26th Annual NXSchool

SYNCHROTRON RADIATION: PRODUCTION & PROPERTIES

DENNIS M. MILLS
Advanced Photon Source

National School for Neutron and X-ray Scattering
July/August 2024

AGENDA

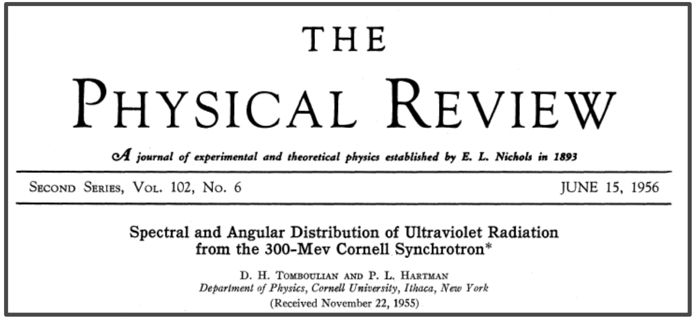
1. Some synchrotron radiation history
2. Properties of radiation from relativistic electrons
3. Insertion devices (primarily undulators)
4. Transverse properties of electron beams in storage rings
5. Next generation light sources

SOME HISTORY

DISCOVERY OF SYNCHROTRON RADIATION

- Synchrotron Radiation (SR) - was first observed in 1947 from a 70 MeV synchrotron at GE in Schenectady\, NY

*"On April 24,[1947] Langmuir and I [Herbert Pollack] were running the machine and as usual were trying to push the electron gun and its associated pulse transformer to the limit. Some intermittent sparking had occurred … At first we thought it might be due to Cerenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk radiation."**



THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

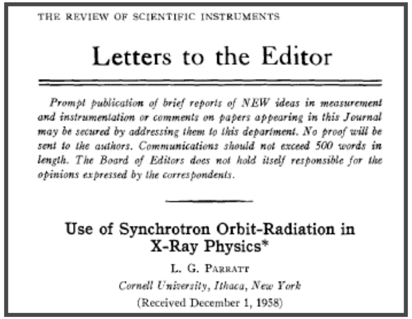
SECOND SERIES, VOL. 102, No. 6 — JUNE 15, 1956

Spectral and Angular Distribution of Ultraviolet Radiation from the 300-Mev Cornell Synchrotron*

D. H. TOMBOULIAN AND P. L. HARTMAN

Department of Physics, Cornell University, Ithaca, New York

(Received November 22, 1955)



THE REVIEW OF SCIENTIFIC INSTRUMENTS

Letters to the Editor

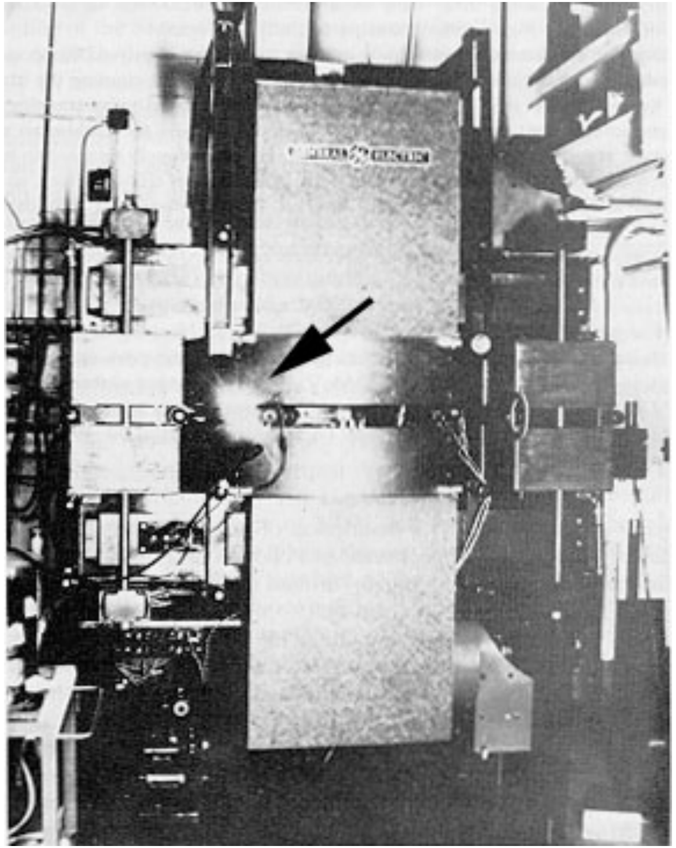
Prompt publication of brief reports of NEW ideas in measurement and instrumentation or comments on papers appearing in this Journal may be secured by addressing them to this department. No proof will be sent to the authors. Communications should not exceed 500 words in length. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Use of Synchrotron Orbit-Radiation in X-Ray Physics*

L. G. PARRATT

Cornell University, Ithaca, New York

(Received December 1, 1958)

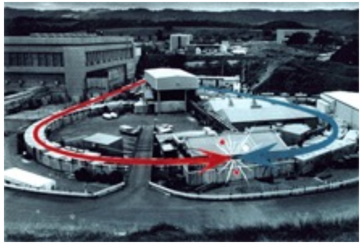


* *The discovery of synchrotron radiation,* Herbert C. Pollock, Am. J. Phys. 51 (3), March 1983

- Storage rings, accelerators that store electrons at a constant energy, provided a far more attractive source. We now use the name synchrotron radiation to describe radiation that is emitted from charged particles traveling at relativistic speeds, regardless of the accelerating source.

TIMELINE FOR X-RAY SR SOURCES IN THE US

SSRP



CHESS

ALS

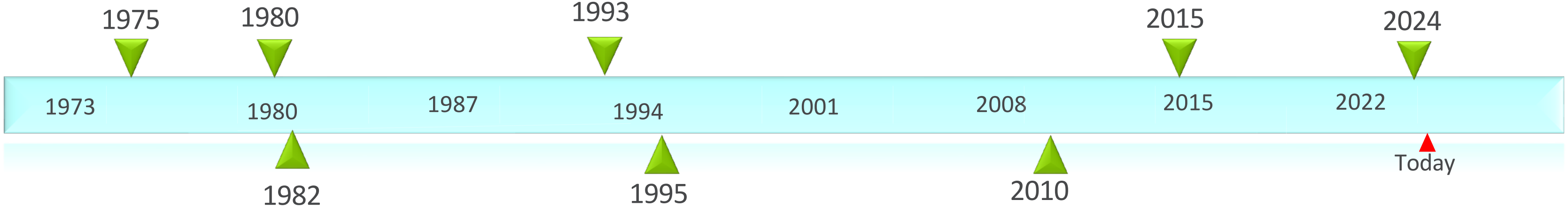
NSLS II

APS-U

1st Generation (1974-1980’s)

3rd Generation (1993-2015)

MBA lattice SR (2016-…)



2nd Generation (1981-1986)

3rd Generation (1993-2015)

4th Gen Free Electron Lasers (FELs) (2010- …)

NSLS I

APS

LINAC Coherent Light Source (LCLS)

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SCHEMATIC OF A MODERN STORAGE RING SOURCE

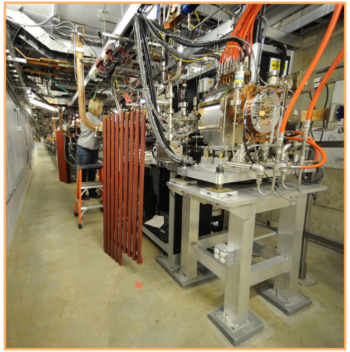
(A) electron source (gun)

