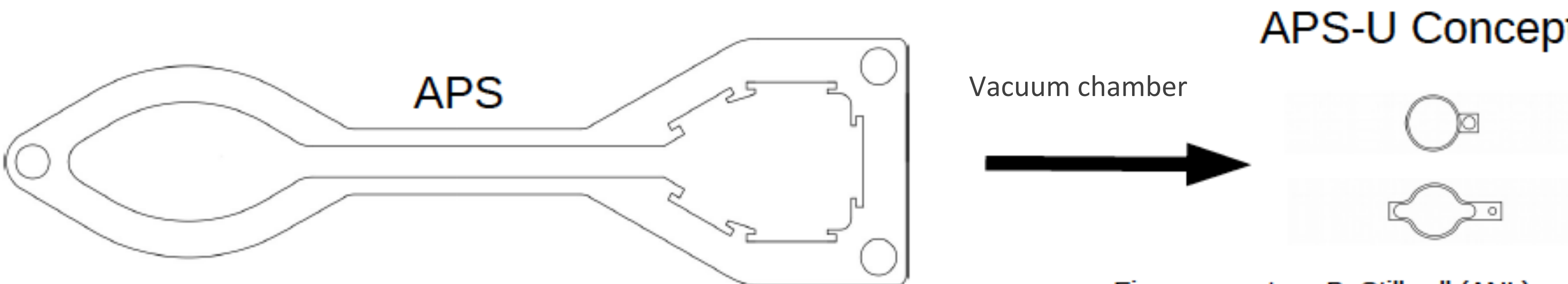


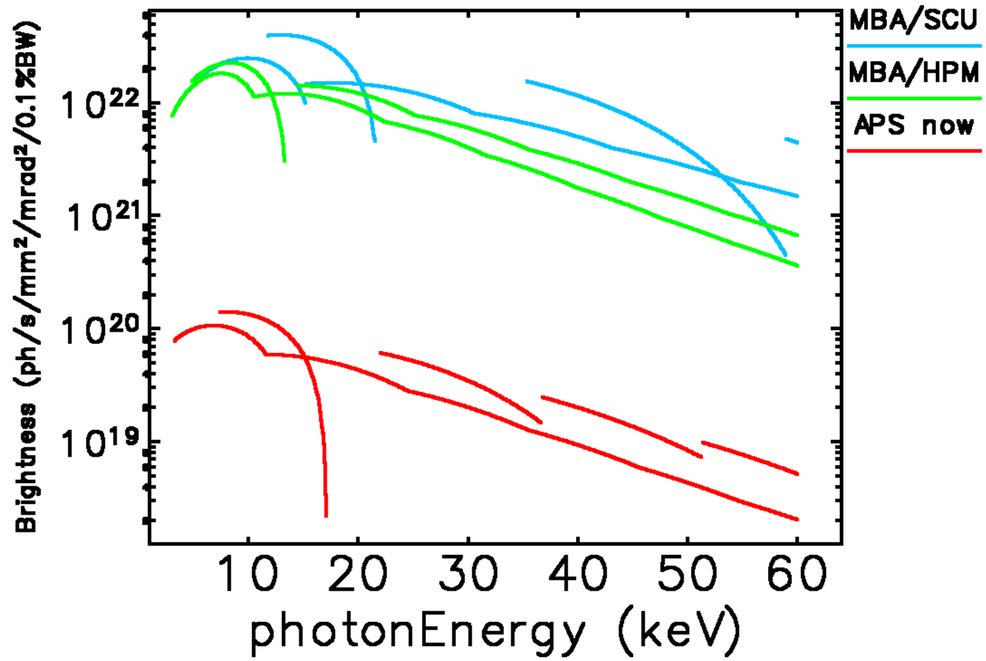
Figures courtesy B. Stillwell (ANL).

  - Need stronger focusing magnetic fields than can be reached with iron magnets with present electron beam energy and size of the storage ring vacuum chamber *(reduce the size of the vacuum chamber)*
  - Present bend magnet configuration limits *emittance (change from 2 bend magnets per sector to multiple bend geometry, i.e., a larger number of shorter bending magnets)*

