25th Annual NXSchool

SYNCHROTRON RADIATION: PRODUCTION & PROPERTIES

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National School for Neutron and X-ray Scattering
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AGENDA

1. A short history related to x-rays
2. Properties of radiation from relativistic electrons
3. Transverse properties of electron beams in storage rings
4. Next generation light sources

SOME HISTORY

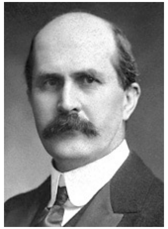
X-RAYS WERE A PUZZLE

- Röntgen discovery of X-rays was made by in 1895 and he first thought they might be some sort of **longitudinal vibrations of the ether.**
- These ”rays” were not deflected by magnetic fields, so carried no charge, and, at the time, refraction by a prism could not be measured (we’ll see why in the talk about x-ray optics).
- Others, including Charles Barkla who studied the polarization of x-rays (Physics Nobel prize winner in 1917 for this and other work on x-ray scattering), thought they were an **extension of the visible light spectrum.**
- William Bragg (Physics Nobel Prize winner in was a strong proponent of x-rays being **”particles” or corpuscular in nature**.
- In 1896 Lord Kelvin sent G. G. Stokes a letter ending thus: “*In respect of the Rontgen X-rays, are you a longitudinalist, or an ultravioletist, or a tertium quidist? “*
- This finally got sorted out – x-rays were electromagnetic waves – but apparently the discussion was lively between the two future Nobel laurates; Bragg (1915) and Barkla (1917).
- In his 1927 Nobel acceptance speech, Compton stated**:** **“*Many will recall also the heated debate between Barkla and Bragg, as late as 1910, one defending the idea that X-rays are waves like light, the other that they consist of streams of little bullets….*”.**

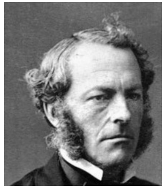
Wilhelm Röntgen
1845 – 1923

Charles Barkla
1877 – 1944

William Bragg
1862 – 1942



G. G. Stokes
1819 – 1903



Lord Kelvin
1824 – 1907

AND SPEAKING OF COMPTON…

2023 is the 100th Anniversary of Compton's Paper on X-rays Scattered by Light Elements

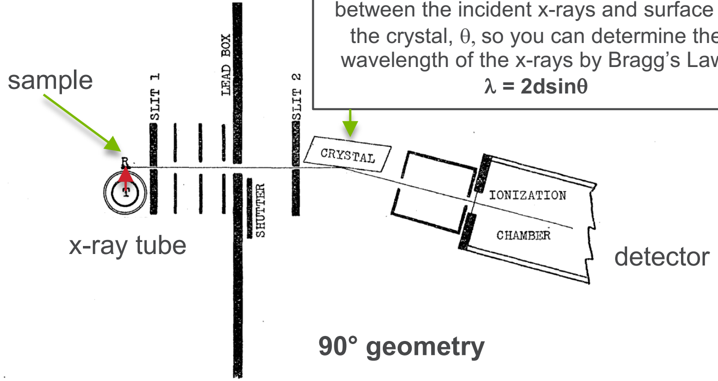
- In the early 1920's, Arthur Compton begins to study the scattering of x-rays from low-Z materials.

- Using a crystal spectrometer, he found that there is a peak at the energy corresponding to that of the incident x-ray energy plus a peak at a lower x-ray energy (longer wavelength) – and that the energy of the peak varies with scattering angle.

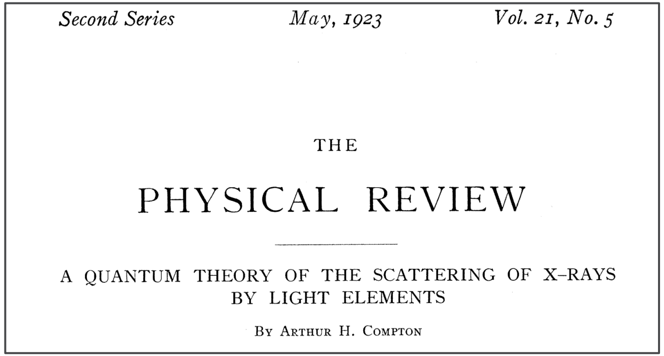
Arthur Compton
9/10/1892 – 3/15/ 962

Crystal Spectrometer

Crystal Analyzer – you know the crystal d-spacing and you can measure the angle between the incident x-rays and surface of the crystal, θ, so you can determine the wavelength of the x-rays by Bragg's Law:
λ = 2dsinθ



90° geometry



Second Series *May, 1923* *Vol. 21, No. 5*

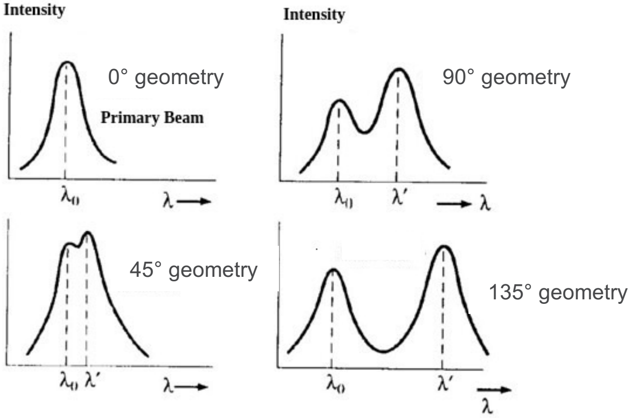
THE

PHYSICAL REVIEW

A QUANTUM THEORY OF THE SCATTERING OF X-RAYS BY LIGHT ELEMENTS

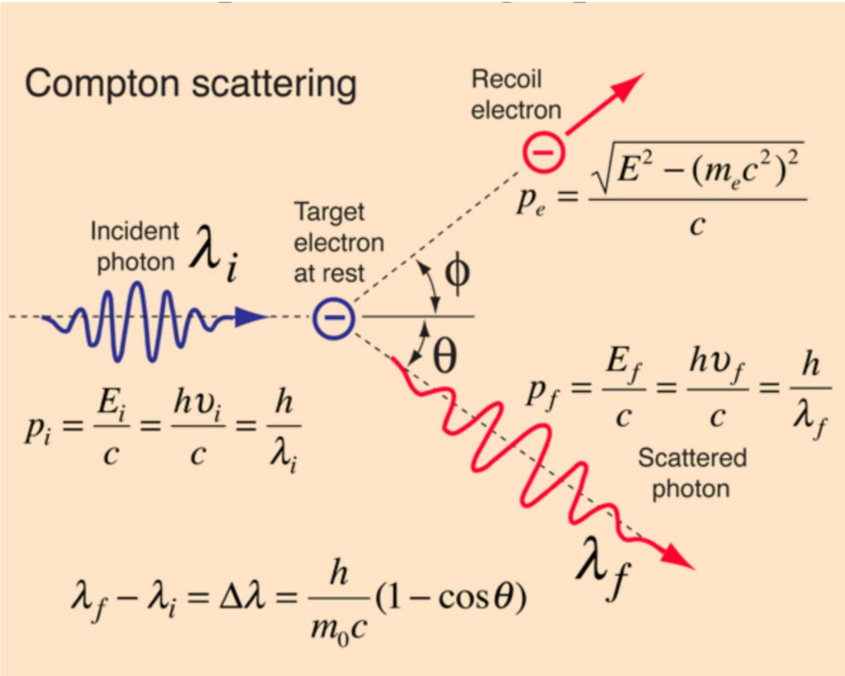
By Arthur H. Compton

AIP Emilio Segrè Visual Archives



WAVE-PARTICLE DUALITY OF LIGHT

Compton realized that the classical theory of an electromagnetic wave cannot explain shifts in wavelength when scattered.



Compton’s experiments represented a fundamental “turning point in physics,” says historian Roger Stuewer of the University of Minnesota in Minneapolis, author of a book on Compton’s work.