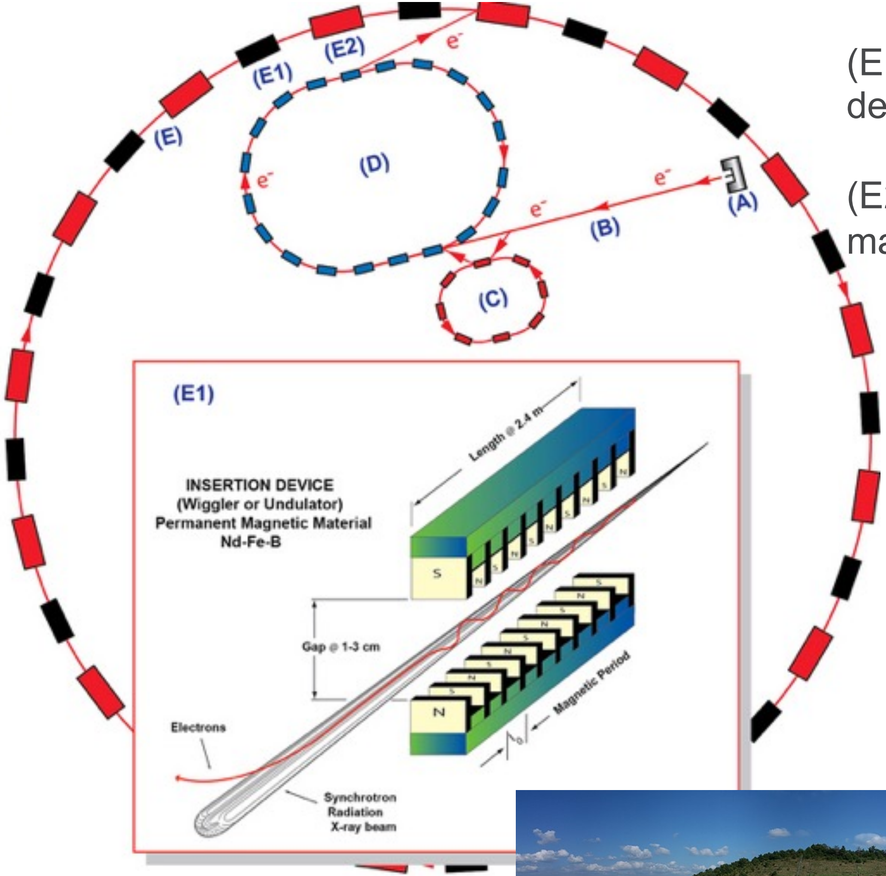
(B) LINAC (450 MeV)

(C) accumulator ring

(D) booster synchrotron (450 MeV to 7 GeV)

(E) Storage ring (7 GeV)





(E1) insertion devices

(E2) bend magnets

PROPERTIES OF SYNCHROTRON RADIATION

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NATIONAL LABORATORY

RADIATION PATTERNS FROM ACCELERATING CHARGES

Definitions:

$$\beta = v/c$$

$$\gamma = 1/(1-\beta^2)^{1/2} = E/m_oc^2$$

Where:

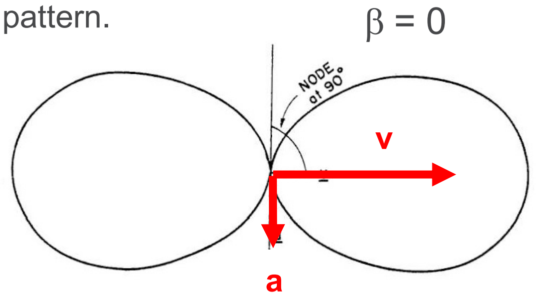
v = velocity of the electron

c = speed of light

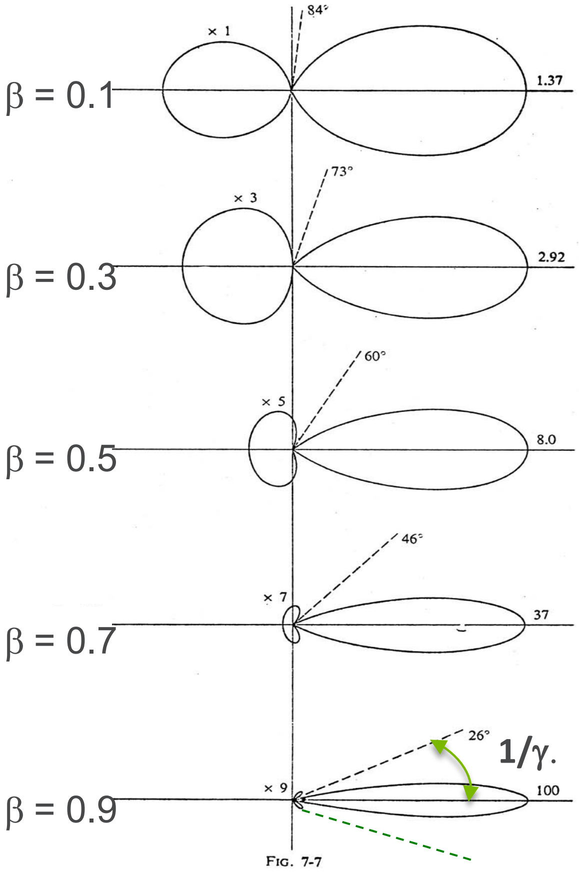
E = electron energy

m_oc^2 = electron rest mass

When an electron is accelerated and its velocity, v, much less the speed of light, c, ($\beta \approx 0$), the distribution of the emitted radiation is a classical dipole pattern.



This drawing is for an electron going in a circle – acceleration, a, is inward and instantaneous velocity, v, is in the direction of a tangent to that circle.



But as β approaches 1, the shape of the radiation pattern changes; it is more forward directed

The angular divergence of the emitted radiation (sometimes called the opening angle of the radiation) ≈ **1/γ**.

RADIATION FROM HIGHLY RELATIVISTIC *(γ >>1)* PARTICLES IN A CIRCULAR ORBIT

At the APS with E = 7 GeV, [6 GeV]

γ = E / m_oc^2 = 7 GeV / 0.511 MeV

γ = 1.4 [1.2] x 10^4

$\beta \approx 1 - 1/(2\gamma^2)$

$\beta \approx 1 - (2 \times 10^{-9}) = 0.999999998$

[$1 - (4 \times 10^{-9}) = 0.999999996$]

v = 0.999999998c

[v = 0.999999996c]

Recall that the divergence (or opening angle) of the x-ray beam is given by $\approx 1/\gamma$:

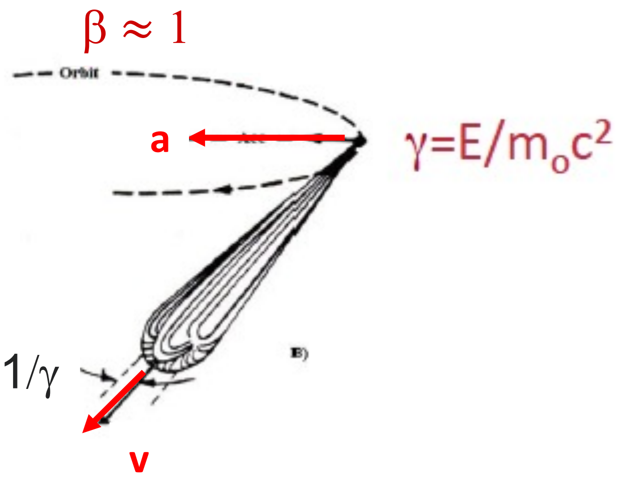
$1/\gamma = 73 \times 10^{-6}$ (73 μradians)

[85×10^{-6} (85 μradians)]

or about

15 [17] arc-seconds

[APS-U]

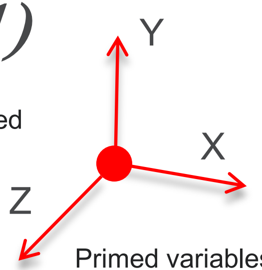


Relativistic velocities are good!!

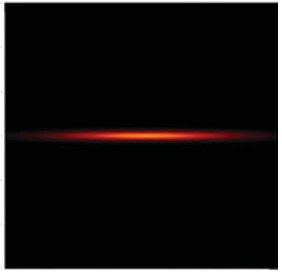
- radiation forward directed
- radiated power $\propto E^4$

See Appendix 1 for more details

Coordinate system used in this presentation.

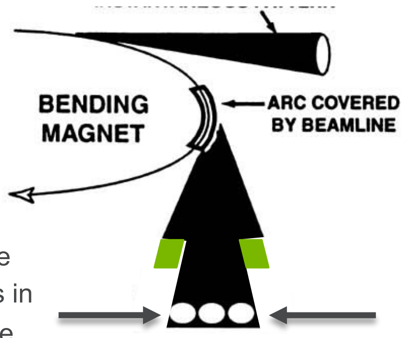


Primed variables (x', y') are angular coordinates.

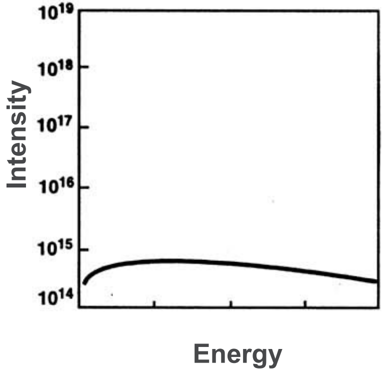


APS electron beam, which is the source of the x-rays, is about 10μ in the vertical (y) direction and about 300μ in the horizontal (x)-direction.

BEND MAGNET (BM) X-RAY RADIATION PROPERTIES



Horizontal opening angle determined by apertures in the front-end or beamline.



You get this for "free" since you need the bending magnets to keep the electrons orbiting in the storage ring.

Bend Magnet Radiation

Spectrum characterized by the critical energy:

E_c[keV] = 0.066 B[kG] E^2 [GeV]   ($E_c \approx$ 19.5 keV @ APS)

[$E_c \approx$ 15.6 keV @ APS-U]

All the BMs sources have the same B-field so spectrum from each BM source is identical.