An MBA-Lattice at APS: A New Generation of Storage Ring Sources

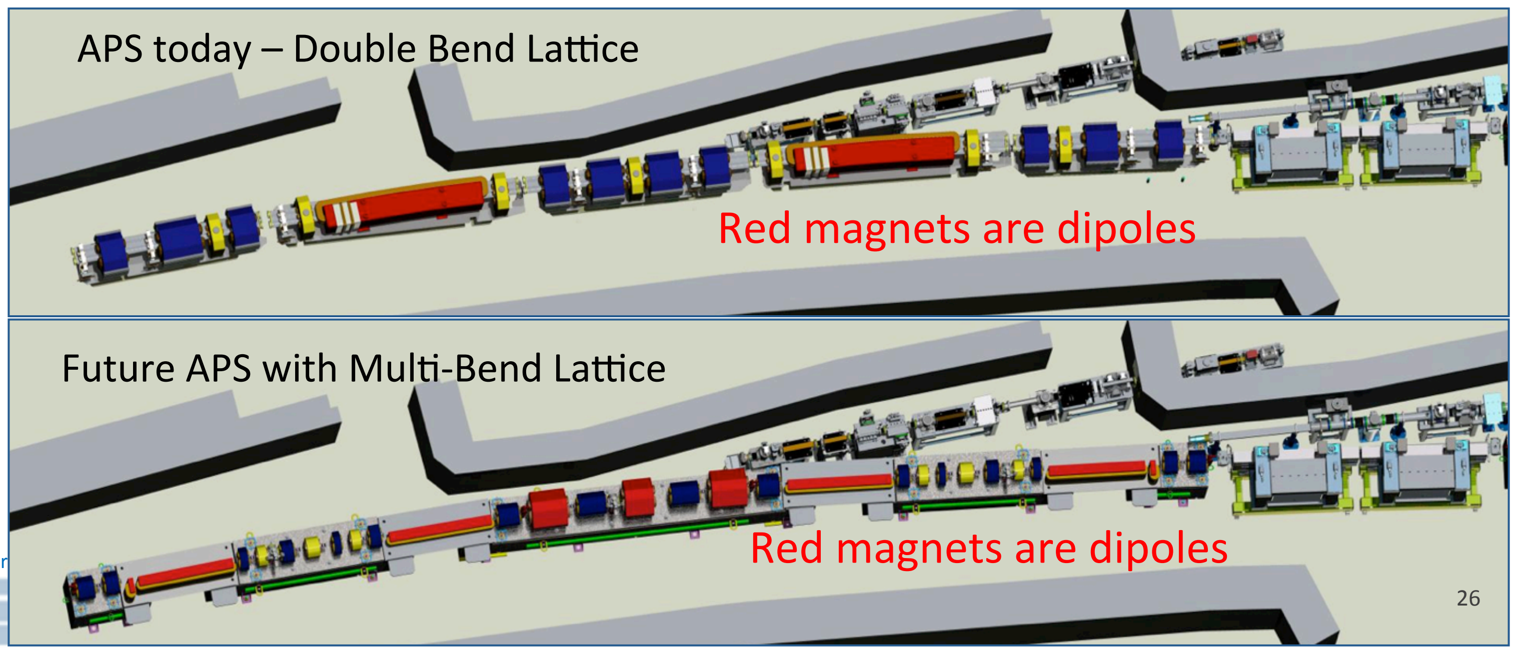
$$\varepsilon_x = C_L \frac{E^2}{N_D^3}$$

C_L = constant
E = beam energy
N_d = dipoles per sector



APS MBA Upgrade:
Beam energy: 7 GeV => 6 GeV
Dipoles per sector: 2 => 7

~50-fold reduction in horizontal emittance



MBA Lattice Upgrades – a Competitive Landscape

ESRF (France)

- Council approved second phase of upgrade, which incorporates an MBA lattice
- ESRF plans to resume operation in 2020, complete four state-of-the-art beamlines by 2022

SPRING-8 (Japan)

- Capable of upgrading in 2020's

MAX-IV (Sweden)

- Construction underway; inauguration June 2016

SIRIUS (Brazil)

- Completing final design; operational in 2018

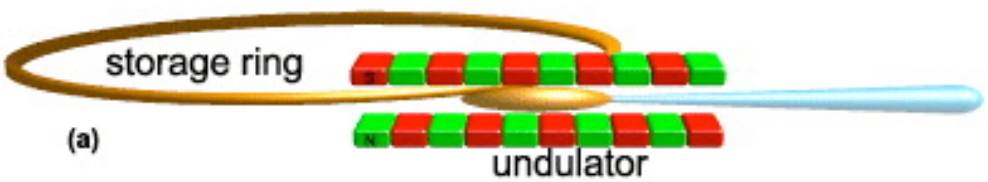
The uncertain U.S. position within the competitive landscape was a central theme in the July 2013 BESAC Report:

*'…It is essential that the facilities this science community [of storage ring users] relies on remain **internationally competitive** in the face of the innovative developments**…** The Office of Basic Energy Sciences should ensure that U.S. storage ring x-ray sources **reclaim their world leadership position**…'*

Our plans for the APS Upgrade enable the U.S. to establish a world leadership position in storage-ring-based x-ray sources