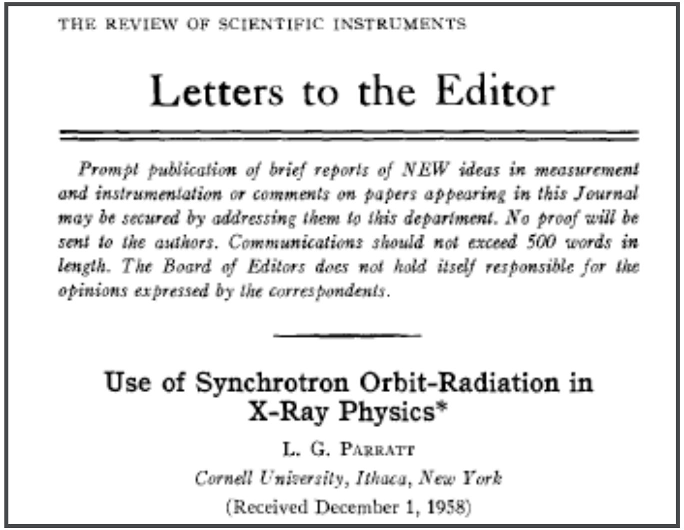
Impact of his work:

- Compton’s explanation of his measurements is important because it **demonstrates that light cannot be described purely as a wave phenomenon**.
- For a shift in wavelength to occur, radiation must behave as a particle (what we now call a photon – he did not use that term in his 1923 paper).
- This was an experimental proof that established the validity of the quantum theory of light, stating that light is constituted of individual units which can carry certain quantities of energy and momentum depending on its wavelength.
- Compton was awarded the 1927 Nobel Prize in physics *"for his discovery of the effect named after him"*

OPPENHEIMER
In 1941 Compton led a committee to look into the feasibility of a uranium-based weapon. Compton was Director of the Metallurgical Lab at the U of Chicago when in June 1942 the US Army Corp of Engineers assumed control of the US nuclear weapons program and Compton's Met Lab became part of the Manhattan Project. That month, Compton asked Robert Oppenheimer (Berkeley) to coordinate weapon theory and ongoing fast-neutron research at half a dozen universities. Later that month, General Leslie Groves appointed Oppenheimer as the scientific director of the Manhattan Project.

DISCOVERY OF SYNCHROTRON RADIATION

- Synchrotron Radiation (SR) - was first observed from a 70 MeV synchrotron at GE in Schenectady in 1947.



THE REVIEW OF SCIENTIFIC INSTRUMENTS

Letters to the Editor

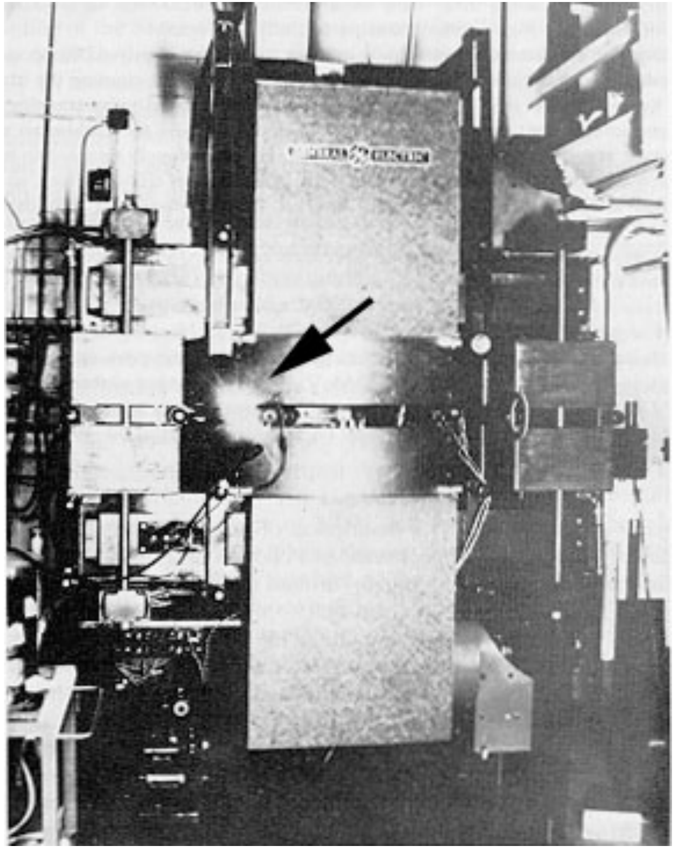
Prompt publication of brief reports of NEW ideas in measurement and instrumentation or comments on papers appearing in this Journal may be secured by addressing them to this department. No proof will be sent to the authors. Communications should not exceed 500 words in length. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Use of Synchrotron Orbit-Radiation in X-Ray Physics*

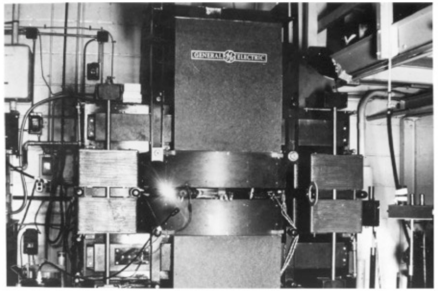
L. G. Parratt

Cornell University, Ithaca, New York

(Received December 1, 1958)

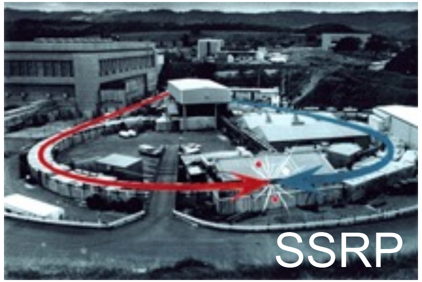


- Storage rings, accelerators that store electrons at a constant energy, provided a far more attractive source. We now use the name synchrotron radiation to describe radiation that is emitted from charged particles traveling at relativistic speeds, regardless of the accelerating source.



General Electric Research Lab

1947



1st Generation (1974-1980's)

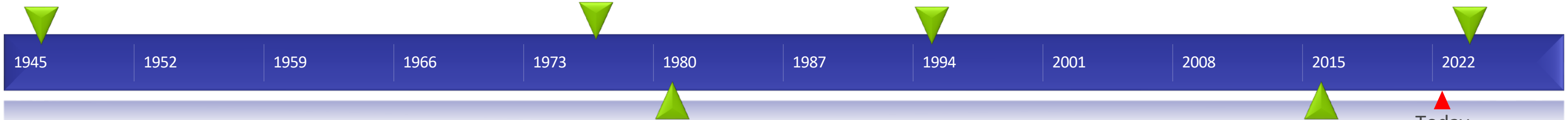
APS

3rd Generation (1994-2007)

1995

APS-U

2024



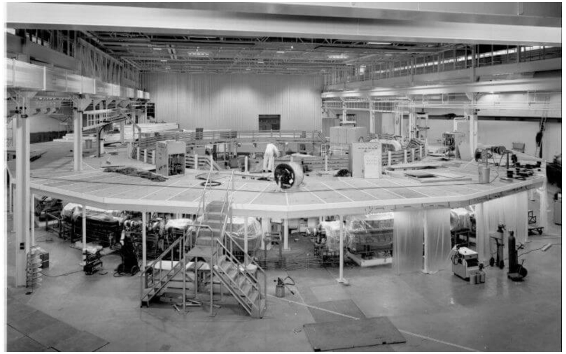
1981

2nd Generation (1981-1986)

2016

4th Generation (2016- ...)