The APS-U lattice has several types of BMs but the source for the BM beamlines is the same at each location so spectrum for each BL is the same.

Recall: λ[Å] = 12.4/E[keV]   so 19.5 keV is 0.64Å

[15.6 keV is 0.79Å]

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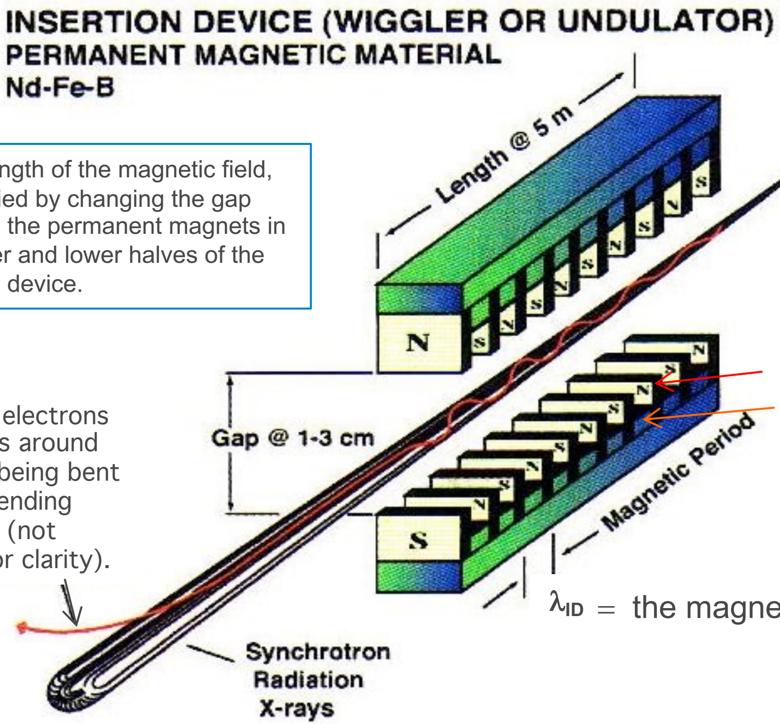
PLANAR INSERTION DEVICES (IDs)

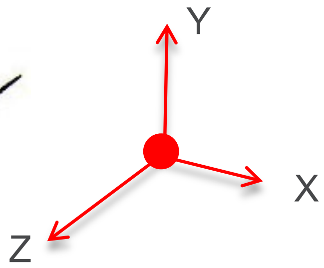
IDs are "inserted" in straight sections of the storage ring between bend magnets.

- Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.
- "Planar" refers to the magnetic field being in one direction (in this case vertical or y-direction).
- IDs can have fields in both the vertical and horizontal directions to produce circularly polarized x-rays and for other applications.

Magnetic field is in the y-direction for a "planar" ID (i.e., no field in the x-direction).

The strength of the magnetic field, B, is varied by changing the gap between the permanent magnets in the upper and lower halves of the insertion device.





Beam of electrons continues around the ring being bent by the bending magnets (not shown for clarity).

Steel or permandur poles with permanent magnets between them supplying the magnetic field.

λ_{ID} = the magnetic period of the ID

CHARACTERIZING INSERTION DEVICES

- IDs are characterized by the so-called field index or deflection parameter, K

Important thing here is K is proportional to the peak magnetic field, B_o since the period of the insertion device, λ_{ID} is fixed for a given device.

$$K = eB_o\lambda_{ID}/2\pi m_o c = 0.0934\ \lambda_{ID}[cm]\ B_o[kG]$$

where λ_{ID} **is the magnetic period** of the insertion device and B_o **the peak magnetic field**. The length of the insertion device, L, is equal to the number of periods, N, times the length of the period, i.e.,

$$L = N\lambda_{ID}\ .$$

- The maximum deflection angle of the particle beam, $x'_{ID,max}$, is given by:

$$x'_{ID,\ max} = \pm(K/\gamma)$$

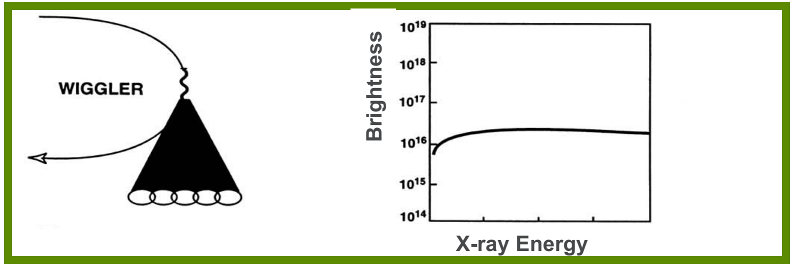
See Appendix 2 for more details on K

WIGGLER (K>> 1)

Horizontal deflection angle of electrons given by $x'_{ID,\,max} = (K/\gamma)$

$$x'_{ID,\,max} \gg 1/\gamma$$

Radiation spectrum looks like 2N dipole sources (N = number of periods)

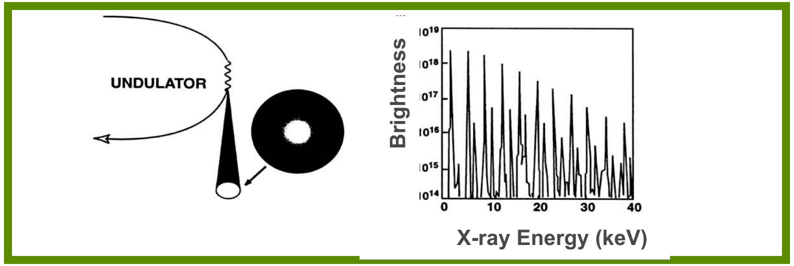


UNDULATOR (K ≈ 1)

When the maximum deflection of the electron beam is on the order of the natural opening angle of the emitted radiation, i.e. $K \approx 1$

$$x'_{ID,\,max} \approx 1/\gamma\ ,$$

So now the radiation from each of the magnetic poles overlaps.



UNDULATOR RADIATION SPECTRA

$\gamma \propto E_{electron}$

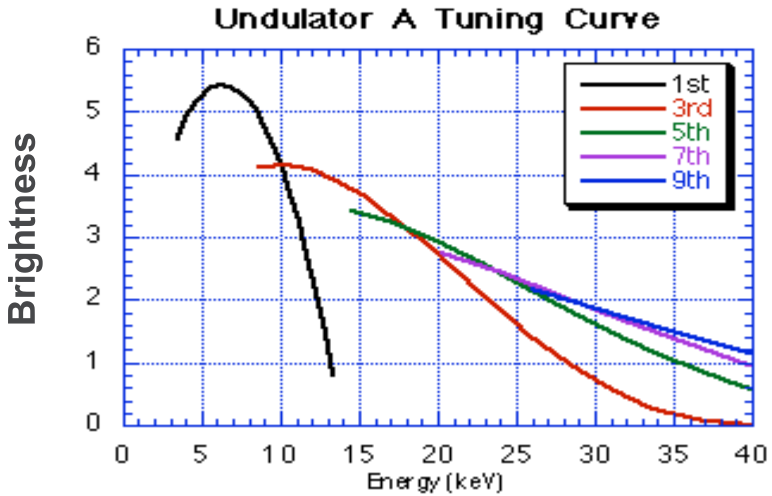
The x-ray wavelengths where constructive interference occurs → $\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$

n is the harmonic number: n=1,3,5…

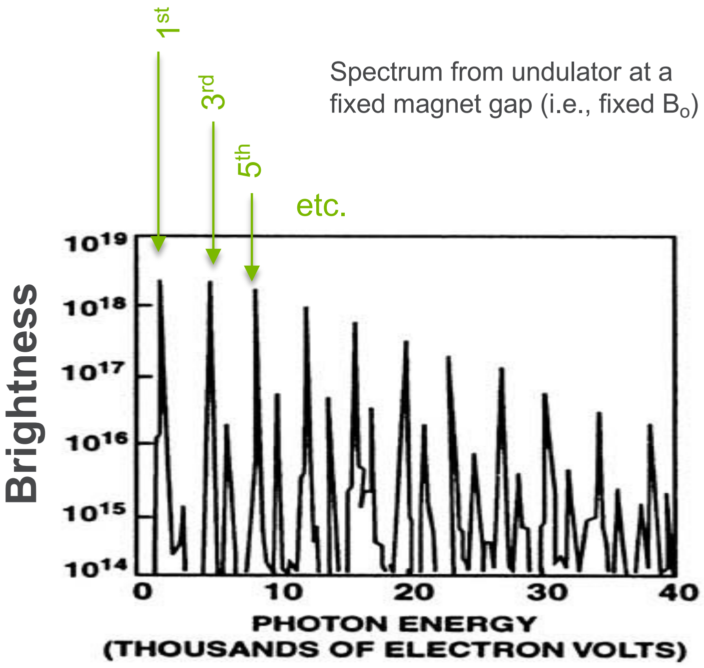
K is proportional to the magnetic field strength for fixed λ_{ID}

The spectrum from an undulator is spikey, but peaks are tunable in energy by varying K, since $K \propto B_0$.