Non-Storage Ring Coherent X-ray Sources - XFELs

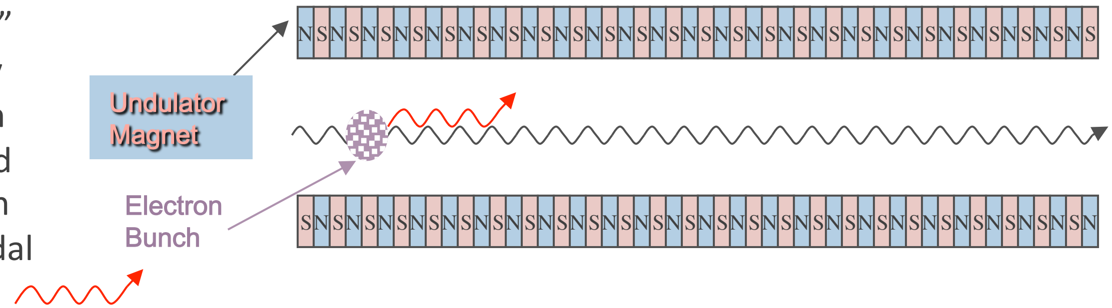
- Another way to reduce the particle beam emittance is through linac-based x-ray free electron lasers (XFELs)
  - Full transverse (spatial) coherence and femtosecond pulses
- An **x-ray FEL** uses the high brightness of an **electron gun** coupled to an **emittance-preserving linac.**
- The problem with a linac is you only use the electrons once, and so have to accelerate new electrons for each pulse (in the storage ring you "recycle" the electrons after they have been accelerated).



- The gain in the laser is obtained through a process called **Self-Amplified Spontaneous Emission** *or* **SASE.**

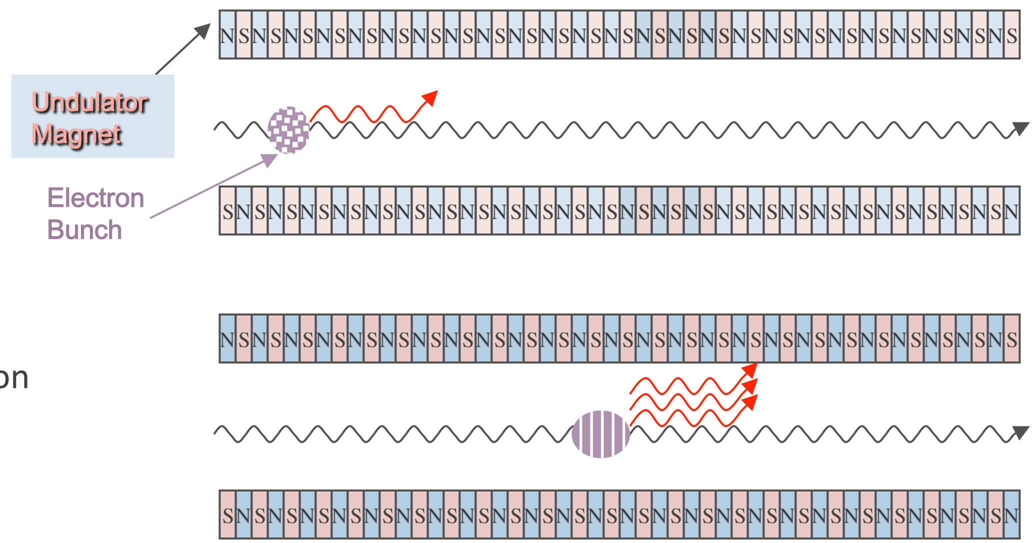
Self-Amplified Spontaneous Emission – Undulator Radiation

The LCLS produces extraordinarily bright pulses of synchrotron radiation in a process called "self-amplified spontaneous emission" (SASE). In this process, an intense and highly collimated electron beam travels through an undulator magnet. The alternating north and south poles of the magnet force the electron beam to travel on an approximately sinusoidal trajectory, emitting synchrotron radiation as it goes.



Linac Coherent Light Source (LCLS) at SLAC is an x-ray FEL using 1 km of the 3 km linac there to produce 15Å to 1.5Å beams.

Self-Amplified Spontaneous Emission - Energy Modulation ➔ Density Modulation (micro-bunching)



The electron beam and its synchrotron radiation are so intense that the electron motion is modified by the electric and magnetic fields of its own emitted synchrotron light. Under the influence of both the undulator magnet and its own synchrotron radiation, the electron beam is forced to form micro-bunches,

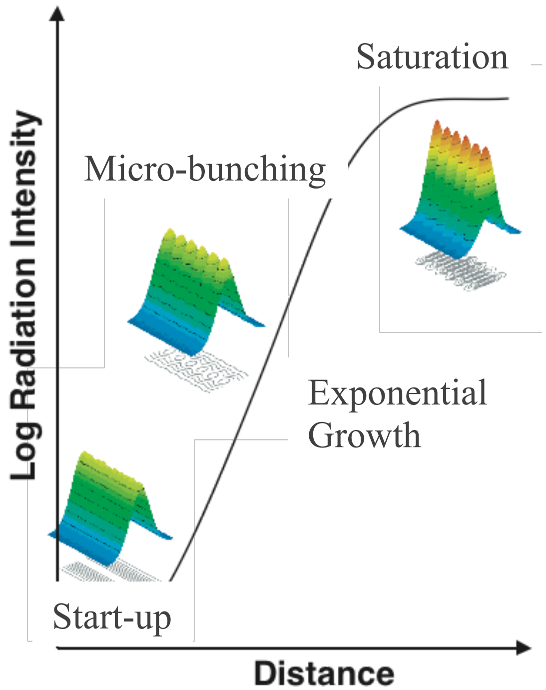
separated by a distance equal to the wavelength of the emitted radiation.