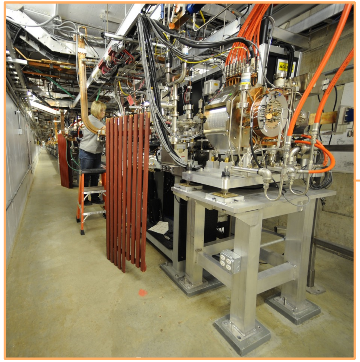
SRS

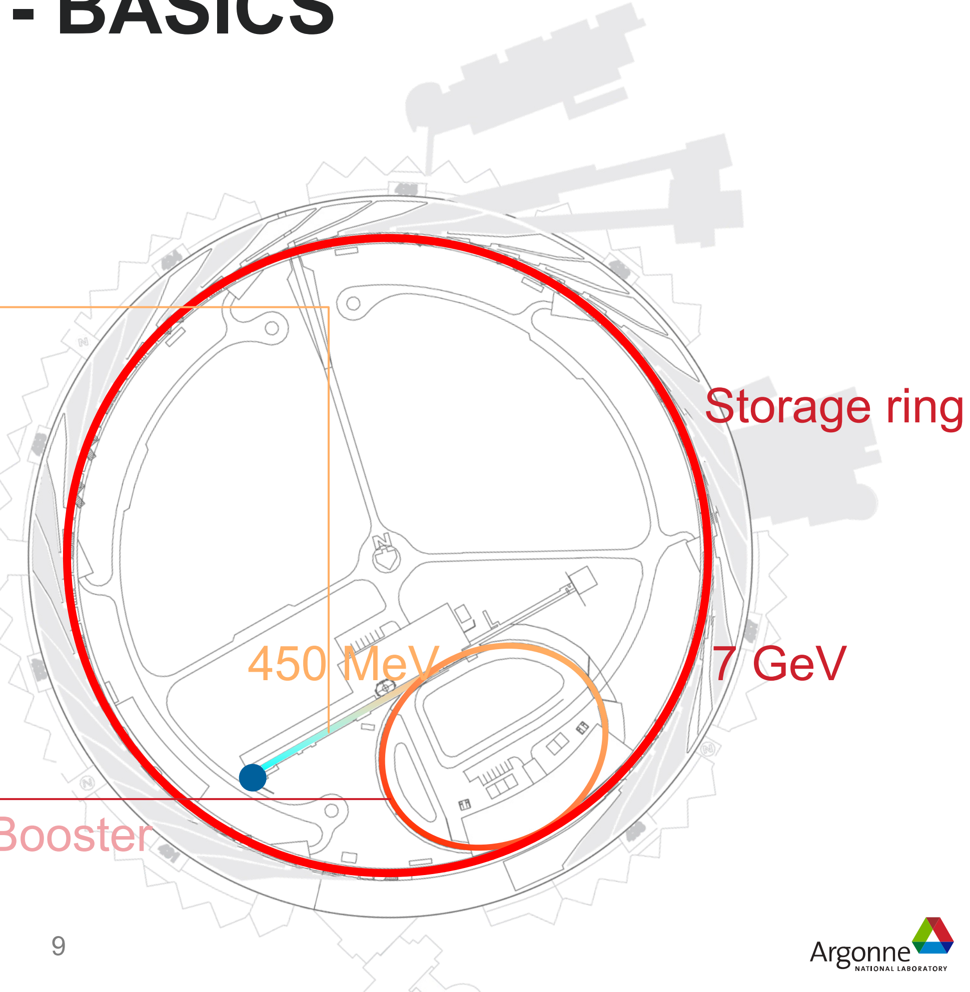


Max-IV

For early history : Blewett, J. P. (1998). 50 YEARS OF SR Synchrotron Radiation ± Early History. *Journal of Synchrotron Radiation*, *5*, 135–139.

X-RAY SYNCHROTRON - BASICS





PROPERTIES OF SYNCHROTRON RADIATION

RADIATION PATTERNS FROM ACCELERATING CHARGES

Definitions:

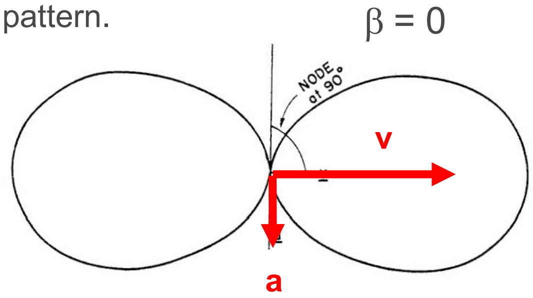
$$\beta = v/c$$

$$\gamma = 1/(1-\beta^2)^{1/2} = E/m_oc^2$$

Where:

v = velocity of the electron
c = speed of light
E = electron energy
m_oc^2 = electron rest mass

When an electron is accelerated and its velocity, v, much less the speed of light, c, ($\beta \approx 0$), the distribution of the emitted radiation is a classical dipole pattern.



This drawing is for an electron going in a circle – acceleration, a, is inward and instantaneous velocity, v, is in the direction of a tangent to that circle.

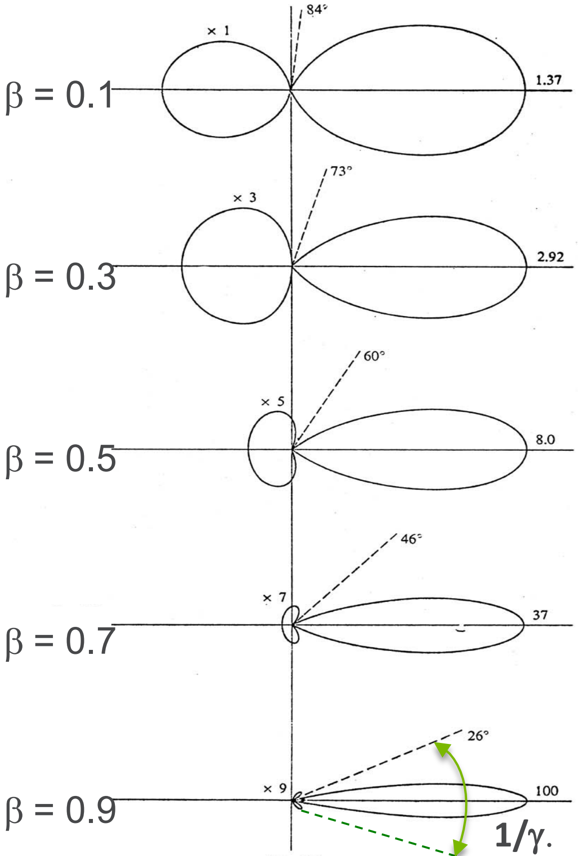


FIG. 7-7

But as β approaches 1, the shape of the radiation pattern changes; it is more forward directed

The angular divergence of the emitted radiation (sometimes called the opening angle of the radiation) ≈ **1/γ.**

RADIATION FROM HIGHLY RELATIVISTIC $(\gamma >> 1)$ PARTICLES IN A CIRCULAR ORBIT

LET'S LOOK AT AN ACCELERATOR WITH THE PARAMETERS OF THE ADVANCED PHOTON SOURCE

At the APS with E = 7 GeV,

$\gamma = E / m_o c^2 = 7\ \text{GeV} / 0.511\ \text{MeV}$

$\gamma = 1.4 \times 10^4$

$\beta \approx 1 - 1/(2\gamma^2)$

$\beta \approx 1 - (2 \times 10^{-9}) = 0.999999998$

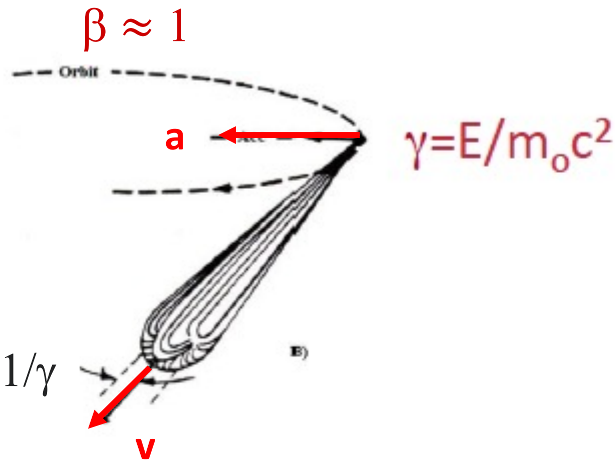
$v = 0.999999998c$

Recall that the divergence (or opening angle) of the x-ray beam is given by $1/\gamma$:

$1/\gamma = 73 \times 10^{-6}$ (73 μradians)

or about

15 arc-seconds

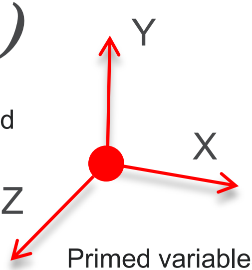


Relativistic velocities are good!!

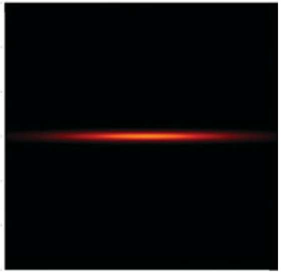
- radiation forward directed
- radiated power ∝ E^4

See Appendix 1 for more details

Coordinate system used in this presentation.

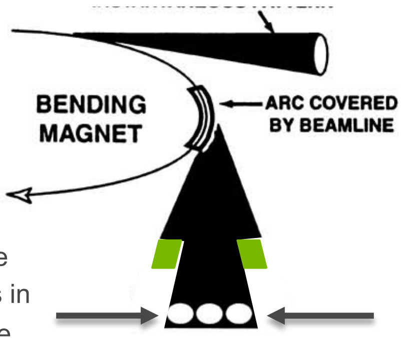


Primed variables (x', y') are angular coordinates.