Spectrum from undulator by varying the magnet gap (i.e., B_o)



Spectrum from undulator at a fixed magnet gap (i.e., fixed B_o)



TRANSVERSE PROPERTIES OF THE ELECTRON BEAM

TRANSVERSE (X & Y) ELECTRON BEAM PROPERTIES

- Although the flux from undulators can be determined without knowledge of the source size and divergence, one very important characteristic of the x-ray beam, namely **brightness,** requires a more detailed knowledge of the particle beam's size and divergence.

Brightness has units of: photons/sec/0.1% BW/source area / source divergence

$$\mathrm{Flux}/4\pi^2\,\Sigma_x\,\Sigma_y\,\Sigma_x'\,\Sigma_y'$$

this is the monochromaticity of the beam

where Σ_i (Σ_i') is the **effective** one sigma value of the source size (divergence) in the i^{th} direction. The total source size and divergence is the quadrature sum of the electron beam and the radiation beam, namely:

We previously determined $\Delta x'_{rad}$ for undulator radiation and can calculate Δx_{rad} from the relationship for the diffraction limit of light (see next couple of slides).

$$\Sigma_x = [\underbrace{\Delta x^2_{rad}}_{\text{radiation beam size}} + \underbrace{\Delta x^2_{electron}}_{\text{electron beam size}}]^{1/2}$$

and a similar expression for Σ_y

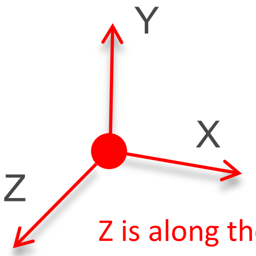
and

$$\Sigma_x' = [\underbrace{\Delta x'^2_{rad}}_{\text{radiation beam divergence}} + \underbrace{\Delta x'^2_{electron}}_{\text{electron beam divergence}}]^{1/2}$$

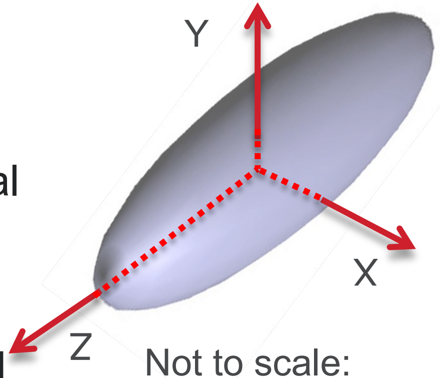
and a similar expression for Σ_y'

ELECTRON BEAM PROPERTIES ARE KEY

- So far, this discussion has not taken into account the properties of the source of the x-rays – namely the electron beam.
- The electron beam has both a size and divergence in the x- and y-directions (transverse to the velocity of the beam).
- The electron beam also has a finite length in the z-direction (along the direction of the velocity of the beam – sometimes called the longitudinal properties).
- To fully understand the properties of the emitted x-ray beams, we need to look a little more closely at the (transverse) properties of the electron beams.



Z is along the instantaneous velocity vector of the electrons



Not to scale:
$\Delta y_{electron}$ ≈10's μ
$\Delta x_{electron}$ ≈100's μ
$\Delta z_{electron}$ ≈ cm

See Appendix 4 for details on longitudinal properties/bunch length of the electron beam and Appendix 5 for x-ray pulse structure from storage rings.

ELECTRON BEAM PHASE SPACE AND EMITTANCE

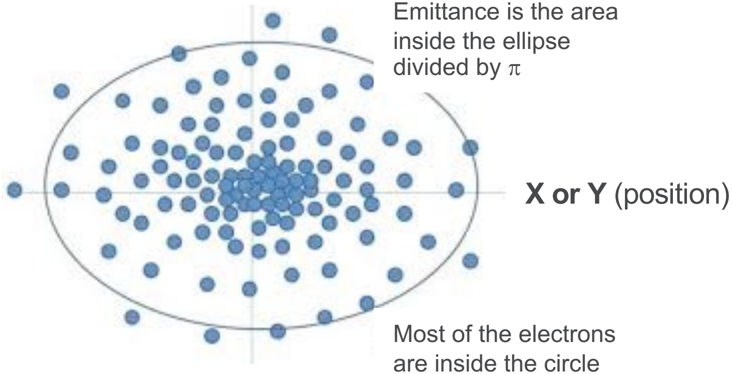
- The area bounded by an ellipse that captures some given fraction of electrons in the xx' (yy') phase space is proportional to what is called the horizontal (or vertical) emittance of the electron beam.

- There is a separate horizontal emittance (ε_x) and vertical emittance (ε_y). In today's storage rings:

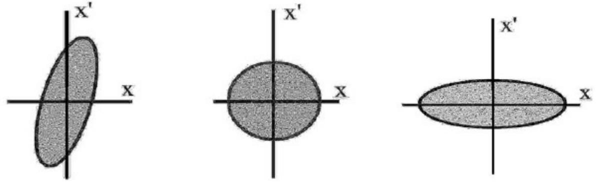
  ε_y is typically 1% of ε_x (the percentage is called the coupling)

- The emittance is a constant of the storage ring, although one can trade off electron beam size for divergence as long as the area of the phase-space remains constant.

- To determine the x-ray beam brightness, you need to know the electron beam source size and divergence at the spot in the storage ring where the undulator is located and add those values - in quadrature - with source size and divergence of the radiation.



This **IS NOT** the electron beam cross-section. It is the so-called **PHASE SPACE** of the beam in the x or y plane.



This is a consequence of Liouville's Theorem – the conditions for which are satisfied in most accelerators.

THE DIFFRACTION LIMIT FOR LIGHT

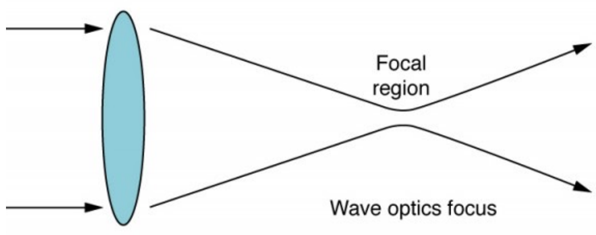
- Just as the Uncertainty Principle sets a lower limit for the product of the size, Δx, and momentum Δp_x, this relationship can be re-written in terms of size Δx_{rad} (Δy_{rad}) and angular divergence $\Delta x'_{rad}$ ($\Delta y'_{rad}$) for radiation.

$$\Delta x \, \Delta p_x, \geq h / 4\pi \Rightarrow \Sigma_x \Sigma_x' \geq \lambda / 4\pi \quad \text{and} \quad \Sigma_y \Sigma_y' \geq \lambda / 4\pi$$

See Appendix 3 for details

- A light source that satisfies $\Sigma_x \Sigma_x' = \lambda / 4\pi$ is called **diffraction limited.** When a radiation beam has this property, it is a **fully coherent source**.

- Another way to define a **diffraction limited beam** is if its potential to be focused to small spots is as high as possible for its wavelength.



What's the diffraction limit for an x-ray with wavelength 1 Å ?

$$\lambda / 4\pi = 1\text{Å} / 4\pi = 10^{-10} \text{ meters} / 4\pi$$

$$\Sigma_x \Sigma_x' \approx 10 \times 10^{-12} \text{ m}$$

10 picometers-radian (radians are dimensionless)

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DIFFRACTION LIMITED SOURCES AND COHERENCE

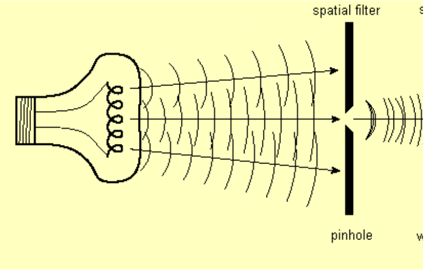
- If both the horizontal and vertical emittance of the particle beam were small compared to the diffraction limit of the light (λ / 4π) , then the x-rays that are emitted would **have full transverse coherence**.

$\mathcal{E}_x \approx 3$ x 10^{-9} *m-rad or* ***3000 pm-rad***

$\mathcal{E}_y \approx 0.03$ x 10^{-9} *m-rad or* ***30 pm-rad***

- But at the APS, as with most 3rd generation synchrotron radiation sources, the electron beam emittance dominates - see figure below.
- Hence the radiation is **partially coherent**
- Partially coherent sources are characterized by the **coherent fraction**, the fraction of the x-ray flux that is coherent.
- Coherent fraction = ratio of diffraction-limited emittance to actual beam emittance.
- For the APS at 1Å, the coherent fraction is ≈ 10^{-3} – useful for experiments that rely on coherence but at the expense of throwing away a lot of (incoherent) flux.