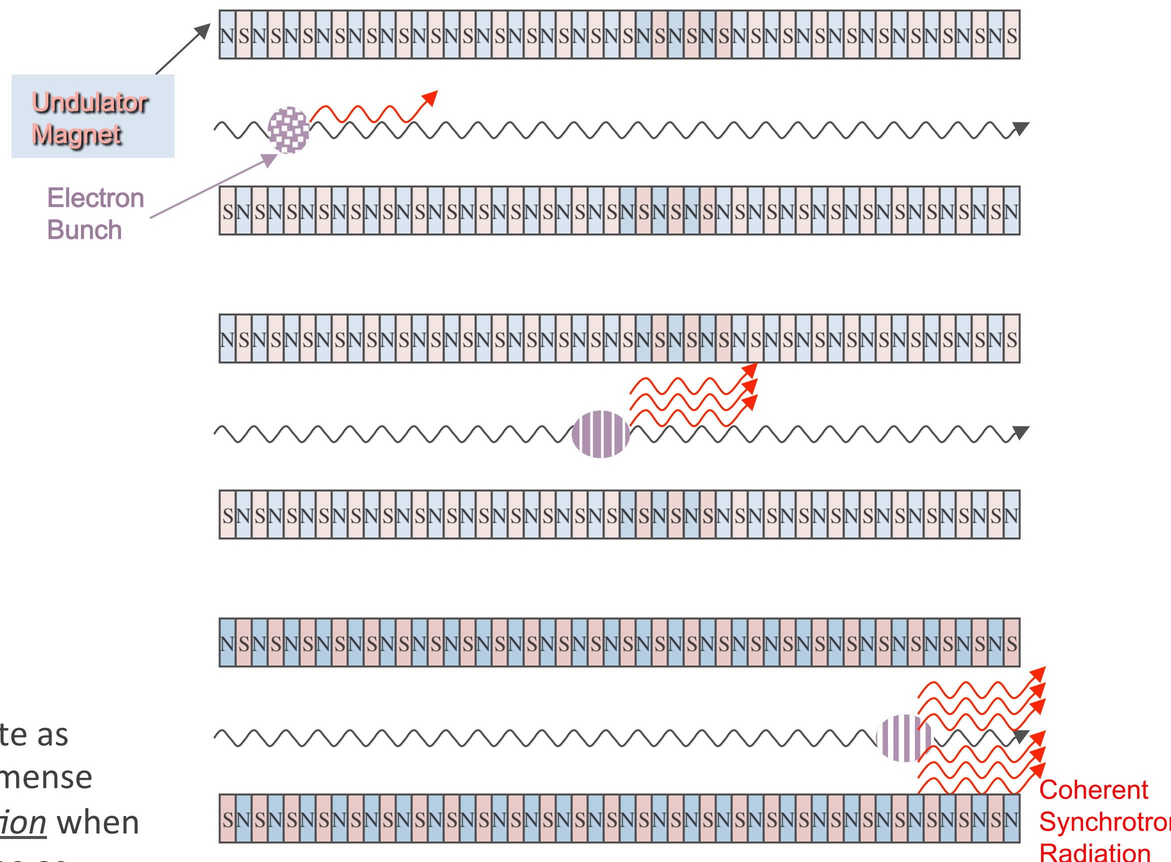
Energy modulation ➔ density modulation

$$Power\ Radiated = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}$$

John Galayda, LCLS

Self-Amplified Spontaneous Emission – Exponential Growth of Intensity





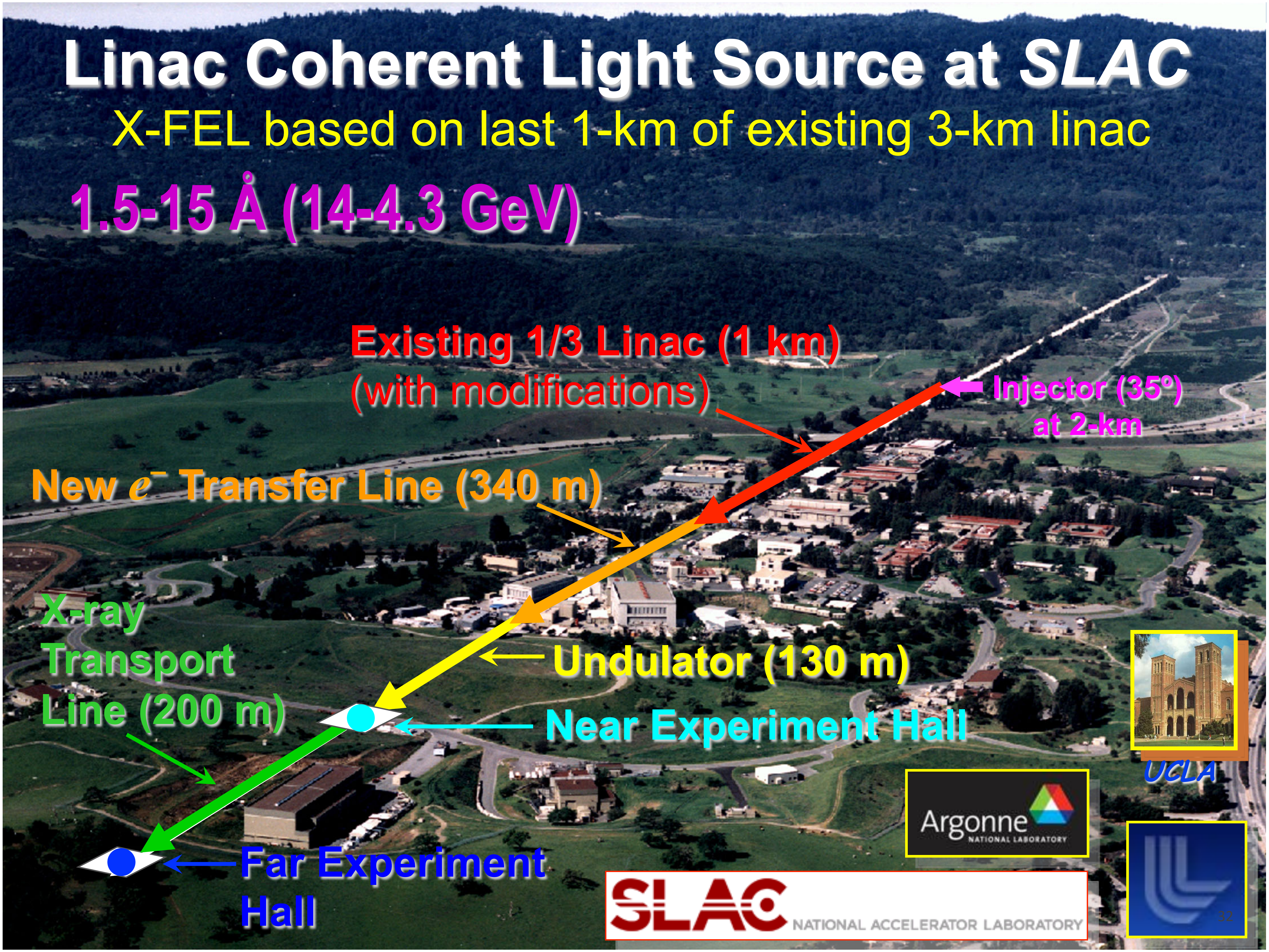