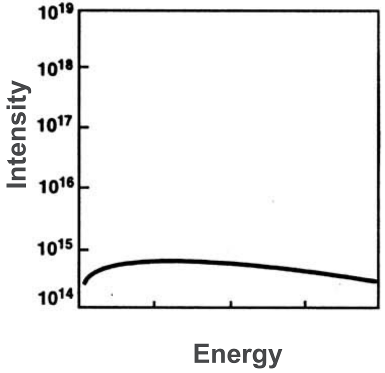


The cross-section of the APS electron beam, which is the source of the x-rays, is about 10μ in the vertical (y) direction and about 300μ in the horizontal (x)-direction.

BEND MAGNET (BM) X-RAY RADIATION PROPERTIES



Horizontal opening angle determined by apertures in the front-end or beamline.



You get this for “free” since you need the bending magnets to keep the electrons orbiting in the storage ring.

Bend Magnet Radiation

Spectrum characterized by the critical energy:

E_c[keV] = 0.066 B[kG] E^2 [GeV]   ($E_c \approx$ 20 keV @ APS)

All the BMs have the same B-field to maintain a closed orbit, so spectrum from each BM is identical.

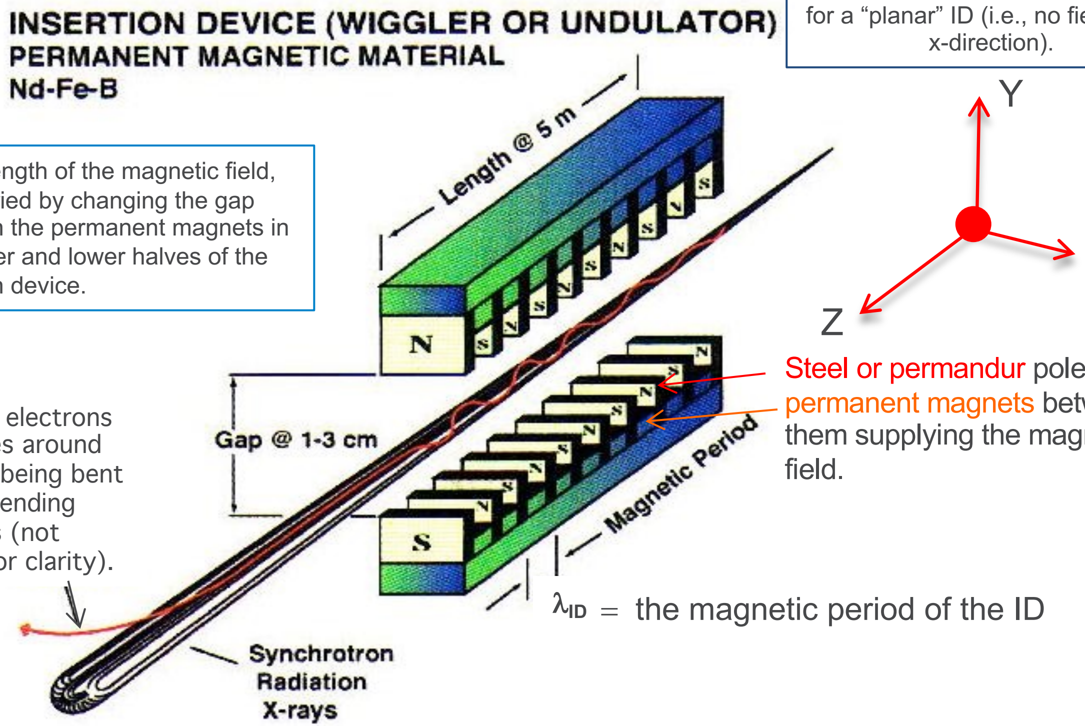
Recall: λ[Å] = 12.4/E[keV] so 20 keV is 0.62Å

See Appendix 2 for more details

PLANAR INSERTION DEVICES (IDs)

IDs are “inserted” in straight sections of the storage ring between bend magnets.

- Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.
- “Planar” refers to the magnetic field being in one direction (in this case vertical or y-direction).
- IDs can have fields in both the vertical and horizontal directions to produce circularly polarized x-rays and for other applications.



Magnetic field is in the y-direction for a “planar” ID (i.e., no field in the x-direction).

The strength of the magnetic field, B, is varied by changing the gap between the permanent magnets in the upper and lower halves of the insertion device.

Beam of electrons continues around the ring being bent by the bending magnets (not shown for clarity).

Steel or permandur poles with permanent magnets between them supplying the magnetic field.

λ_{ID} = the magnetic period of the ID

CHARACTERIZING INSERTION DEVICES

- IDs are characterized by the so-called field index or deflection parameter, K

Important thing here is K is proportional to the peak magnetic field, B_o.

$$K = eB_o\lambda_{ID}/2\pi m_o c = 0.0934\ \lambda_{ID}[\mathrm{cm}]\ B_o[\mathrm{kG}]$$

where λ_{ID} **is the magnetic period** of the insertion device and **B_o the peak magnetic field**. The length of the insertion device, L, is equal to the number of periods, N, times the length of the period, i.e.,

$$L = N\lambda_{ID}\ .$$

- The maximum deflection angle of the particle beam, $x'_{ID,max}$, is given by:

$$x'_{ID,\,max} = \pm(K/\gamma)$$

See Appendix 3 for more details on K

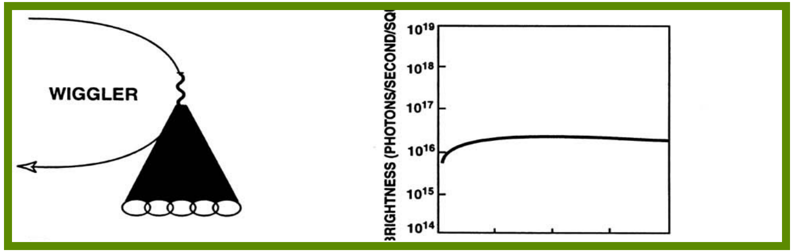
WIGGLER (K>> 1)

Horizontal deflection angle of electrons given by $x'_{\mathrm{ID,\,max}} = (K/\gamma)$

$$x'_{\mathrm{ID,\,max}} >> 1/\gamma$$

Radiation spectrum looks like 2N dipole sources (N = number of periods)

Top View



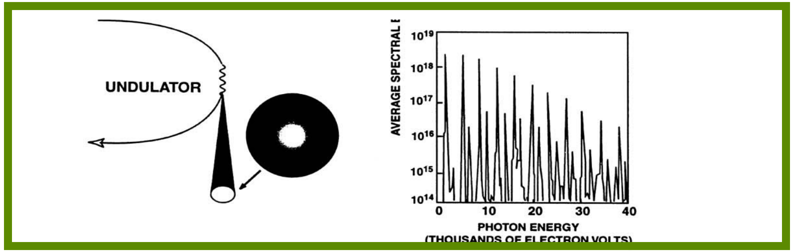
UNDULATOR (K ≈ 1)

When the maximum deflection of the electron beam is on the order of the natural opening angle of the emitted radiation, i.e. K ≈ 1

$$x'_{\mathrm{ID,\,max}} \approx 1/\gamma\ ,$$

So now the radiation from each of the magnetic poles overlaps.

Top View



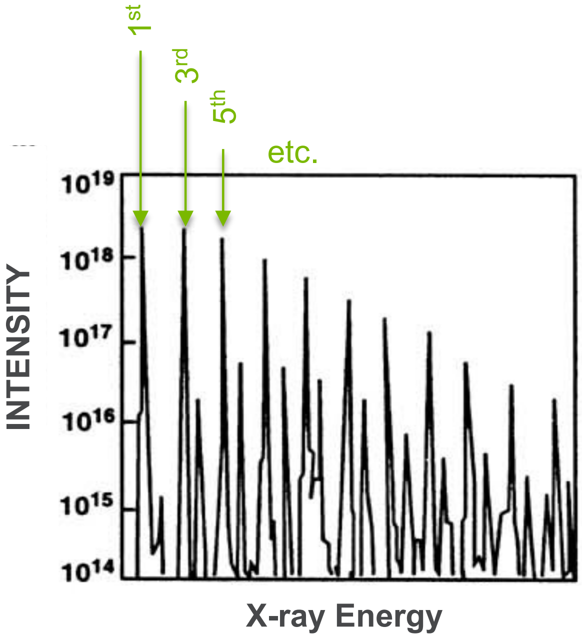
UNDULATOR RADIATION SPECTRA – IT'S COMPLICATED

- This overlap can cause interference effects in the spectrum.
- On-axis (x' = y' = 0), the constructive interference occurs at a particular x-ray energy ($E_n^{x\text{-ray}}$) and its odd harmonics, i.e. n = 1,3, 5… given by:

$$E_n^{x\text{-ray}} = [nhc\,\gamma^2] / [\pi\, \lambda_{ID}(1 + K^2/2)]$$

where: h=Planck's constant and c the velocity of light.