An incandescent light bulb is an example of very incoherent source.



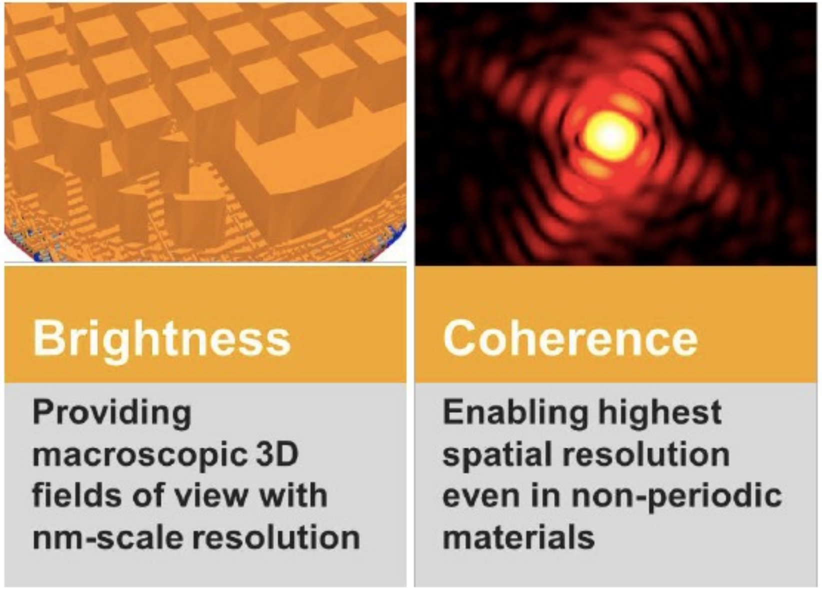
You can produce coherent light from an incoherent source if we are willing to throw away a lot of the light.

THE DRIVE FOR MORE COHERENT SOURCES

APS-Upgrade: a 42 picometer-radian x-ray source

- The goal of the upgrade is to reduce the electron beam emittance from the current 3000 pm-rad to 42 pm-rad.
- This will result in a diffraction limited source below 3 keV and much higher coherence at higher energies (100 to 1000 times).

APS-U will enable multiscale, 3-D exploration of complex materials and chemical systems – may any atom's position, identity and motion.



See lectures by S. Hruszkewycz, L. Lurio, and others on the use of x-ray coherence.

NEXT GENERATION LIGHT SOURCES

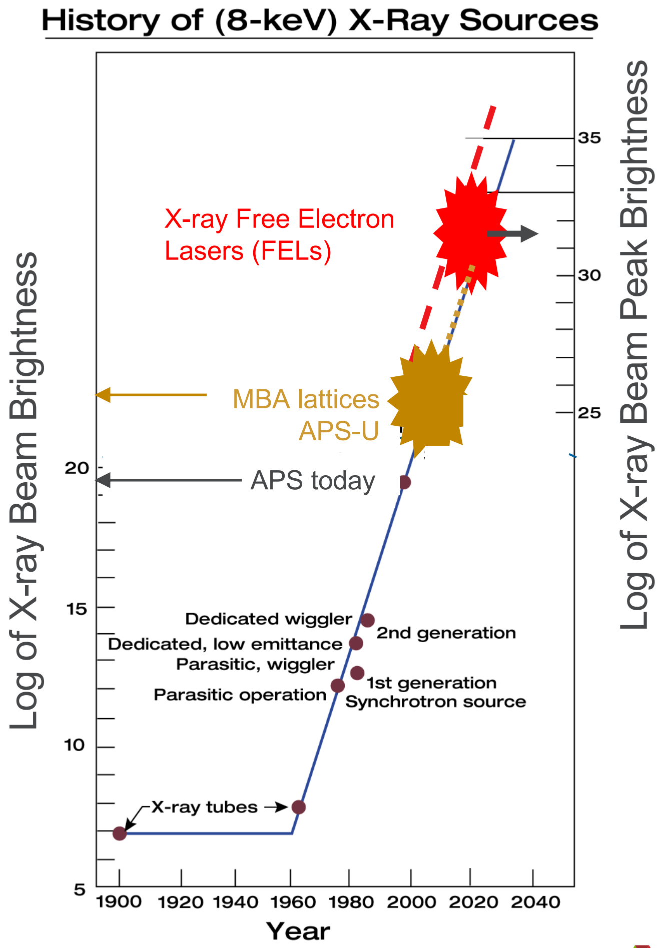
THE DRIVE FOR MORE COHERENT X-RAY SOURCES

- How do we get from where we are today:

  ε_x = 3000 picometer-radian

  to something closer to the 10 picometers-radian emittance for a near-fully coherent (i.e., high brightness) x-ray beam at 1Å?

- Presently there are two approaches to obtain a more coherent x-ray source:
  - Storage rings with so-called Multi-Bend Achromat (MBA) magnet structures
  - Using LINACs that satisfy the conditions for a free electron laser (FEL) *– See Appendix 6*



See presentation by M. Liang for details on FELs

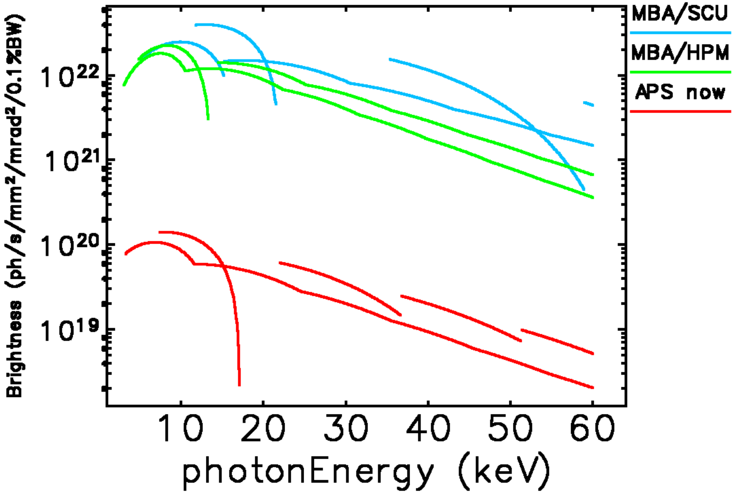
THE APS-U PROJECT: A MULTI-BEND ACHROMAT (MBA) LATTICE

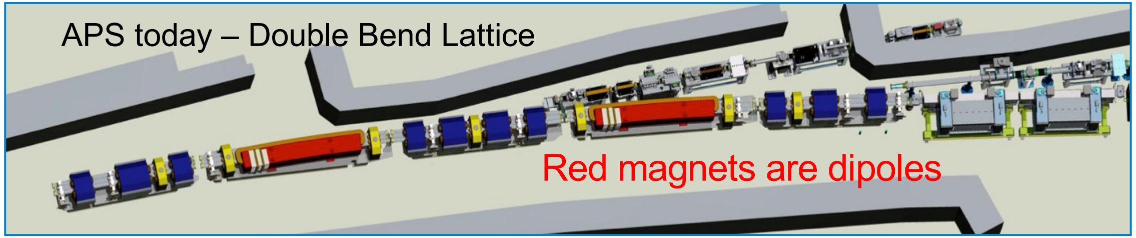
$$\varepsilon_x = C_L \frac{E^2}{N_D^3}$$

C_L = *constant*
E = *beam energy*
N_d = dipoles per sector

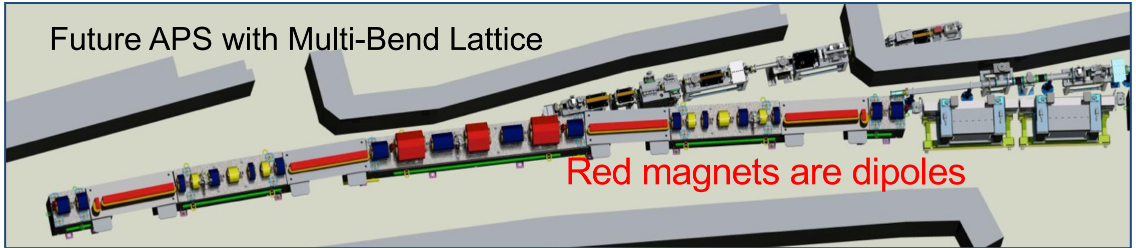
APS MBA Upgrade:
Beam energy: 7 GeV → 6 GeV
Current: 100 mA → 200 mA
Dipoles per sector: 2 → 7