These micro-bunches begin to radiate as if they were single particles with immense charge. The process reaches *saturation* when The micro-bunching process has gone as far as it can go. 

John Galayda, LCLS

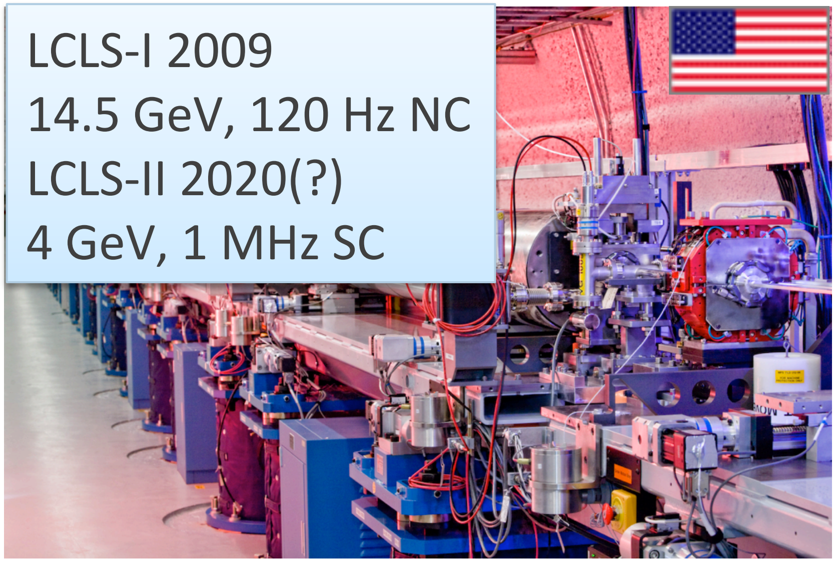
Linac Coherent Light Source at *SLAC*

X-FEL based on last 1-km of existing 3-km linac

1.5-15 Å (14-4.3 GeV)