- Recall that K is proportional to the peak magnetic field generated by the undulator.
  - In permanent magnet undulators vary the gap between the upper and lower poles to change magnetic field
  - In a superconductor undulator, vary the current in the windings to change magnetic field



If you put a pinhole at x'=y'=0, and scan the monochromator, the output will look like this.

UNDULATOR RADIATION OPENING ANGLE

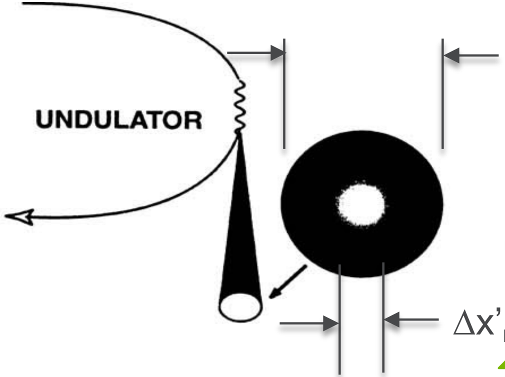
- At the fundamental (1st harmonic or n = 1), the horizontal and vertical opening angles of the radiation is given by:

$$\Delta x'_{rad} = \Delta y'_{rad} \approx (1/\gamma)\,[1/N]^{½}$$

where N = number of periods [typically 100 or more].

- So the opening angle of the harmonics are much less than the natural opening angle, $(1/\gamma)$, by as much as 10 or 12.

- This narrowing of the radiation beam's divergence only occurs at energies, $E_n^{x\text{-ray}}$. This low divergence cone, embedded in the larger power envelope, is sometimes called the central cone of the undulator beam.



Power envelope (integrated over all photon energies)

$\Delta x'_{rad} = \Delta y'_{rad} = (1/\gamma)\,[1/N]^{½}$

I've labeled the x-ray radiation properties with the subscript "rad" to distinguish it from the electron beam source size and divergence that will be discussed later.

THE DIFFRACTION LIMIT FOR LIGHT

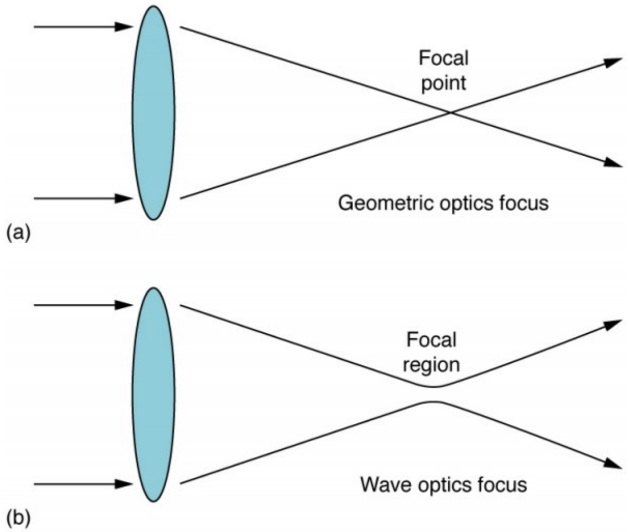
- Just as the Uncertainty Principle sets a lower limit for the product of the size, Δx, and momentum Δp_x, this relationship can be re-written in terms of size Δx_{rad} (Δy_{rad}) and angular divergence $\Delta x'_{rad}$ ($\Delta y'_{rad}$) for radiation.

$$\Delta x \, \Delta p_x, \geq h / 4\pi \longrightarrow \Delta x_{rad} \, \Delta x'_{rad} \geq \lambda / 4\pi \text{ and } \Delta y_{rad} \, \Delta y'_{rad} \geq \lambda / 4\pi$$

See Appendix 4 for details

- A light beam that satisfies $\Delta x_{rad} \, \Delta x'_{rad} = \lambda / 4\pi$ is called **diffraction limited.** When a radiation beam has this property, it is a **fully coherent source**.

- Another way to define a **diffraction limited beam** is if its potential to be focused to small spots is as high as possible for its wavelength.



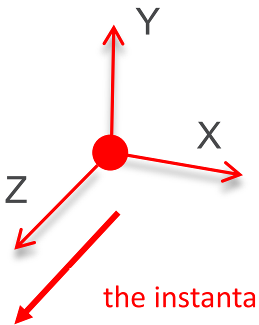
What's the diffraction limit at 1 Å ?

$$\lambda / 4\pi = 1Å / 4\pi = 10^{-10} \text{ meters} / 4\pi$$
$$\lambda / 4\pi \approx 10 \times 10^{-12} \text{ m}$$

10 picometers-radian (radians are dimensionless)

ELECTRON BEAM PROPERTIES ARE KEY

- So far, this discussion has not taken into account the properties of the source of the x-rays – namely the electron beam.
- The electron beam has both a size and divergence in the x- and y-directions (transverse to the velocity of the beam).
- The electron beam also has a finite length in the z-direction (along the direction of the velocity of the beam – sometimes called the longitudinal properties).
- To fully understand the properties of the emitted x-ray beams, we need to look a little more closely at the (transverse) properties of the electron beams.



the instantaneous velocity of the electrons

TRANSVERSE PROPERTIES OF THE ELECTRON BEAM

TRANSVERSE (X & Y) ELECTRON BEAM PROPERTIES

- Although the flux from undulators can be determined without knowledge of the source size and divergence, one very important characteristic of the x-ray beam, namely **brightness,** requires a more detailed knowledge of the particle beam’s size and divergence.

Brightness has units of: photons/sec/0.1% BW/source area / source divergence

$$\text{Flux}/4\pi^2\,\Sigma_x\,\Sigma_y\,\Sigma_x'\,\Sigma_y'$$

this is the monochromaticity of the beam

where Σ_i (Σ_i') is the **effective** one sigma value of the source size (divergence) in the ith direction. The total source size and divergence is the quadrature sum of the electron beam and the radiation beam, namely:

We previously determined Δx_{rad} and $\Delta x'_{rad}$ for undulator radiation

$$\Sigma_x = [\underbrace{\Delta x^2_{rad}}_{\text{radiation beam size}} + \underbrace{\Delta x^2_{electron}}_{\text{electron beam size}}]^{1/2}$$

and

$$\Sigma_x' = [\underbrace{\Delta x'^2_{rad}}_{\text{radiation beam divergence}} + \underbrace{\Delta x'^2_{electron}}_{\text{electron beam divergence}}]^{1/2}$$

and a similar expression for Σ_y

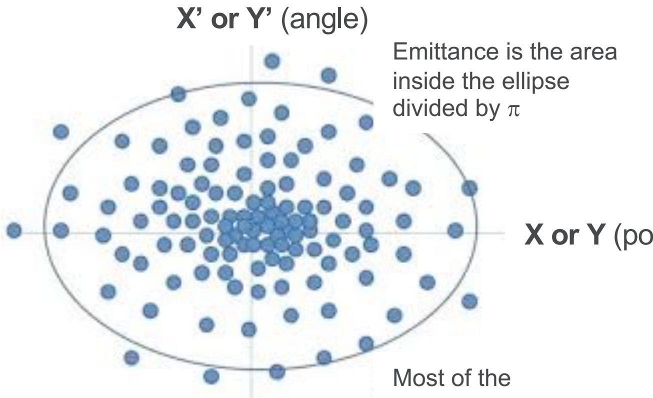
and a similar expression for Σ_y'

ELECTRON BEAM PHASE SPACE AND EMITTANCE

- Accelerator physicists describe the electron beam in terms of its horizontal (x-plane) and vertical (y-plane) position and divergence or it's **phase space**.
- The area bounded by an ellipse that captures some given fraction of electrons in this xx' (yy') phase space is proportional to what is called the horizontal (or vertical) emittance of the electron beam.
- There is a separate horizontal emittance (ε_x) and vertical emittance (ε_y). In today's storage rings:

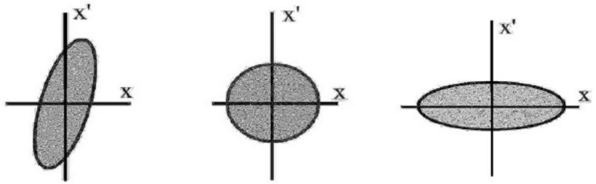
ε_y is typically 1% of ε_x (the percentage is called the coupling)

- The emittance is a constant of the storage ring, although one can trade off electron beam size for divergence as long as the area of the phase-space remains constant.
- To determine the x-ray beam brightness, you need to know the electron beam source size and divergence at the spot in the storage ring where the undulator is located and add those values - in quadrature - with source size and divergence of the radiation.