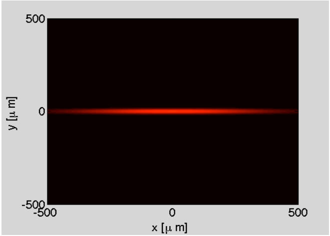
10^2 to10^3 X increase in brightness

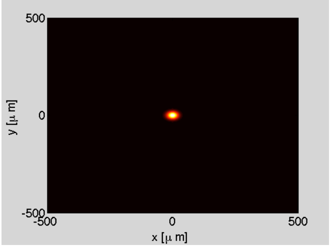




~50-fold reduction in horizontal emittance



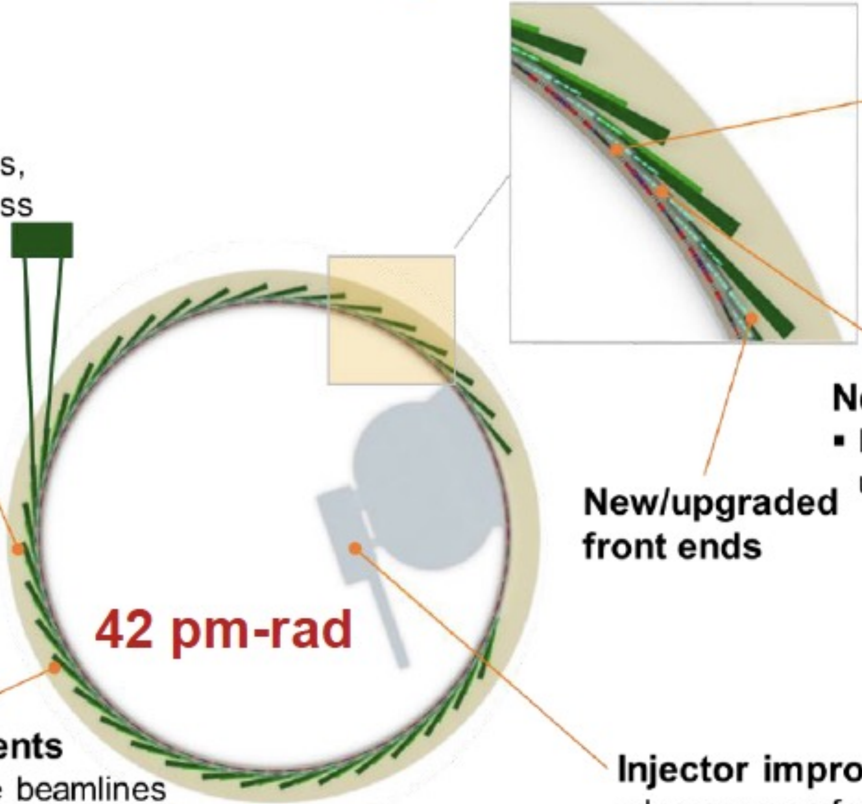




APS-U Project Scope

Feature beamlines
- Suite of beamlines, including long beamlines, designed for best-in-class performance

Total Project Cost - $815M



New storage ring
- 6 GeV with 200 mA, 42 pm-rad emittance
- Hybrid 7BA lattice with reverse bends
- Improved electron and photon stability

New insertion devices
- Including superconducting undulators

New/upgraded front ends

Beamline enhancements
- Improvements to make beamlines "Upgrade Ready"
- Existing beamlines are planned to come back on-line after the upgrade

Injector improvements
- Increase performance beyond present capability

On-axis "swap-out" injection

MBA UPGRADES – A COMPETITIVE LANDSCAPE

ESRF (France)

- ESRF/Extremely Bright Source (EBS) upgrade is operational

MAX-IV (Sweden)

- A new 3 GeV MBA lattice-based storage ring is operational

SIRIUS (Brazil)

- A new 3 GeV MBA lattice-based storage ring is operational

ALS-U (US)

- In the design phase; operational 2027 (?)

Diamond Light Source (UK)

- In the design phase; operational 2027 (?)

SPRING-8 (Japan)

- Planning an MBA upgrading in 2020’s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4 July 2014 Last updated at 06:33 ET
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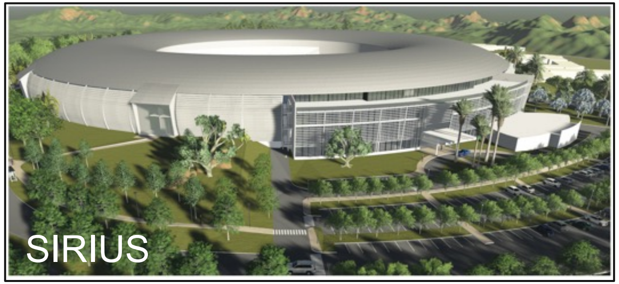
Upgrade for Europe's big X-ray light source

By Jonathan Amos
Science correspondent, BBC News



LIGHT SOURCES OF THE FUTURE

Basic Energy Sciences Advisory Committee (BESAC) on BES Future Facilities (April 2024)

- Future Light Source
  - *Absolutely central to plan facility for future US leadership*
  - science mission/case/design TBD

PHYSICAL REVIEW ACCELERATORS AND BEAMS **26,** 021601 (2023)

Editors' Suggestion

Toward a diffraction limited light source

Pantaleo Raimondi and Simone Maria Liuzzo
ESRF, Grenoble, France

(Received 17 October 2022; accepted 17 January 2023; published 22 February 2023)

This paper introduces a storage ring lattice design for synchrotron radiation sources which fulfills all the required features for the realization of a diffraction limited light source. Insertion devices half gaps of 2.5 mm are assumed and enabled by the design of dedicated straight sections. The use of long straight sections is possible for such a lattice and led to the definition of a set of transparency conditions used to minimize the effect of such insertions for on- and off- energy beam dynamics. The design proposed enables an extremely long Touschek lifetime and beam accumulation. A real case study will be shown for large circumference storage ring upgrade lattices.

Seminar Announcement

Title:	A Corrugated Waveguide-Based Collinear Wakefield Accelerator for a Hard X-ray Free Electron Laser
Presenter:	Sasha Zholents
Date:	Wednesday, July 24, 2024
Time:	3:30 pm

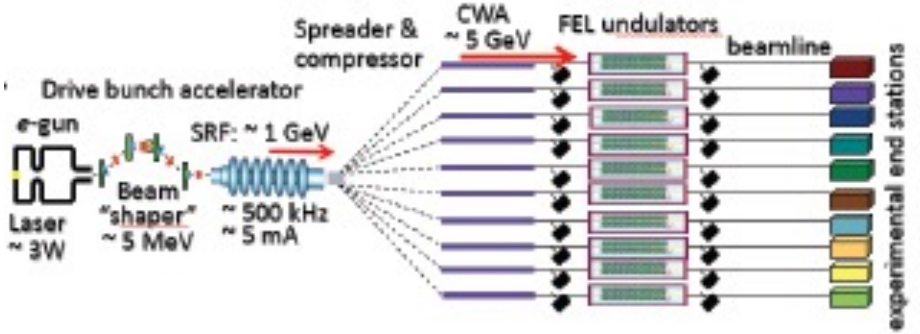
39th Free Electron Laser Conf. FEL2019, Hamburg, Germany JACoW Publishing
ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-TUD04

CAVITY-BASED FREE-ELECTRON LASER RESEARCH AND DEVELOPMENT: A JOINT ARGONNE NATIONAL LABORATORY AND SLAC NATIONAL LABORATORY COLLABORATION

G. Marcus, F.-J. Decker, G. L. Gassner, A. Halavanau, J. Hastings, Z. Huang, Y. Liu, J. MacArthur, R. Margraf, T. Raubenheimer, A Sakdinawat, T. Tan, D. Zhu, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA
J. W. J. Anton, L. Assoufid, K. Goetze, W. Jansma, S. Kearney, K.-J. Kim, R. Lindberg, A. Miceli, X. Shi, D. Shu, Y. Shvyd'ko, J. P. Sullivan, M. White, Argonne National Laboratory, Argonne, IL 60439, USA
B. Lantz, Stanford University, Stanford, CA 94305, USA

A CONCEPTUAL DESIGN OF A COMPACT WAKEFIELD ACCELERATOR FOR A HIGH REPETITION RATE MULTI USER X-RAY FREE-ELECTRON LASER FACILITY*

A. Zholents†, S. Doran, W. Jansma, M. Kasa, R. Kustom, A. Nassiri, J. Power, S. Sorsher, N. Strelnikov, K. Suthar, E. Trakhtenberg, I. Vasserman, G. Waldschmidt, J. Xu, Argonne National Laboratory, Lemont, IL, USA
S.S. Baturin, the University of Chicago, PSD Enrico Fermi Institute, Chicago, IL,USA
H. Perez, Illinois Institute of Technology,Chicago, IL,USA



approx. 100 MV/m

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SUMMARY

- Synchrotron radiation continues to be an important tool for researchers across all scientific and engineering disciplines.
- Storage rings:
  - can produce partially coherent x-ray beams with megahertz repetition rates but with lower peak intensity than free electron lasers (FELs).
- FELs:
  - can provide very short pulses (femtoseconds) of high peak intensity, coherent x-rays but the repetition rate may be limited to hundreds of hertz when using RT radio frequency linacs.
  - Superconducting radio frequency (SCRF) linacs will increase rep rates to MHz (LCLS II and the European XFEL).
- The LCLS II-HE, APS-U, and the ALS-U will keep US x-ray facilities at the cutting edge to produce world class science in the years to come.
- There is a strong science case for a new generation of sources such as:
  - Diffraction limited storage rings
  - X-ray free electron lasers (with longitudinal coherence)
- Also a drive to reduce size (and cost) of sources to allow greater access to these powerful facilities