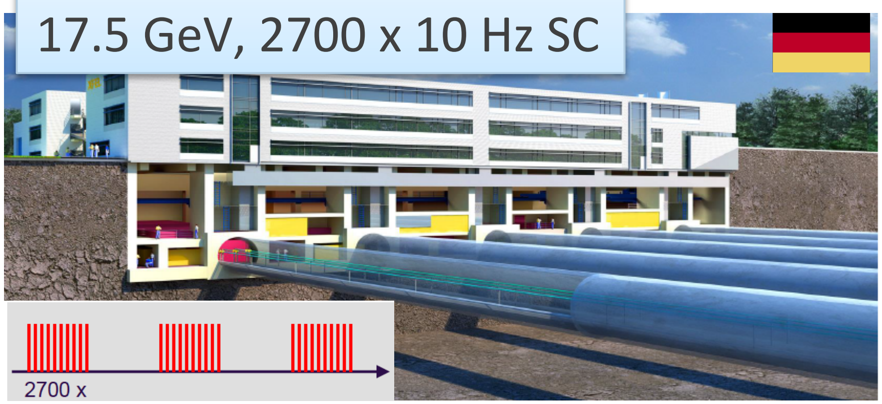
Hard X-Ray FELs in Operation & Under Construction



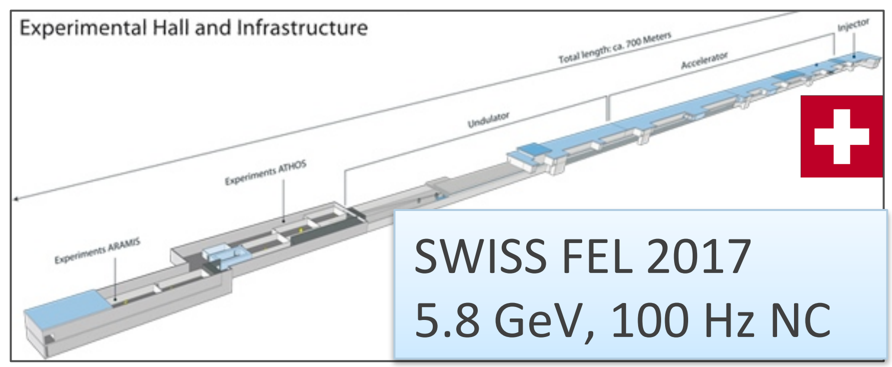
LCLS-I 2009
14.5 GeV, 120 Hz NC
LCLS-II 2020(?)
4 GeV, 1 MHz SC

SACLA 2011
8.5 GeV, 60 Hz NC

PAL XFEL 2015
10 GeV, 100 Hz NC



XFEL 2017
17.5 GeV, 2700 x 10 Hz SC



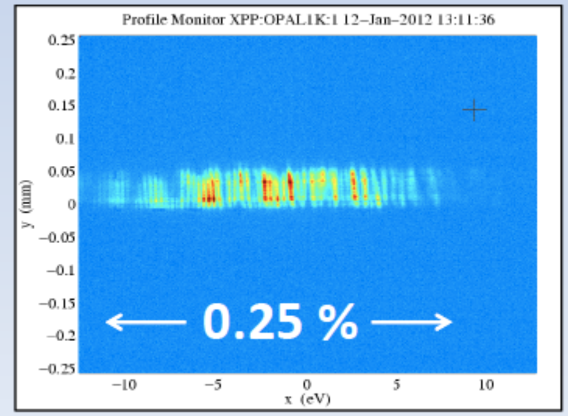
SWISS FEL 2017
5.8 GeV, 100 Hz NC

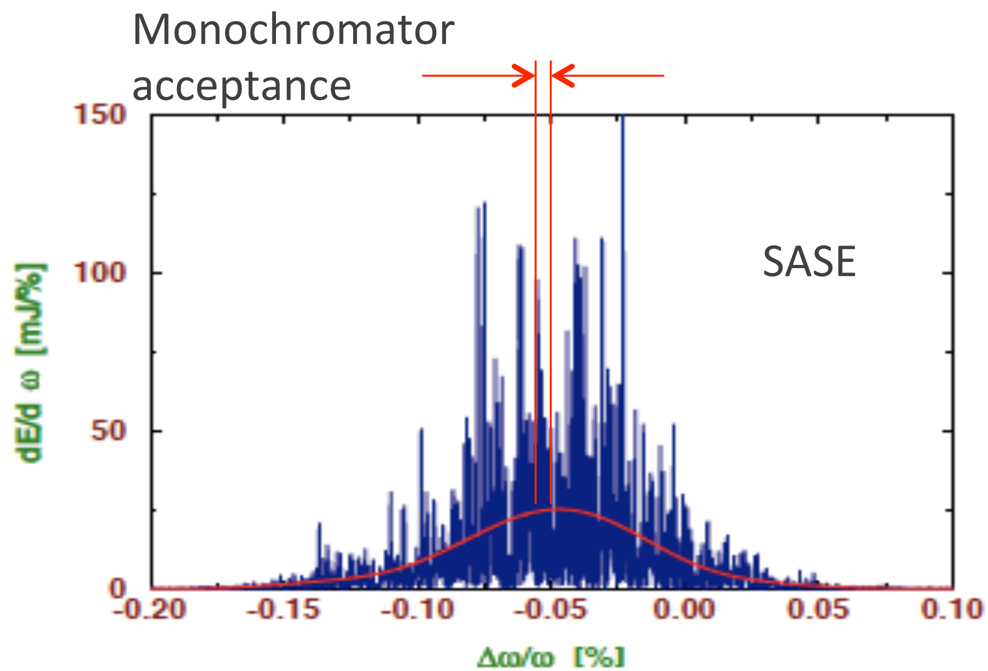
NC = normal conducting LINAC
SC = superconducting LINAC