Emittance is the area inside the ellipse divided by π

Most of the electrons are inside the circle



This is a consequence of Liouville's Theorem – the conditions for which are satisfied in most accelerators.

DIFFRACTION LIMITED SOURCES AND COHERENCE

- If both the horizontal and vertical emittance of the particle beam were small compared to the diffraction limit of the light ($\lambda / 4\pi$) , then the x-rays that are emitted would **have full transverse coherence**.

- But at the APS, as with most 3rd generation synchrotron radiation sources, the electron beam emittance dominates - see figure below.

$\varepsilon_x \approx 3 \times 10^{-9}$ *m-rad or* ***3000 pm-rad***

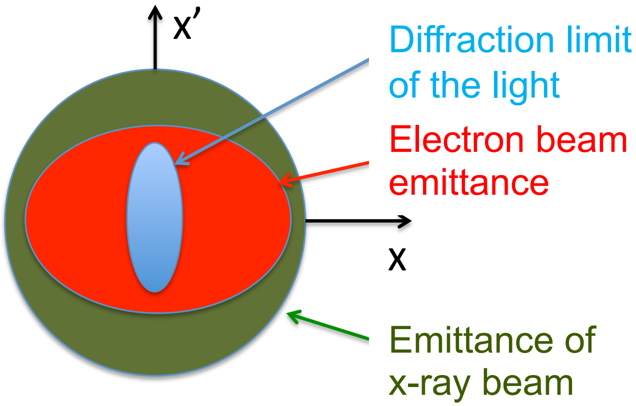
$\varepsilon_y \approx 0.03 \times 10^{-9}$ *m-rad or* ***30 pm-rad***

- Hence the radiation is **partially coherent**

- Partially coherent sources are characterized by the **coherent fraction**, the fraction of the x-ray flux that is coherent.

- Coherent fraction = ratio of diffraction-limited emittance to actual beam emittance.

- For the APS at 1Å, the coherent fraction is ≈ 10^{-3} – useful for experiments that rely on coherence but at the expense of throwing away a lot of (incoherent) flux.



Diffraction limit of the light

Electron beam emittance

Emittance of x-ray beam

See lectures by S. Hruszkewycz, L. Lurio, and others.

NEXT GENERATION LIGHT SOURCES

THE DRIVE FOR MORE COHERENT X-RAY SOURCES

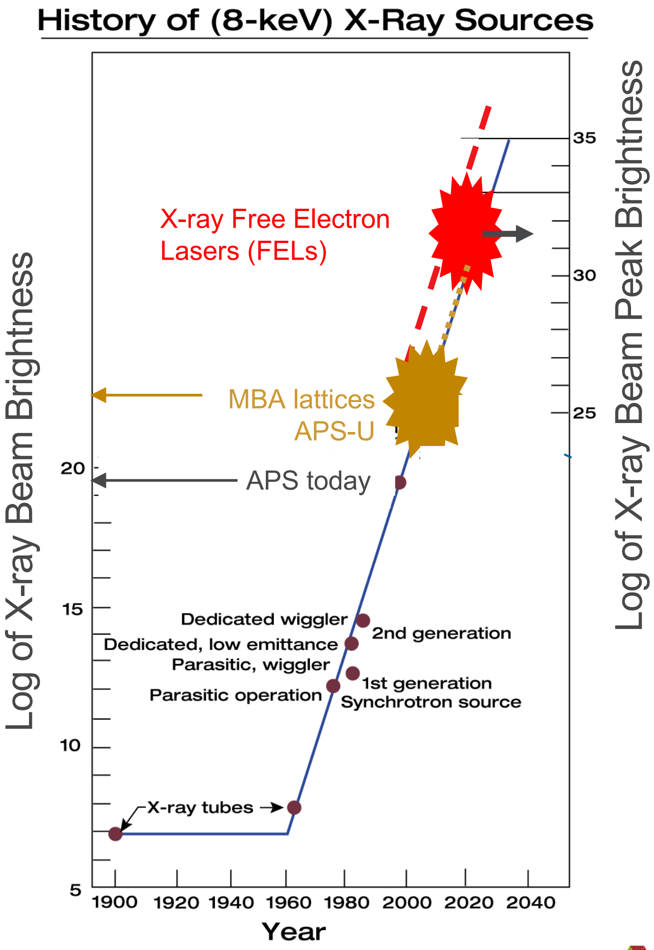
- How do do we get from where we are today:

  ε_x = 3000 picometer-radian

  to something closer to the 10 picometers-radian emittance for a fully coherent (i.e., high brightness) x-ray beam at 1Å?

- Presently there are two approaches to obtain a more coherent x-ray source:
  - Storage rings with so-called Multi-bend Achromat (MBA) magnet structures
  - Using LINACs that satisfy the conditions for a free electron laser (FEL) - – *See Appendix 6*

See presentation by P. Fouss for details on FELs



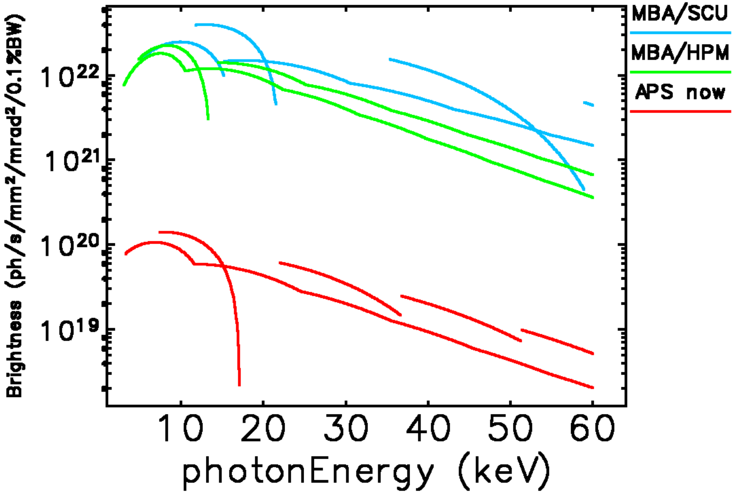
THE APS-U PROJECT: A MULTI-BEND ACHROMAT (MBA) LATTICE

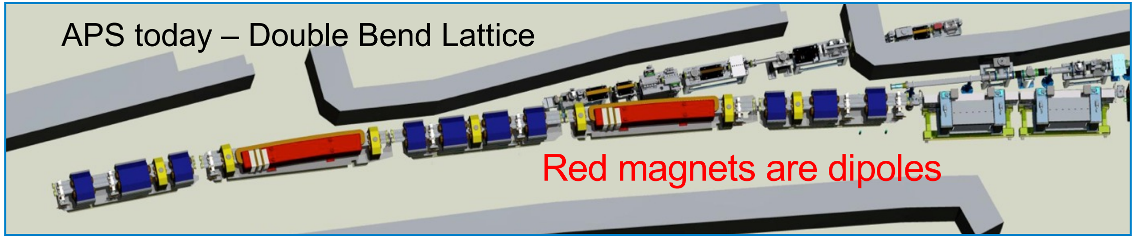
$$\varepsilon_x = C_L \frac{E^2}{N_D^3}$$

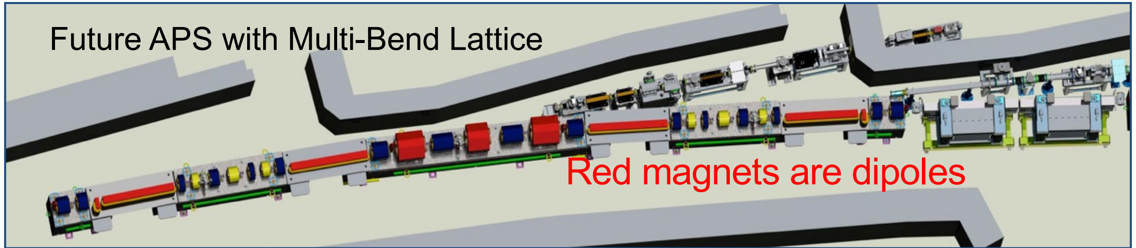
C_L = constant
E = beam energy
N_d = dipoles per sector