REFERENCES

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"Elements of Modern X-ray Physics" by Jens Als-Nielsen and Des McMorrow

"Third-Generation Hard X-Ray Synchrotron Radiation Sources: Source Properties, Optics, and Experimental Techniques", Edited by Dennis M. Mills

"Synchrotron Radiation and Free-Electron Lasers: Principles of Coherent X-Ray Generation", by Kwang-Je Kim, Zhirong Huang and Ryan Lindberg

"Handbook of Accelerator Physics and Engineering", Edited by A. Cho and M. Tigner

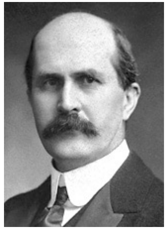
QUESTIONS?

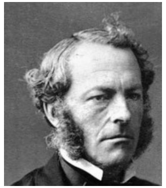
X-RAYS WERE A PUZZLE

- Röntgen discovery of X-rays was made by in 1895 and he first thought they might be some sort of **longitudinal vibrations of the ether.**
- These ”rays” were not deflected by magnetic fields, so carried no charge, and, at the time, refraction by a prism could not be measured (we’ll see why in the talk about x-ray optics).
- Others, including Charles Barkla who studied the polarization of x-rays (Physics Nobel prize winner in 1917 for this and other work on x-ray scattering), thought they were an **extension of the visible light spectrum.**
- William Bragg (Physics Nobel Prize winner in was a strong proponent of x-rays being **”particles” or corpuscular in nature**.
- In 1896 Lord Kelvin sent G. G. Stokes a letter ending thus: “*In respect of the Rontgen X-rays, are you a longitudinalist, or an ultravioletist, or a tertium quidist?* “
- This finally got sorted out – x-rays were electromagnetic waves – but apparently the discussion was lively between the two future Nobel laurates; Bragg (1915) and Barkla (1917).
- In his 1927 Nobel acceptance speech, Compton stated**:** **“*Many will recall also the heated debate between Barkla and Bragg, as late as 1910, one defending the idea that X-rays are waves like light, the other that they consist of streams of little bullets….*”.**


Wilhelm Röntgen
1845 – 1923


Charles Barkla
1877 – 1944


William Bragg
1862 – 1942


G. G. Stokes
1819 – 1903


Lord Kelvin
1824 – 1907

UNDULATOR RADIATION OPENING ANGLE

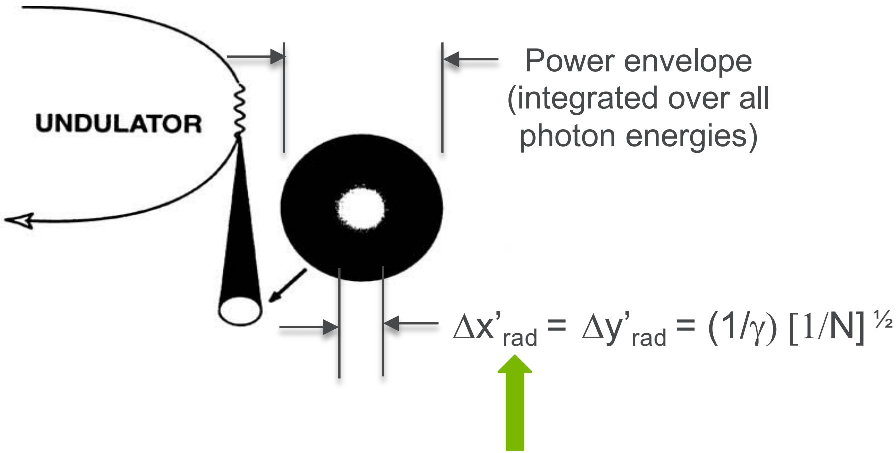
- At the fundamental (1st harmonic or n = 1), the horizontal and vertical opening angles of the radiation is given by:

$$\Delta x'_{rad} = \Delta y'_{rad} \approx (1/\gamma)\,[1/N]^{1/2}$$

where N = number of periods [typically 100 or more].

- So the **opening angle of the harmonics are much less than the natural opening angle**, $(1/\gamma)$, by as much as 10 or 12.

- This narrowing of the radiation beam's divergence only occurs at energies, $E_n^{x\text{-ray}}$. This low divergence cone, embedded in the larger power envelope, is sometimes called the **central cone of the undulator beam.**



I've labeled the x-ray radiation properties with the subscript "rad" to distinguish it from the electron beam source size and divergence that will be discussed later.

APPENDICIES

APPENDIX 1: RADIATED POWER FROM CHARGES AT RELATIVISTIC VELOCITIES

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r, in the non-relativistic case, α is just the centripetal acceleration, v^2/r. In the relativistic case:

$$a = \frac{1}{m_o} \frac{dp}{d\tau} = \frac{1}{m_o} \gamma \frac{d\gamma m_o v}{dt} = \gamma^2 \frac{dv}{dt} = \gamma^2 \frac{v^2}{r}$$

Where $\tau = t/\gamma$ = proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

APPENDIX 1: DEPENDENCE ON PARTICLE MASS AND ENERGY OF RADIATED POWER

$$\boxed{P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e^- with the same total energy, E.)

2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

APPENDIX 2: WHERE DID "K" COME FROM?

$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

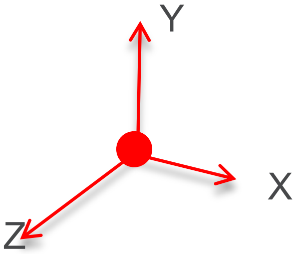
Equation of motion for a relativistic charged particle in a magnetic field

$$\dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) \qquad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi c}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right) = -\frac{eB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c}\left[\frac{\lambda_{ID}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{ID}}\right) = \left[\frac{eB_0}{m_0}\frac{\lambda_{ID}}{2\pi c}\right]\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right) = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

Magnetic field in y-direction and electron traveling in z-direction

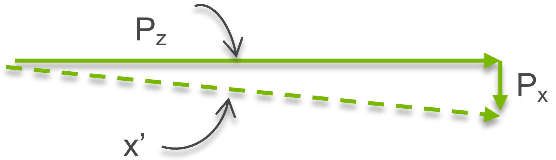


$$x_{\max} = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \quad \text{and} \quad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \quad \text{where} \quad K = \left[\frac{eB_0}{2\pi}\frac{\lambda_{ID}}{m_0 c}\right]$$

APPENDIX 3: THE DIFFRACTION LIMITED OF LIGHT

- The Heisenberg Uncertainty Principle sets a lower limit for the product of the size Δx (Δy), and angular divergence Δx' (Δy'), of radiation. Recall:

primes are used to denote angles



For radiation from relativistic electrons, x' (y') is proportional to P_x (small angle approximation).

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\frac{p_x}{p_z} = x' \text{ or } \frac{\Delta p_x}{p_z} = \Delta x' \text{ and } p_z = \hbar k = \frac{\hbar(2\pi)}{\lambda}$$

$$so: \Delta x \Delta p_x = \Delta x \Delta x' p_z = \Delta x \Delta x' [\frac{\hbar(2\pi)}{\lambda}] \geq \hbar / 2$$

$$\Delta x \Delta x' \geq \lambda / 4\pi$$

- This says, for a given wavelength λ, the product of its size and divergence cannot be is less than λ/4π. When the product is ≈ λ/4π in both the x and y directions, the radiation is **fully coherent and the source is said to be diffraction limited.**