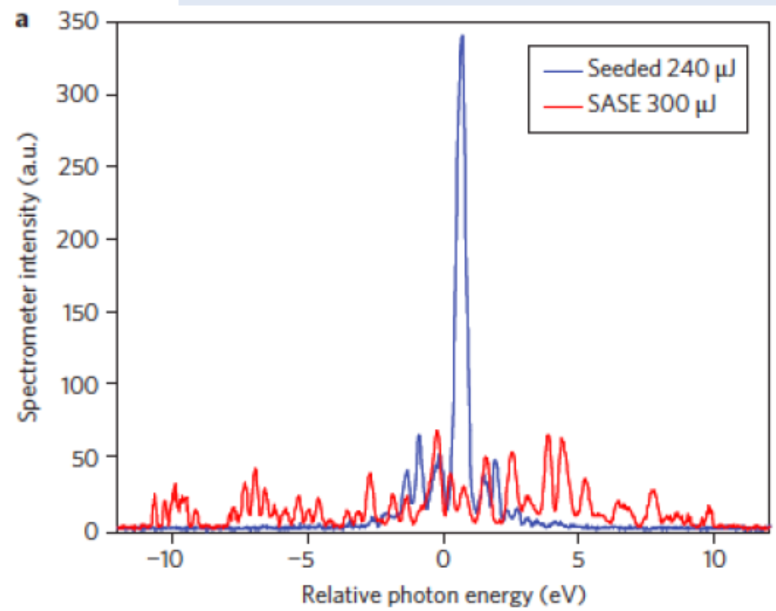
The Self-Amplified Spontaneous Emission (SASE) Process

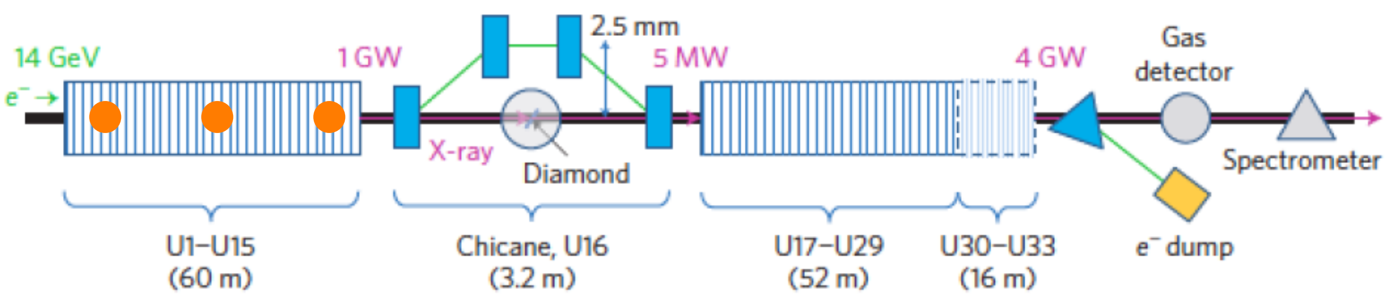
- The lasing starts up from the random micro-bunching (i.e., shot noise) on the electron beam instead of being coherently produced by an input “seed” source.

Measured SASE spectrum (LCLS)









One possible “seeding” geometry is to use a diamond crystal to select the wavelength of interest to amplify.

Summary

- Synchrotron radiation continues to be an important tool for researchers across all scientific and engineering disciplines.

- There is a strong science case for a new generation of sources such as:
  - X-ray free electron lasers
  - Low-emittance storage rings

- FELs can provide very short pulses (femtoseconds) of high peak intensity, coherent x-rays but the repetition rate is currently limited to hundreds of hertz.
  - The Linac Coherent Light Source II (LCLS II), under construction at Stanford, will increase that to MHz.

- Storage rings can produce partially coherent x-ray beams with megahertz repetition rates but with lower peak power.
  - The APS is working to incorporate a low emittance lattice into the proposed upgrade of the facility (APS-U) that will produce beams of high coherence at megahertz rates.

- Both LCLS II and APS-U will keep US SR facilities at the cutting edge to produce world class science in the years to come.