APS MBA Upgrade:
Beam energy: 7 GeV => 6 GeV
Dipoles per sector: 2 => 7

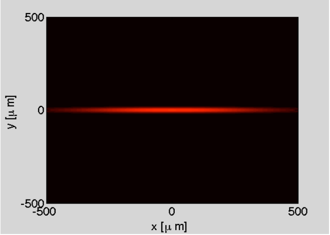
10^2 to10^3 X increase in brightness

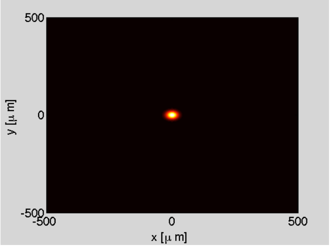




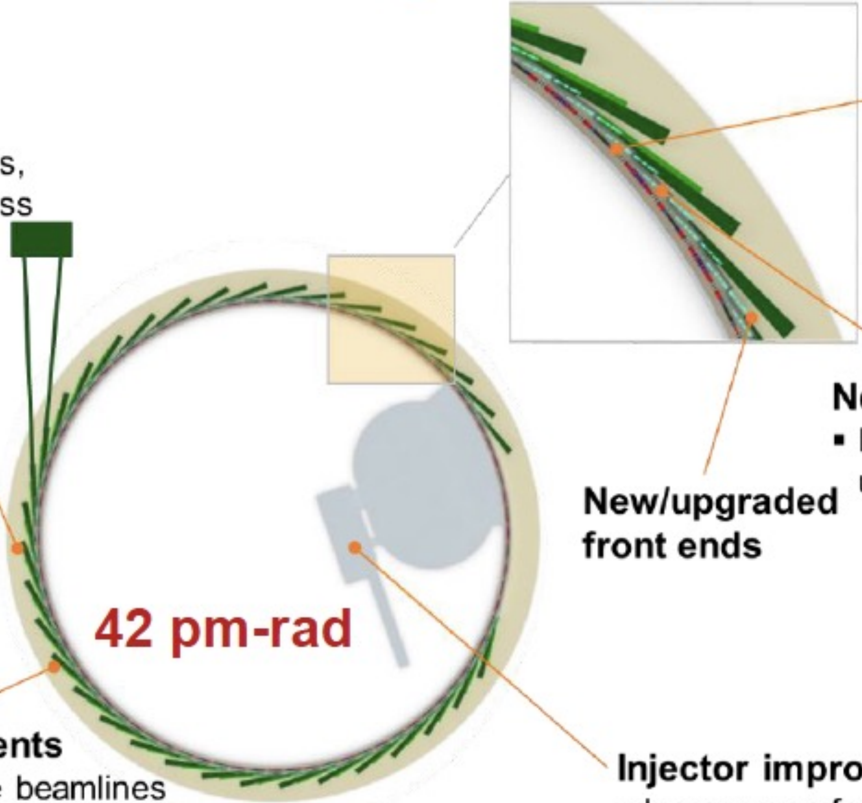


~50-fold reduction in horizontal emittance





APS-U Project Scope



Feature beamlines
- Suite of beamlines, including long beamlines, designed for best-in-class performance

New storage ring
- 6 GeV with 200 mA, 42 pm-rad emittance
- Hybrid 7BA lattice with reverse bends
- Improved electron and photon stability

New insertion devices
- Including superconducting undulators

New/upgraded front ends

Total Project Cost - $815M

Beamline enhancements
- Improvements to make beamlines "Upgrade Ready"
- Existing beamlines are planned to come back on-line after the upgrade

Injector improvements
- Increase performance beyond present capability

On-axis "swap-out" injection

MBA UPGRADES – A COMPETITIVE LANDSCAPE

ESRF (France)

- ESRF/Extremely Bright Source (EBS) is operational

MAX-IV (Sweden)

- A new (green field site) 3 GeV MBA lattice-based storage ring that is operational

SIRIUS (Brazil)

- A new (green field site) 3 GeV MBA lattice-based storage ring that is operational

ALS-U (US)

- In the design phase; operational late 2020's

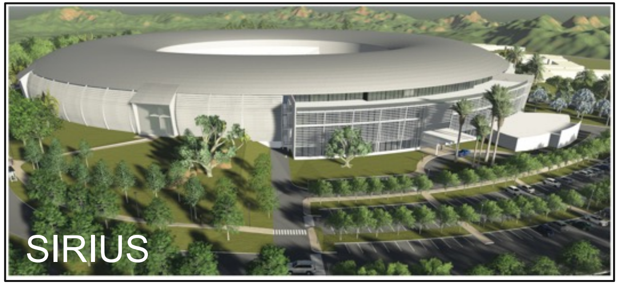
SPRING-8 (Japan)

- Planning an MBA upgrading in 2020's

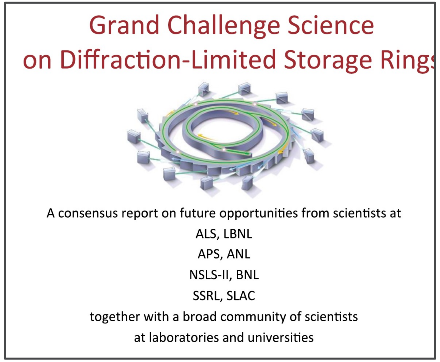
4 July 2014 Last updated at 06:33 ET

Upgrade for Europe's big X-ray light source

By Jonathan Amos
Science correspondent, BBC News



SUMMARY



Grand Challenge Science
on Diffraction-Limited Storage Rings

A consensus report on future opportunities from scientists at
ALS, LBNL
APS, ANL
NSLS-II, BNL
SSRL, SLAC
together with a broad community of scientists
at laboratories and universities

- Synchrotron radiation continues to be an important tool for researchers across all scientific and engineering disciplines.
- There is a strong science case for a new generation of sources such as:
  - Low-emittance storage rings
  - X-ray free electron lasers
- Storage rings can produce partially coherent x-ray beams with megahertz repetition rates but with lower peak intensity than free electron lasers (FELs).
- FELs can provide very short pulses (femtoseconds) of high peak intensity, coherent x-rays but the repetition rate of many FELs limited to hundreds of hertz.
  - Superconducting radio frequency (SCRF) linacs will increase rep rates that to MHz (LCLS II and the European XFEL).
- The LCLS II, APS-U, and the ALS-U will keep US x-ray facilities at the cutting edge to produce world class science in the years to come.

REFERENCES

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"Elements of Modern X-ray Physics" by Jens Als-Nielsen and Des McMorrow

"Third-Generation Hard X-Ray Synchrotron Radiation Sources: Source Properties, Optics, and Experimental Techniques", Edited by Dennis M. Mills

"Synchrotron Radiation and Free-Electron Lasers: Principles of Coherent X-Ray Generation", by Kwang-Je Kim, Zhirong Huang and Ryan Lindberg

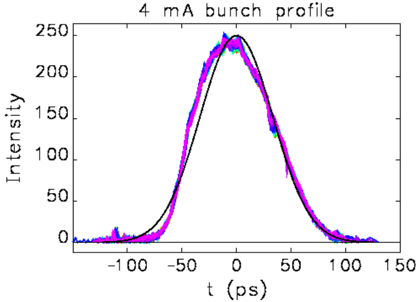
"Handbook of Accelerator Physics and Engineering", Edited by A. Cho and M. Tigner

QUESTIONS?