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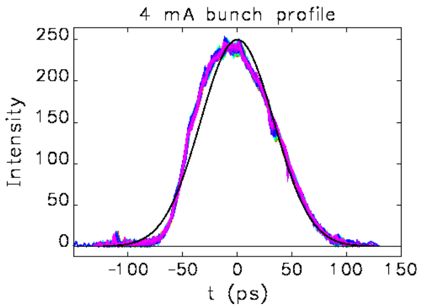
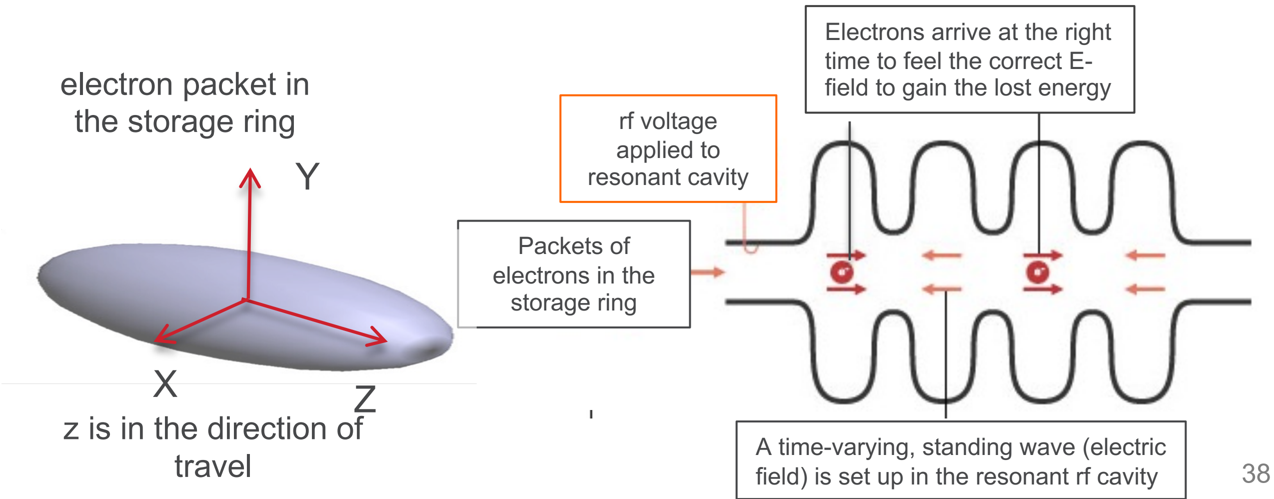
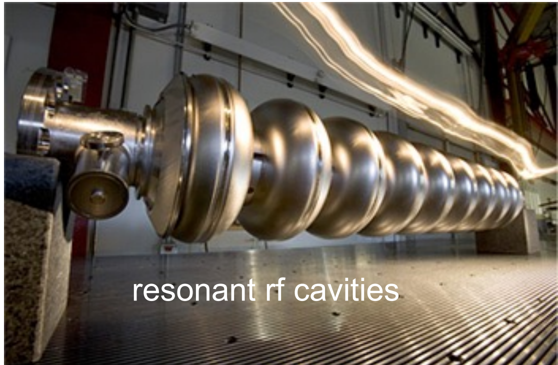
APPENDIX 4: LONGITUDINAL PROPERTIES OF THE ELECTRON BEAM - PULSE DURATION

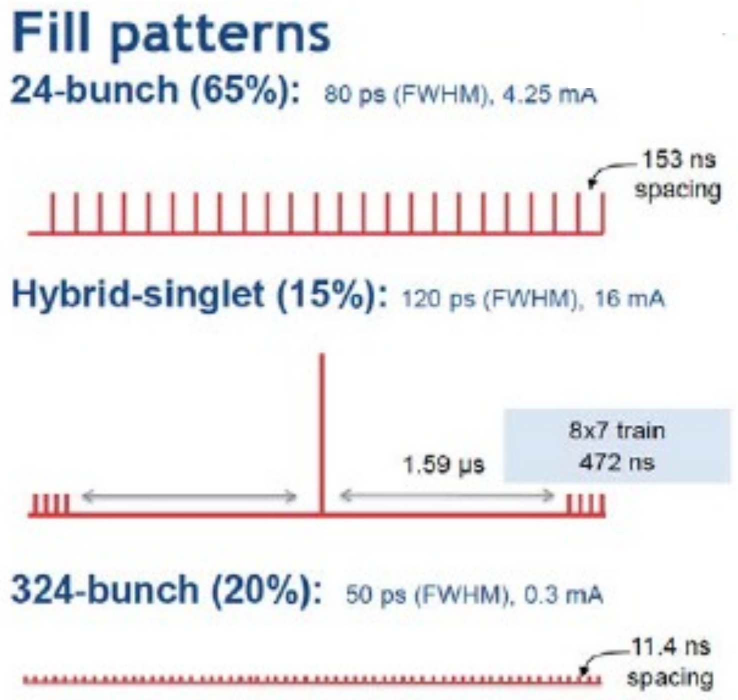
- Because the electrons are radiating x-rays, they are constantly losing energy, and so to restore the energy loss on each revolution, radio-frequency (RF) resonant cavities are installed in the storage ring to replenish the radiative energy losses.
- Particles are grouped together by the action of the radio-frequency (RF) cavities into bunches. At APS:
  - 1104 m circumference (3.68 microsecond period)
  - there are 1296 evenly spaced "RF buckets" (stable orbit positions) around the ring
  - the bunch length of the electron packet in one of these "buckets" is about 3 cm in length, corresponding to a **pulse duration** of about 100 psec [230 psec]







APPENDIX 5: APS TIME STRUCTURE DEPENDS ON FILL PATTERNS (100 mA [200 MA] TOTAL CURRENT)



- The time structure is determined by which of the rf buckets are filled with electrons.

24 equally spaced bunches (about 4 mA/bunch)

- compromise between quasi-continuous source and pulsed source [this will go to 48 bunches for a total of 200 mA with APS-U]

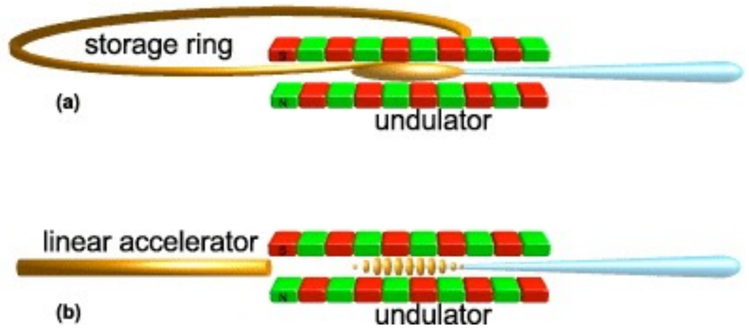
1 + 8x7 (12-14 mA in one bunch and the rest of the 100 mA distributed in 7 trains of 8 closely spaced bunches on the other side of the ring) [this mode will not be supported in the APS-U]

324 equally spaced bunches (about 0.3 mA/bunch)

- [this will remain with total of 200 mA with APS-U]

APPENDIX 6: NON-STORAGE RING COHERENT X-RAY SOURCES – *X-RAY FREE ELECTRON LASERS (XFELS)*

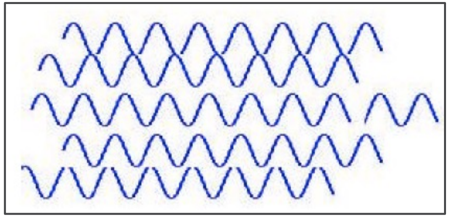
- Another way to reduce the particle beam emittance is through linac-based x-ray free electron lasers (XFELs)
  - Full transverse (spatial) coherence and femtosecond pulses
- An **x-ray FEL** uses the high brightness of an **electron gun** coupled to an **emittance-preserving linac.** You don't have to deal with the equilibrium beam size that occurs in a storage ring so can have a very low emittance and **very short pulses** (femtoseconds).



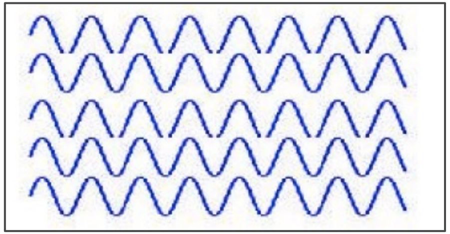
- The problem with a linac is you only use the electrons once, and so have to accelerate new electrons for each pulse. The acceleration process is what takes most the electrical power so some different physics is required.

- That new paradigm is to generate "gain" in the emission process to get more x-rays from a single bunch of electrons - gain is obtained through a process called **Self-Amplified Spontaneous Emission** *or* **SASE.**

APPENDIX 6: GETTING THE ELECTRONS TO RADIATE IN PHASE



- When the electrons in a storage ring go through the undulator, there is no correlation between their positions on the scale of the radiation wavelength. As a result, the fields they generate superimpose at random and its intensity is proportional to the number of electrons, N_e.



- If we could order the electrons so that they are all lined up in thin sheets, periodically grouped with spacing equal to the radiation wavelength, the radiation fields would superimpose in phase. The intensity is then proportional to N_e^2.

- Since N_e is a billion or more, one could obtain a huge gain.

- In this process, an intense and highly collimated electron beam travels through a long undulator magnet (≈ 100 m) producing x-rays.

- If the synchrotron radiation is sufficiently intense (i.e., the undulator is long enough), the electron motion is modified by the E&M fields of its own emitted light.

- The electron beam forms micro-bunches, separated by a distance equal to the wavelength of the emitted radiation.