References

"Introduction to Synchrotron Radiation" by Giorgio Margaritondo

"Synchrotron Radiation Sources – A Primer" by Herman Winick

"Undulators, Wigglers, and their Applications" Edited by H. Onuki and P. Elleaume

"Elements of Modern X-ray Physics" by Jens Als-Nielsen and Des McMorrow

"Third-Generation Hard X-Ray Synchrotron Radiation Sources: Source Properties, Optics, and Experimental Techniques", Edited by Dennis M. Mills

"Handbook od Accelerator Physics and Engineering", Edited by A. Cho and M. Tigner

Appendix 1a: Radiated Power from Charges at Relativistic Velocities

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r, in the non-relativistic case, α is just the centripetal acceleration, v^2/r. In the relativistic case:

$$a = \frac{1}{m_o}\frac{dp}{d\tau} = \frac{1}{m_o}\gamma\frac{d\gamma m_o v}{dt} = \gamma^2\frac{dv}{dt} = \gamma^2\frac{v^2}{r}$$

Where $\tau = t/\gamma$ = proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}$$

Appendix 1b: Dependence on Mass and Energy of Radiated Power

$$P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e^- with the same total energy, E.)

2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

Appendix 2: BM Spectral Distribution

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c.

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and ρ ισ the radius of curvature of the trajectory. Note that the critical energy scales as γ^3. In practical units, the critical energy can be written as:

$$E_c[keV] = 2.218\ E^3[GeV]/\ \rho[m] = 0.06651\ B[kG]\ E^2[GeV]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at E = 7 GeV . The critical energy of the radiation emitted from the BM is:

$$E_c[keV] = 0.06651\ B[kG]\ E^2[GeV]$$

or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5\ keV}\ \text{or}\ \underline{0.64\ Å.}$$

Appendix 3: Where did "K" come from?

Equation of motion for a relativistic charged particle in a magnetic field

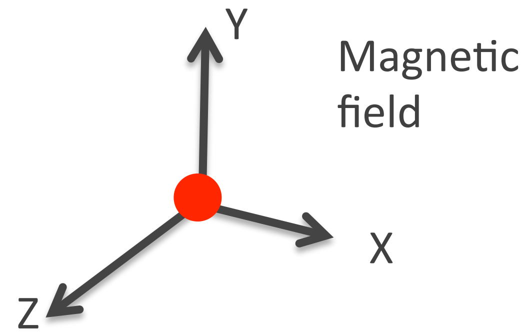
$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$\dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) \qquad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi c}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right) = -\frac{eB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c}\left[\frac{\lambda_{ID}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{ID}}\right) = \left[\frac{eB_0}{m_0}\frac{\lambda_{ID}}{2\pi c}\right]\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right) = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$x_{\max} = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \quad and \quad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \quad where \quad K = \left[\frac{eB_0}{2\pi}\frac{\lambda_{ID}}{m_0 c}\right]$$



Appendix 4: How Do You Get 1 Å X-rays from a 3 cm Period Magnetic Field?

Where does the $1/\gamma^2$ come from in the equation: $\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$?

1) Consider the electron in its rest frame:

- It does not see a static magnetic field from the undulator, but rather a time-varying B-field and associated E-field (due to the relativistic transformation of the magnetic field of the device).
- The period of the E and B field are Lorentz contracted so that: $\lambda_{e\text{-}frame} = \lambda_{ID} / \gamma$ and so the electron oscillates (and hence radiates) with that same period driven by the EM fields.

2) Back in the lab frame:

- Due to the fact that the electron is traveling towards us, the radiation emitted by the electron is Doppler shifted to higher frequencies (shorter wavelengths). The relativistic Doppler shift goes as $\sqrt{1-\beta} / \sqrt{1+\beta} \approx 1/2\gamma$, and so the wavelength observed in the lab is:

$$\lambda_{lab} \approx (\lambda_{ID}/\gamma)(1/2\gamma) = (\lambda_{ID}/2\gamma^2)$$

Appendix 5: Calculating X-ray Beam Brightness

APS Electron Beam Parameters

APS operates with $\varepsilon_H = 3 \times 10^{-9}$ m-rad and a coupling (ratio of vertical emittance to horizontal emittance) of 0.9% , therefore

$$\varepsilon_V = 0.025 \times 10^{-9} \text{ m-rad.}$$

The particle beam source size and divergence at the locations of the IDs are:

$$\sigma_H = 270 \text{ microns}$$
$$\sigma'_H = 11 \text{ microradians}$$
$$\sigma_V = 9 \text{ microns}$$
$$\sigma'_V = 3 \text{ microradians}$$

APS Undulator X-ray Beam Parameters

APS Undulator A has a length of 2.4 meters. For 1Å radiation the natural opening angle is:

$$\sigma'_{x\text{-ray}} = 1/\gamma\,(1/N)^{1/2}$$
$$= [\lambda/2L]^{1/2} = 4.5\text{x}10^{-6} \text{ rad}$$

The corresponding source size of the radiation is:

$$\sigma_{x\text{-ray}} = [\lambda L/8\pi^2]^{1/2} = 1.7 \text{ microns.}$$

For source size $\Sigma = [\sigma_{x\text{-ray}}{}^2 + \sigma_{electron}{}^2]^{1/2}$ and source divergence $\Sigma' = [\sigma'_{x\text{-ray}}{}^2 + \sigma'_{electron}{}^2]^{1/2}$

$$\mathcal{B} = \text{Flux}/4\pi^2\, \Sigma_H\, \Sigma_V\, \Sigma_H'\, \Sigma_V'$$