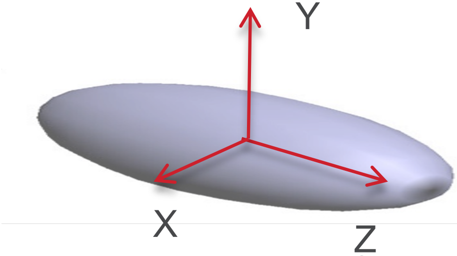
EXTRA

LONGITUDINAL PROPERTIES - PULSE DURATION

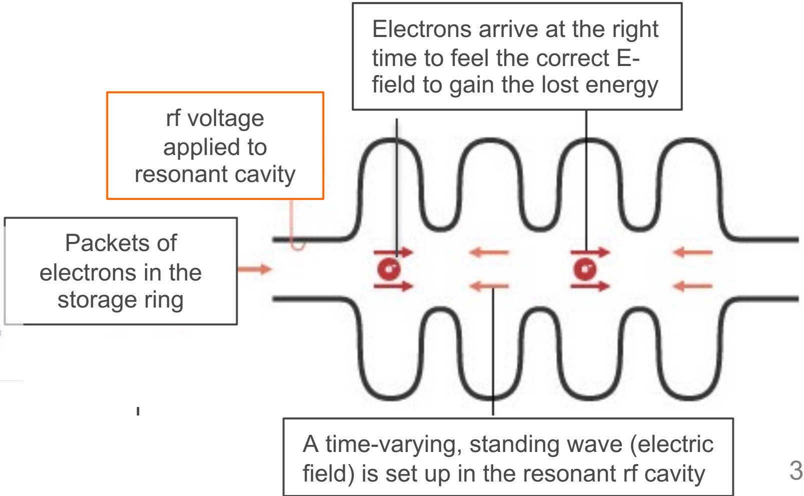
- Because the electrons are radiating x-rays, they are constantly losing energy, and so to restore the energy loss on each revolution, radio-frequency (RF) resonant cavities are installed in the storage ring to replenish the radiative energy losses.
- Particles are grouped together by the action of the radio-frequency (RF) cavities into bunches. At APS:
  - 1104 m circumference (3.68 microsecond period)
  - there are 1296 evenly spaced "RF buckets" (stable orbit positions) around the ring
  - the bunch length of the electron packet in one of these "buckets" is about 3 cm in length, corresponding to a **pulse duration** of about 100 psec



electron packet in the storage ring



z is in the direction of travel

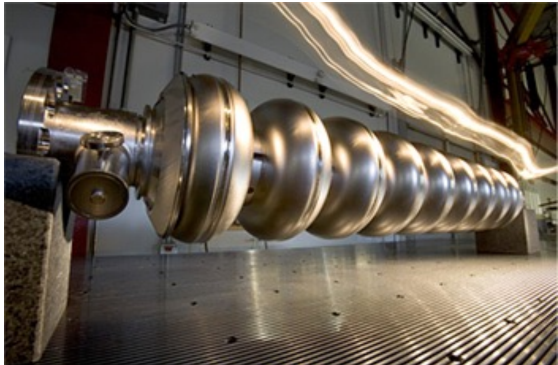


Electrons arrive at the right time to feel the correct E-field to gain the lost energy

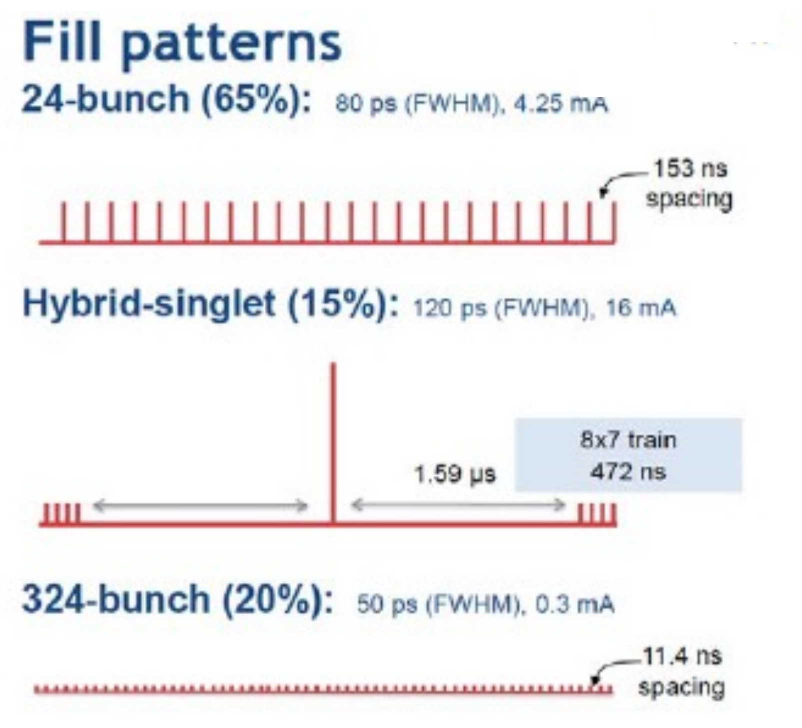
rf voltage applied to resonant cavity

Packets of electrons in the storage ring

A time-varying, standing wave (electric field) is set up in the resonant rf cavity



APS TIME STRUCTURE DEPENDS ON FILL PATTERNS (100 mA TOTAL CURRENT)



- The time structure is determined by which of the rf buckets are filled with electrons.

24 equally spaced bunches (about 4 mA/bunch)
  - compromise between quasi-continuous source and pulsed source

1 + 8x7 (12-14 mA in one bunch and the rest of the 100 mA distributed in 7 trains of 8 closely spaced bunches on the other side of the ring)
  - timing experiments

324 equally spaced bunches (about 0.3 mA/bunch)
  - approximates a quasi-continuous source

APPENDIX 1: RADIATED POWER FROM CHARGES AT RELATIVISTIC VELOCITIES

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r, in the non-relativistic case, α is just the centripetal acceleration, v^2/r. In the relativistic case:

$$a = \frac{1}{m_o} \frac{dp}{d\tau} = \frac{1}{m_o} \gamma \frac{d\gamma m_o v}{dt} = \gamma^2 \frac{dv}{dt} = \gamma^2 \frac{v^2}{r}$$

Where $\tau = t/\gamma$ = proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

DEPENDENCE ON PARTICLE MASS AND ENERGY OF RADIATED POWER

$$\boxed{P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e^- with the same total energy, E.)

2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

APPENDIX 2: BM SPECTRAL DISTRIBUTION

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c.

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and r the radius of curvature of the trajectory. Note that the critical energy scales as γ^3. In practical units, the critical energy can be written as:

$$E_c[\text{keV}] = 2.218\ E^3[\text{GeV}]/\ \rho[\text{m}] = 0.06651\ B[\text{kG}]\ E^2[\text{GeV}]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at E = 7 GeV . The critical energy of the radiation emitted from the BM is:

$$E_c[\text{keV}] = 0.06651\ B[\text{kG}]\ E^2[\text{GeV}]$$

or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5\ \text{keV}}\ \text{or}\ \underline{0.64\ \text{Å}}.$$

APPENDIX 3: WHERE DID “K” COME FROM?

Equation of motion for a relativistic charged particle in a magnetic field

$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v}\times\vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

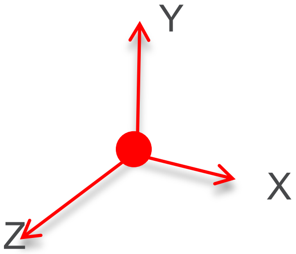
$$\dot{v}_x = \frac{ecB_0}{\gamma m_0}\sin\left(\frac{2\pi z}{\lambda_{ID}}\right) \qquad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi c}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right) = -\frac{eB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c}\left[\frac{\lambda_{ID}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{ID}}\right) = \left[\frac{eB_0}{m_0}\frac{\lambda_{ID}}{2\pi c}\right]\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right) = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$x_{\max} = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \qquad \text{and} \qquad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \qquad \text{where} \qquad K = \left[\frac{eB_0}{2\pi}\frac{\lambda_{ID}}{m_0 c}\right]$$

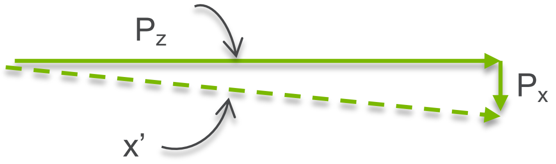
Magnetic field in y-direction and electron traveling in z-direction



APPENDIX 4: THE DIFFRACTION LIMITED OF LIGHT

- The Heisenberg Uncertainty Principle sets a lower limit for the product of the size Δx (Δy), and angular divergence $\Delta x'$ ($\Delta y'$), of radiation. Recall:

primes are used to denote angles



For radiation from relativistic electrons, x' (y') is proportional to P_x (small angle approximation).

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\frac{p_x}{p_z} = x' \text{ or } \frac{\Delta p_x}{p_z} = \Delta x' \text{ and } p_z = \hbar k = \frac{\hbar(2\pi)}{\lambda}$$

$$so: \Delta x \Delta p_x = \Delta x \Delta x' p_z = \Delta x \Delta x' [\frac{\hbar(2\pi)}{\lambda}] \geq \hbar / 2$$

$$\Delta x \Delta x' \geq \lambda / 4\pi$$

- This says, for a given wavelength λ, the product of its size and divergence cannot be is less than $\lambda/4\pi$. When the product is ≈ $\lambda/4\pi$ in both the x and y directions, the radiation is **fully coherent and the source is said to be diffraction limited.**